CHAPTER V

NEUTROSOPHIC PARAMETERIZED SOFT SETS

Definition:5.1

Let U be an initial universe. Let $P(U)$ denotes the power set of U. Let E be the set of parameters and $A \subset E$. Then a **neutrosophic parameterized soft set**

$$\widetilde{F}_A = \{(< e, T_A(e), I_A(e), F_A(e) >, g_A(e)) : e \in E\}$$

where $T_A(e) : E \to [0,1]$, $I_A(e) : E \to [0,1]$, $F_A(e) : E \to [0,1]$ and $g_A(e) : E \to P(U)$ such that $g_A(e) = \phi$ if $T_A(e) = 0, I_A(e) = 1$ and $F_A(e) = 1$ here T_A be the degree of truth

membership , I_A be the degree of indeterminacy membership and F_A be the degree of falsity membership function of neutrosophic parameterized soft set respectively.

Example:5.2

Assume that $U=\{x_1,x_2,x_3\}$ is a universal set and $E=\{e_1,e_2\}$ be the set of parameters. If $A=\{<e_1,0.2,0.3,0.4>,<e_2,0.3,0.5,0.4>\}$ and $g_A(e_1)=\{x_2,x_3\}$ $g_A(e_2)=U$ .Then a neutrosophic parameterized soft set $\widetilde{F}_A$ is written by

$$\widetilde{F}_A=\{(<e_1,0.2,0.3,0.4>,\{x_2,x_3\}),(<e_2,0.3,0.5,0.4>,U)\}$$

Definition:5.3

Let $\widetilde{F}_A$ be a neutrosophic parameterized soft set. If $g_A(e)=U, T_A(e)=0$ $I_A(e)=1,$ and $F_A(e)=1$ for all $e\in E$. Then $\widetilde{F}_A$ is called **A-empty neutrosophic parameterized soft set** and is denoted by $\widetilde{F}_{\phi_A}$. If $A=\phi$, then the A-empty neutrosophic parameterized soft set is called empty neutrosophic parameterized soft set and is denoted by $\widetilde{F}_{\phi}$.

Definition:5.4

Let $\widetilde{F}_A$ be a neutrosophic parameterized soft set. If $g_A(e)=U, T_A(e)=1$ $I_A(e)=0,$ and $F_A(e)=0$ for all $e\in E$. Then $\widetilde{F}_A$ is called **A-universe neutrosophic parameterized soft set** and is denoted by $\widetilde{F}_A$. If $A=E$, then the A-universe neutrosophic parameterized soft set is called universe neutrosophic parameterized soft set and is denoted by $\widetilde{F}_E$.

Definiion:5.5

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two neutrosophic parameterized soft sets. Then $\widetilde{F}_A$ is said to be **neutrosophic parameterized soft subset** of $\widetilde{G}_B$ if and only if

$T_A(e) \leq T_B(e), I_A(e) \geq I_B(e)$ and $F_A(e) \geq F_B(e)$ and $g_A(e) \subseteq h_B(e)$ for all $e \in E$

and is denoted by $\widetilde{F}_A \subseteq \widetilde{G}_B$.

$\widetilde{F}_A$ is said to be **neutrosophic parameterized soft super set** of $\widetilde{G}_B$ if $\widetilde{G}_B$ is a neutrosophic parameterized soft subset of $\widetilde{F}_A$.This relationship is denoted by $\widetilde{F}_A \supseteq \widetilde{G}_B$.

Definition:5.6

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two neutrosophic parameterized soft sets. Then $\widetilde{F}_A$ is said to be **neutrosophic parameterized soft equal** of $\widetilde{G}_B$ if and only if $\widetilde{F}_A \subseteq \widetilde{G}_B$ and $\widetilde{G}_B \subseteq \widetilde{F}_A$ and is denoted by $\widetilde{F}_A = \widetilde{G}_B$.

Definition:5.7

Let $\widetilde{F}_A$ be neutrosophic parameterized soft set. Then the **complement of neutrosophic parameterized soft set** $\widetilde{F}_A$ is denoted by $\widetilde{F}_A^C$ and is defined by

$\widetilde{F}_A^C = \{(< e, T_A(e), I_A(e), F_A(e) >, g_{A^C}(e)) : e \in E\}$ where $g_{A^C}(e) = U / g_A(e)$

Definition:5.8

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two neutrosophic parameterized soft sets. Then **neutrosophic parameterized soft union** of $\widetilde{F}_A$ and $\widetilde{G}_B$ is denoted by $\widetilde{F}_A \cup \widetilde{G}_B$ and is defined by

$\widetilde{F}_A \cup \widetilde{G}_B = \{(< e, \max\{T_A(e), T_B(e)\}, \min\{I_A(e), I_B(e)\}, \min\{F_A(e), F_B(e)\} >, g_{A\cup B}(e)) : e \in E\}$
where $g_{A\cup B}(e) = g_A(e) \cup g_B(e)$.

Definition:5.9

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two neutrosophic parameterized soft sets. Then **neutrosophic parameterized soft intersection** of $\widetilde{F}_A$ and $\widetilde{G}_B$ is denoted by $\widetilde{F}_A \cap \widetilde{G}_B$ and is defined by

$\widetilde{F}_A \cap \widetilde{G}_B = \{(< e, \min\{T_A(e), T_B(e)\}, \max\{I_A(e), I_B(e)\}, \max\{F_A(e), F_B(e)\} >, g_{A\cap B}(e)) : e \in E\}$
where $g_{A\cap B}(e) = g_A(e) \cap g_B(e)$.

Example:5.10

Let $U = \{x_1, x_2, x_3, x_4\}$ is a universal set and $E = \{e_1, e_2, e_3\}$ be the set of parameters. Then two neutrosophic parameterized soft sets $\widetilde{F}_A$ and $\widetilde{G}_B$ are

$\widetilde{F}_A = \{(< e_1, 0.2, 0.3, 0.4 >, \{x_1, x_2\}), (< e_2, 0.3, 0.5, 0.4. >, \{x_2, x_3\})\}$

$\widetilde{G}_B = \{(< e_2, 0.1, 0.2, 0.4 >, \{x_3, x_4\}), (< e_3, 0.5, 0.2, 0.3. >, \{x_3\})\}$. Then

$\widetilde{F}_A \cup \widetilde{G}_B = \{(< e_1, 0.2, 0.3, 0.4 >, \{x_1, x_2\}), (< e_2, 0.3, 0.2, 0.4. >, \{x_2, x_3, x_4\}), (< e_3, 0.5, 0.2, 0.3 >, \{x_3\})\}$

$\widetilde{F}_A \cap \widetilde{G}_B = \{(< e_2, 0.1, 0.5, 0.4. >, \{x_3, x_4\})\}$

$\widetilde{F}_A{}^C = \{(< e_1, 0.4, 0.3, 0.2 >, \{x_3, x_4\}), (< e_2, 0.4, 0.5, 0.3. >, \{x_1, x_4\})\}$

Example:5.11

Let $U = \{x_1, x_2, x_3, x_4\}$ is a universal set and $E = \{e_1, e_2, e_3\}$ be the set of parameters. Then

$\widetilde{F}_A = \{(< e_1, 0.2, 0.3, 0.4 >, \{x_1, x_2\}), (< e_2, 0.3, 0.5, 0.4 >, \{x_2\})\}$
$\widetilde{G}_B = \{(< e_1, 0.1, 0.2, 0.4 >, \{U\}), (< e_2, 0.5, 0.2, 0.3. >, \{x_1, x_4\})\}$.

It can be seen that $\widetilde{F}_A \subseteq \widetilde{G}_B$, but every element of $\widetilde{F}_A$ is not an element of $\widetilde{G}_B$.

Theorem:5.12

Let $\widetilde{F}_A$ be neutrosophic parameterized soft set. Then

1. $\widetilde{F}_A \subseteq \widetilde{F}_E$
2. $\widetilde{F}_\phi \subseteq \widetilde{F}_A$
3. $\widetilde{F}_A \subseteq \widetilde{F}_A$

Theorem:5.13

Let $\widetilde{F}_A$, $\widetilde{G}_B$ and $\widetilde{K}_D$ are three neutrosophic parameterized soft sets. Then

1. $\widetilde{F}_A = \widetilde{G}_B$ and $\widetilde{G}_B = \widetilde{K}_D \Leftrightarrow \widetilde{F}_A = \widetilde{K}_D$
2. $\widetilde{F}_A \subseteq \widetilde{G}_B$ and $\widetilde{G}_B \subseteq \widetilde{F}_A \Leftrightarrow \widetilde{F}_A = \widetilde{G}_B$
3. $\widetilde{F}_A \subseteq \widetilde{G}_B$ and $\widetilde{G}_B \subseteq \widetilde{K}_D \Leftrightarrow \widetilde{F}_A \subseteq \widetilde{K}_D$.

Theorem:5.14

Let $\widetilde{F}_A$ be neutrosophic parameterized soft set. Then

1. $(\widetilde{F}_A^C)^C = \widetilde{F}_A$
2. $\widetilde{F}_A^C = \widetilde{F}_E$

Theorem:5.15

Let $\widetilde{F}_A$, $\widetilde{G}_B$ and $\widetilde{K}_D$ be three neutrosophic parameterized soft sets. Then

1. $\widetilde{F}_A \cup \widetilde{F}_A = \widetilde{F}_A$

2. $\widetilde{F}_A \cup \widetilde{F}_\phi = \widetilde{F}_A$

3. $\widetilde{F}_A \cup \widetilde{F}_E = \widetilde{F}_E$

4. $\widetilde{F}_A \cup \widetilde{G}_B = \widetilde{G}_B \cup \widetilde{F}_A$

5. $(\widetilde{F}_A \cup \widetilde{G}_B) \cup \widetilde{K}_D = \widetilde{F}_A \cup (\widetilde{G}_B \cup \widetilde{F}_A)$

Theorem:5.16

Let $\widetilde{F}_A$, $\widetilde{G}_B$ and $\widetilde{K}_D$ be three neutrosophic parameterized soft sets. Then

1. $\widetilde{F}_A \cap \widetilde{F}_A = \widetilde{F}_A$

2. $\widetilde{F}_A \cap \widetilde{F}_\phi = \widetilde{F}_\phi$

3. $\widetilde{F}_A \cap \widetilde{F}_E = \widetilde{F}_A$

4. $\widetilde{F}_A \cap \widetilde{G}_B = \widetilde{G}_B \cap \widetilde{F}_A$

5. $(\widetilde{F}_A \cap \widetilde{G}_B) \cap \widetilde{K}_D = \widetilde{F}_A \cap (\widetilde{G}_B \cap \widetilde{F}_A)$

Theorem:5.17

Let $\widetilde{F}_A$, $\widetilde{G}_B$ and $\widetilde{K}_D$ be three neutrosophic parameterized soft sets. Then

1. $\widetilde{F}_A \cup (\widetilde{G}_B \cap \widetilde{K}_D) = (\widetilde{F}_A \cup \widetilde{G}_B) \cap (\widetilde{F}_A \cup \widetilde{K}_D)$

2. $\widetilde{F}_A \cap (\widetilde{G}_B \cup \widetilde{K}_D) = (\widetilde{F}_A \cap \widetilde{G}_B) \cup (\widetilde{F}_A \cap \widetilde{K}_D)$

Theorem:5.18

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two neutrosophic parameterized soft sets. Then

1. $(\widetilde{F}_A \cup \widetilde{G}_B)^C = \widetilde{F}_A{}^C \cap \widetilde{G}_B^C$

2. $(\widetilde{F}_A \cap \widetilde{G}_B)^C = \widetilde{F}_A{}^C \cup \widetilde{G}_B^C$

Definition:5.19

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two neutrosophic parameterized soft sets. Then '**OR**' operation of neutrosophic parameterized soft sets $\widetilde{F}_A$ and $\widetilde{G}_B$ is denoted by $\widetilde{F}_A \vee \widetilde{G}_B$ and is defined as

$$\widetilde{F}_A \vee \widetilde{G}_B = \{(<(e,h),(\max\{T_A(e),T_B(h)\},\min\{I_A(e),I_B(h)\},\min\{F_A(e),F_B(h)\})>,g_{A\cup B}(e,h)) : e,h \in E\}$$

where $g_{A\cup B}(e,h) = g_A(e) \cup g_B(h)$.

Definition:5.20

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two neutrosophic parameterized soft set. Then '**AND**' operation of neutrosophic parameterized soft set $\widetilde{F}_A$ and $\widetilde{G}_B$ is denoted by $\widetilde{F}_A \wedge \widetilde{G}_B$ and is defined by

$$\widetilde{F}_A \wedge \widetilde{G}_B = \{(<(e,h),(\min\{T_A(e),T_B(h)\},\max\{I_A(e),I_B(h)\},\max\{F_A(e),F_B(h)\})>,g_{A\cap B}(e,h)) : e,h \in E\}$$

where $g_{A\cup B}(e,h) = g_A(e) \cup g_B(h)$.

Theorem:5.21

Let $\widetilde{F}_A$, $\widetilde{G}_B$ and $\widetilde{K}_D$ be three neutrosophic parameterized soft sets. Then

1. $\widetilde{F}_A \wedge \widetilde{F}_\phi = \widetilde{F}_\phi$

2. $(\widetilde{F}_A \wedge \widetilde{G}_B) \wedge \widetilde{K}_D = \widetilde{F}_A \wedge (\widetilde{G}_B \wedge \widetilde{K}_D)$

3. $(\widetilde{F}_A \vee \widetilde{G}_B) \vee \widetilde{K}_D = \widetilde{F}_A \vee (\widetilde{G}_B \vee \widetilde{K}_D)$

Definition:5.22

Let $\widetilde{F}_A$ be neutrosophic parameterized soft set. Then a neutrosophic parameterized soft aggregation operator of $\widetilde{F}_A$, denoted by $\widetilde{F}_A{}^{agg}$ and is defined by

$$\widetilde{F}_A^{agg} = \{(< x, T_A^{agg}(x), I_A^{agg}(x), F_A^{agg}(x) >: x \in U)\}$$

which is a neutrosophic parameterized soft set over U, where

$$T_A{}^{agg} : U \to [0,1] \text{ and } T_A{}^{agg}(x) = \frac{1}{|U|} \sum_{\substack{e \in E \\ x \in U}} T_A(e) K_{g_{A(e)}}(x)$$

$$I_A{}^{agg} : U \to [0,1] \text{ and } I_A{}^{agg}(x) = \frac{1}{|U|} \sum_{\substack{e \in E \\ x \in U}} I_A(e) K_{g_{A(e)}}(x)$$

$$F_A{}^{agg} : U \to [0,1] \text{ and } F_A{}^{agg}(x) = \frac{1}{|U|} \sum_{\substack{e \in E \\ x \in U}} F_A(e) K_{g_{A(e)}}(x)$$

and where

$$K_{g_{A(e)}}(x) = \begin{cases} 1, e \in g_A(e) \\ 0, otherwise \end{cases}$$

$|U|$ is the cardinality of U.

Definition:5.23

Let $\widetilde{F}_A$ be neutrosophic parameterized soft set and $\widetilde{F}_A{}^{agg}$ is an aggregation neutrosophic parameterized soft set, then a reduced fuzzy set of $\widetilde{F}_A{}^{agg}$ is a fuzzy set over U denoted by

$$\widetilde{F}_A^{agg} = \left\{ \frac{T_{\widetilde{F}_{A_g}}^{agg}(x)}{e} : x \in U \right\} \text{ where } T_{\widetilde{F}_{A_g}}{}^{agg}(x) : U \to [0,1] \text{ and}$$

$$T_{\widetilde{F}_{A_g}}^{agg}(x) = T_A^{agg}(x) + I_A^{agg}(x) - F_A^{agg}(x)$$

Neutrosophic Parameterized (NP)- Decision Methods

A NP- soft decision making method by the following algorithm to produce a decision fuzzy set from a crisp set of the alternatives.

Algorithm:

i.Constructs a feasible Neutrosophic subsets A over the parameters set E,

ii.Constructs a NP- soft set $\widetilde{F}_A$ over the alternatives set U,

iii.Computes the aggregation Neutrosophic parameterized soft set $\widetilde{F}_A^{agg}$ of $\widetilde{F}_A$,

iv.Computes the reduced fuzzy set $T_{\widetilde{F}_{Ag}}{}^{agg}(x)$ of $\widetilde{F}_A^{agg}$,

v.Chooses the element of $T_{\widetilde{F}_{Ag}}{}^{agg}(x)$ that has maximum membership degree.

Now, an example for the NP- soft decision making method

Example:5.24

Assume that a company wants to fill a position. There are four candidates who fill in a form in order to apply formally for the position. There is a decision maker (DM) that is from the department of human resources. He wants to interview the candidates, but it is very difficult to make it all of them. Therefore, by using the NP- soft decision making method, the number of candidates are reduced to a suitable one. Assume that the set of candidates $U=\{x_1,x_2,x_3,x_4\}$ which may be characterized by a set of parameters $E=\{e_1,e_2,e_3\}$ for i=1,2,3 the parameter i stand for experience, computer knowledge and young age, respectively. Now, apply the method as follows:

Step: 1

Assume that DM constructs a feasible Neutrosophic subsets A over the parameters set E as follows:

$A = \{< e_1, 0.2, 0.3, 0.4 >, < e_2, 0.3, 0.2, 0.4. >, < e_3, 0.5, 0.2, 0.3 >\}$

Step: 2

DM constructs an NP- soft set $\widetilde{F}_A$ over the alternatives set U as follows:

$\widetilde{F}_A = \{(< e_1, 0.2, 0.3, 0.4 >, \{x_1, x_2\}), (< e_2, 0.3, 0.2, 0.4. >, \{x_2, x_3, x_4\}), (< e_3, 0.5, 0.2, 0.3 >, \{x_3\})\}$

Step: 3

DM computes the aggregation neutrosophic parameterized soft set $\widetilde{F}_A^{agg}$ of $\widetilde{F}_A$ as follows:

$\widetilde{F}_A^{agg} = \{< e_1, 0.05, 0.075, 0.1 >, < e_2, 0.1, 0.125, 0.2 >, < e_3, 0.2, 0.1, 0.175 >, < e_4, 0.125, 0.05, 0.075 >\}$

Step: 4

Computes the reduced fuzzy set $T_{\widetilde{F}_{A_g}}{}^{agg}(x)$ of $\widetilde{F}_A^{agg}$ as follows:

$T_{\widetilde{F}_{A_g}}{}^{agg}(x_1) = 0.025$

$T_{\widetilde{F}_{A_g}}{}^{agg}(x_2) = 0.025$

$T_{\widetilde{F}_{A_g}}{}^{agg}(x_3) = 0.125$

$T_{\widetilde{F}_{A_g}}{}^{agg}(x_4) = 0.1$

Step: 5

Finally, DM chooses x_3 for the position from $T_{\tilde{F}_{A_g}}{}^{agg}(x)$ since it has the maximum degree 0.125 among others.