CHAPTER - I

PRELIMINARY DEFINITIONS AND NOTATIONS

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Definition : 1.1

A pair (F, E) is called a **soft set** over U if and only if F is a mapping of E into the set of all subsets of the set U.

In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(\varepsilon)$, $\varepsilon \in E$, from this family may be considered as the set of ε - approximate elements of the soft set.

Example : 1.2

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and $E = \{e_1$(expertise in english), e_2(expertise in mathematics), e_3(expertise in chemistry), e_4(expertise in computer science)$\}$ be the set of parameters and $A = \{e_1, e_3, e_4\} \subseteq E$, Then $(F, A) = \{F(e_1) = \{S_1, S_3\}, F(e_3) = \{S_1, S_2, S_4\}, F(e_4) = \{S_4\}\}$ is the soft set representing over all expertness of the students.

Definition : 1.3

A pair (F, A) is called a **fuzzy soft set** over U where $F : A \rightarrow P(U)$ is a mapping from A into P(U).

Example : 1.4

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and $E = \{e_1$(expertise in english), e_2(expertise in mathematics), e_3(expertise in chemistry), e_4(expertise in computer science)$\}$ be the set of parameters and $A = \{e_1, e_3, e_4\} \subseteq E$, Then

$(F, A) = \{F(e_1) = \{S_1/0.3, S_2/0.5, S_3/0.1, S_4/0.7\},$

$F(e_3) = \{S_1/0.4, S_2/0.2, S_3/0.6, S_4/0.1\},$

$F(e_4) = \{S_1/0.2, S_2/0.7, S_3/0.2, S_4/0.3\}\}$ is the fuzzy soft set representing overall expertness of the students.

Definition : 1.5

Let U be a universe and E a set of attributes. Then the pair (U, E) denotes the collection of all fuzzy soft sets on U with attributes from E and is called a **fuzzy soft class**.

Definition : 1.6

A soft set (F, A) over U is said to be **null fuzzy soft set** denoted by φ if $\forall \varepsilon \in A$, $F(\varepsilon)$ is the null fuzzy set $\overline{0}$ of U where $\overline{0}(x) = 0 \ \forall \ x \in U$.

Definition : 1.7

A soft set (F, A) over U is said to be **absolute fuzzy soft set** denoted by A if $\forall \ \varepsilon \in A$, $F(\varepsilon)$ is the null fuzzy set $\overline{1}$ of U where $\overline{1}(x) = 1 \ \forall \ x \in U$.

Definition : 1.8

For two fuzzy soft sets (F, A) and (G, B) in a fuzzy soft class (U, E), we say that (F, A) is a **fuzzy soft subset** of (G, B), if

1) $A \subseteq B$
2) For all $\varepsilon \in A$, $F(\varepsilon) \subseteq G(\varepsilon)$ and is written as $(F, A) \subseteq (G, B)$.

Example : 1.9

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and $E = \{e_1$(expertise in english), e_2(expertise in mathematics), e_3(expertise in chemistry), e_4(expertise in computer science)$\}$ be the set of parameters and $A = \{e_1, e_2\} \subseteq E$ and $B = \{e_1, e_2, e_4\} \subseteq E$. We consider the fuzzy soft sets

$(F, A) = \{F(e_1) = \{S_1/0.7, S_2/0.1, S_3/0.2, S_4/0.6\},$

$F(e_2) = \{S_1/0.8, S_2/0.6, S_3/0.1, S_4/0.5\}\}$ and

$(G, B) = \{G(e_1) = \{S_1/0.7, S_2/0.2, S_3/0.3, S_4/0.7\},$

$G(e_2) = \{S_1/0.9, S_2/0.7, S_3/0.3, S_4/1\},$

$G(e_3) = \{S_1/0.1, S_2/0.2, S_3/0.7, S_4/0.6\}\}$

Here $A \subseteq B$ and for all $\varepsilon \in A$, $F(\varepsilon) \subseteq G(\varepsilon)$

Thus $(F, A) \subseteq (G, B)$.

Definition : 1.10

Let (F, A) and (G, B) be two fuzzy soft sets in a soft class (U, E) with $A \cap B \neq \varphi$. Then **intersection** of two fuzzy soft sets (F, A) and (G, B) in a soft class (U, E) is a fuzzy soft set (H, C) where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$.

We write $(F, A) \cap (G, B) = (H, C)$.

Example : 1.11

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and $E = \{e_1$(expertise in english), e_2(expertise in mathematics), e_3(expertise in chemistry), e_4(expertise in computer science)$\}$ be the set of parameters and $A = \{e_1, e_2, e_4\} \subseteq E$ and $B = \{e_1, e_5\} \subseteq E$. Then

$(F, A) = \{F(e_1) = \{S_1/0.9, S_2/0.2, S_3/0.6, S_4/0.8\},$

$F(e_2) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.8\},$

$F(e_4) = \{S_1/0.1, S_2/0.7, S_3/0.3, S_4/0.2\}\}$

$(G, B) = \{G(e_1) = \{S_1/0.8, S_2/0.2, S_3 /0.5, S_4/0.8\}$

$G(e_5) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.7\}\}$

Then (F, A) $\cap$ (G, B) = (H, C), where C = A $\cap$ B = {e_1}

Thus (H, C) = {H(e_1) = {S_1/0.8, S_2/0.2, S_3/0.5, S_4/0.8}}

Definition : 1.12

Union of two fuzzy soft sets (F, A) and (G, B) in a fuzzy soft class (U, E) is a fuzzy soft set (H, C) where C = A $\cup$ B and $\forall \varepsilon \in$ C,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

and is written as (F, A) $\cup$ (G, B) = (H, C).

Example : 1.13

Let U = {S_1, S_2, S_3, S_4} be the set of students under consideration and E = {e_1(expertise in english), e_2(expertise in mathematics), e_3(expertise in chemistry), e_4(expertise in computer science)} be the set of parameters and A = {e_1, e_3, e_4} $\subseteq$ E and B = {e_1, e_5} $\subseteq$ E. Then

(F, A) = {F(e_1) = {S_1/0.9, S_2/0.2, S_3/0.6, S_4/0.8},

F(e_3) = {S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.8},

F(e_4) = {S_1/0.1, S_2/0.7, S_3/0.3, S_4/0.2}}

(G, B) = {G(e_1) = {S_1/0.8, S_2/0.2, S_3/0.5, S_4/0.8},

G(e_5) = {S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.7}}

Then (F, A) $\cup$ (G, B) = (H, C), where C = A $\cup$ B = {e_1, e_3, e_4, e_5}

(H, C) = {H(e_1) = {S_1/0.9, S_2/0.2, S_3/0.6, S_4/0.8},

H(e_3) = {S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.8},

$H(e_4) = \{S_1/0.1, S_2/0.7, S_3/0.3, S_4/0.2\}$,

$H(e_5) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.7\}\}$

Definition : 1.14

The **complement** of a fuzzy soft set (F, A) is denoted by $(F, A)^C$ and is defined by $(F, A)^C = (F^C, A)$ where $F^C: A \to P(U)$ is a mapping given by $F^C(\alpha) = [F(\alpha)]^C$, $\forall \alpha \in A$.

Example : 1.15

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and $E = \{e_1$(expertise in english), e_2(expertise in mathematics), e_3(expertise in chemistry), e_4(expertise in computer science)} be the set of parameters and $A = \{e_1, e_2\} \subseteq E$, Then

$(F, A) = \{F(e_1) = \{S_1/0.7, S_2/0.1, S_3/0.2, S_4/0.6\}$,

$F(e_2) = \{S_1/0.8, S_2/0.6, S_3/0.1, S_4/0.5\}\}$

$(F, A)^C = \{F^C(e_1) = \{S_1/0.3, S_2/0.9, S_3/0.8, S_4/0.4\}$,

$F^C(e_2) = \{S_1/0.2, S_2/0.4, S_3/0.9, S_4/0.5\}\}$

Definition : 1.16

Let $A = [a_{ij}] \in FSM_{m\times n}$. Then we define $A^T = [a_{ij}^T] \in FSM_{m\times n}$, where $a_{ij}^T = a_{ij}$.

Definition : 1.17

Then the set $O_s = \{ j : c_j = \max\{c_i : i = 1,2,3,\ldots,m \}\}$ is called the **optimum subscript set**, $O_d = \{u_j : u_j \in U \text{ and } j \in O_s\}$ is called the **optimum decision set** of U.

Definition : 1.18

Given an universal set U and a set of parameters E, for $A \subseteq E$, the pair (F, A) is called an **intuitionistic fuzzy soft set** over U if F is a mapping from A to the set of all Intuitionistic Fuzzy subsets of U.

Definition : 1.19

Let U be an initial Universe set and E be the set of parameters. Let $A \subseteq E$. A pair (F, A) is called **intuitionistic fuzzy soft set** over U where F is a mapping given by $F : A \rightarrow IF^U$, where IF^U denotes the collection of all intuitionistic fuzzy subsets of U.

Example : 1.20

Suppose that $U = \{S_1, S_2, S_3, S_4\}$ is a set of students and $E = \{e_1, e_2, e_3\}$ is a set of parameters, which stand for result, conduct and sports performances respectively. Consider the mapping from parameters set $A \subseteq E$ to the set of all intuitionistic fuzzy subsets of power set U. Then soft set (F, A) describes the character of the students with respect to the given parameters, for finding the best student of an academic year. Consider $A = \{e_1, e_2\}$ then intuitionistic fuzzy soft set is

$(F, A) = \{F(e_1) = \{(S_1,0.8,0.1), (S_2,0.3,0.6), (S_3,0.8,0.2), (S_4,0.9,0.0)\},$

$F(e_2) = \{(S_1,0.8,0.1), (S_2,0.9,0.1), (S_3,0.4,0.5), (S_4,0.3,0.6)\}\}.$