# *INTRODUCTION*

## Introduction

Topology is a major area of Mathematics and it includes many subfields. The most basic and traditional division of topology is the point set topology which establishes the foundational aspects of topology and discusses about compactness and connectedness. Topology has shown its fruitfulness in both pure and applied domains. Topological structures are suitable mathematical models for formulating both qualitative and quantitative data. The influence of topological spaces can be observed in the fields like computer science, digital topology, computational topology for geometric and molecular design, data mining, information systems, quantum physics, particle physics, high energy physics and superstring theory. In the study of topological spaces many concepts of topology have been generalized by introducing the concept of semi open sets due to Levine(1963) instead of open sets. Njastad (1965) introduced α-open sets and Zaitsev (1968) proposed the concept of π-closed sets and studied their properties and characteraization. On the other hand Levine (1970) introduced the notion of generalized closed (briefly g-closed ) sets in topological spaces and showed that compactness, countably compactness, para compactness and normality are all g-closed hereditary. He also introduced a separation axiom called $T_{1/2}$ between $T_1$ and $T_0$. Mashhour et al (1982), Andirjevic (1986), Bhattacharya and Lahari (1987) and Arya and Nour (1990) introduced and investigated respectively pre-open sets, semi preopen sets, semi-generalized open sets, generalized semi open sets which are some of the weaker forms of open sets and complements of these sets are called the same type of closed sets respectively. Noiri and Dontchev (2000) introduced the notion of π generalized closed sets and obtained characterizations of quasi normal spaces. Park (2006) developed the notion of π generalized pre-closed sets and investigated compactness and connectedness. Aslim, Guler and Noiri (2006) introduced the concept of π generalized semi-closed sets and analyzed the basic properties. Sanjay Tahiliani (2010) studied the notion of πgβ-closed sets in topological spaces.

Strong forms of continuous maps have been introduced and investigated by several mathematicians. Strongly continuous maps, perfectly continuous maps, completely continuous maps and weak almost continuous maps were introduced by Levine (1970), Noiri (1984, 1973). In 1980 Noiri introduced a new concept called strong θ-continuity which is stronger than continuity. Sundaram and Nagaveni (1998) introduced and studied weakly generalized continuous maps, weakly generalized closed maps and weakly generalized irresolute maps .

Biswas (1969) introduced and investigated semi homeomorphisms. Crossely and Hildebrand (1972) introduced and studied irresolute functions which are stronger than semi-continuous maps but are independent of continuous maps. Sundaram (1991) introduced and discussed semi generalized continuous maps. Balachandran et al (1991) have introduced and studied generalized continuous maps. Devi et al (1995) have introduced and analyzed semi-generalized homeomorphisms and generalized semi homeomorphisms in topological spaces. Maki et al (1991) have introduced generalized homeomorphisms and studied its properties.

˘Cech closure spaces were introduced by ˘Cech (1966) and studied by many authors. Bi˘Cech closure spaces were introduced by Chandrasekhara Rao, Gowri and Swaminathan (2008). In 2010 Devi et al. introduced the concept of $\alpha\psi$- closed sets in Bi˘Cech closure spaces.

Noiri (2006) introduced the concept of minimal structure on a nonempty set and the concept of mg-closed set in minimal structures which is analogus to a g-closed set in topological spaces is introduced by Levine. Popa and Noiri (2000) introduced and studied m-continuous functions. Boonpok (2010) enhanced the concept of biminimal structure space and studied the fundamental properties of $m_X^1$ $m_X^2$-closed sets, $m_X^1$ $m_X^2$-open sets and M-continuous functions. Noiri and Valeriu Popa (2011) introduced the notion of almost M continuous multifunctions.

Choquet (1947) initiated the idea of grills on topological spaces. This concept is a powerful supporting tool like nets and filters for getting a deeper insight into further study of topological spaces. Roy and Mukherjee (2007) and Roy et al.(2008) defined and studied a topology by associating the existing topology and a grill.

In the literature, we find various types of stronger and weaker forms of open and closed sets, stronger and weaker forms of continuous maps, irresolute maps, open maps and closed maps and generalized homeomorphisms which play important roles in the study of topological spaces. Several authors have studied various classes of g-closed sets and introduced weaker forms of continuity, irresolute and homeomorphic mappings with respect to the corresponding generalized closed sets.

The present study focuses on the following concepts:

1. Generalized π-closed ( gπ-closed) sets in topological spaces
2. Quasi generalized π-functions in topological spaces.
3. Generalized π-closed sets in Bi- ˇ Cech closure spaces.
4. Generalized π-closed sets in Biminimal Structure spaces.
5. Generalized π-closed sets and grills.

Chapter 1 deals with preliminary definitions and results that are needed for our study.

In Chapter 2, we have introduced the concept of generalized π-closed sets, generalized π-open sets and obtained the relationships among gπ-closed sets, regular closed sets, π-closed sets, g-closed sets, πg-closed sets, πgp-closed sets, πgs-closed sets, αg-closed sets, gp-closed sets and gs-closed sets. The family of gπ- closed sets lies between those of π -closed sets and g-closed sets. Using the notion of gπ-closed sets, the concepts of gπ-continuous functions, gπ-irresolute functions, contra

gπ-continuous functions, almost contra gπ-continuous functions and $_{g\pi}T_{1/2}$ spaces are introduced and their properties are discussed.

A subset A of a topological space (X, $\tau$) is called **a generalized π- closed (briefly gπ-closed )** set in (X, $\tau$) if π cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, $\tau$).

Some of the intrestings results obtained in this chapter are as follows

1. The union of two gπ-closed subsets of X is a gπ-closed subset of X. But the intersection of two gπ-closed sets fails to be gπ-closed is substantiated by counter examples. Where as, the intersection of a gπ-closed set and π-closed set is a gπ-closed set.

2. Let A be a gπ-closed set of (X, $\tau$). Then π cl(A) – A does not contain any non-empty closed set.

3. If A is a gπ-closed set in a space (X, $\tau$) and A $\subseteq$ B $\subseteq$ π cl(A), then B is also a gπ-closed set.

Some characterizations discussed here are as follows:

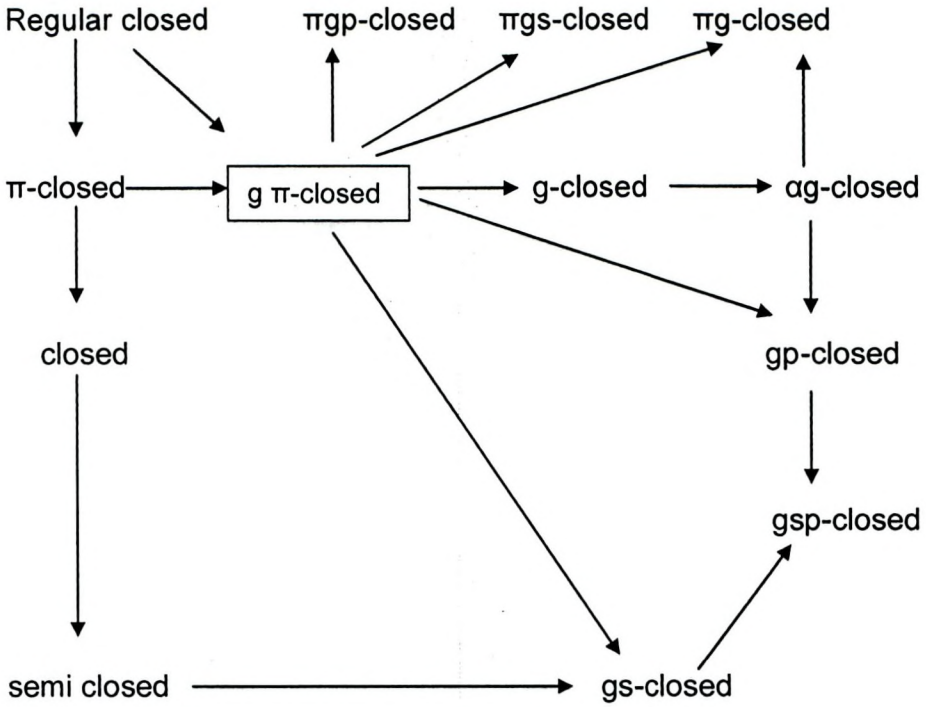
1. Let A be a gπ-closed set of (X, $\tau$). Then A is π-closed if and only if πcl(A) –A is closed.

2. A subset A of (X, $\tau$) is gπ-open if and only if F$\subseteq$ πint(A) whenever F is closed and F$\subseteq$ A.

As an application, a new space $_{g\pi}$ $T_{1/2}$ space is introduced as follows

A topological space (X, $\tau$) is called a $_{g\pi}$ **$T_{1/2}$ space** if every gπ-closed set is π closed.

The following diagram shows the relationships of gπ-closed sets with other sets.




where A ⟶ B represents A implies B but not conversely

A characterization of $_{g\pi}T_{1/2}$ space is derived as, A topological space $(X, \tau)$ is $_{g\pi}T_{1/2}$ if and only if each singleton of X is either closed or π-open.

Then, the continuous function and irresolute function in terms of gπ-open sets are defined as follows:

A function $f : (X,\tau) \to (Y,\sigma)$ is said to be **gπ-continuous** if the inverse image of every open set in $(Y,\sigma)$ is gπ-open in $(X,\tau)$.

A function $f : (X, \tau) \to (Y, \sigma)$ is called **gπ-irresolute** if $f^{-1}(V)$ is gπ-closed in $(X, \tau)$ for every gπ closed set V of $(Y, \sigma)$.

Many important properties of gπ-continuous functions are proved analogous to continuous functions.

It is proved that, the compositions of various functions form to be gπ-continuous function.

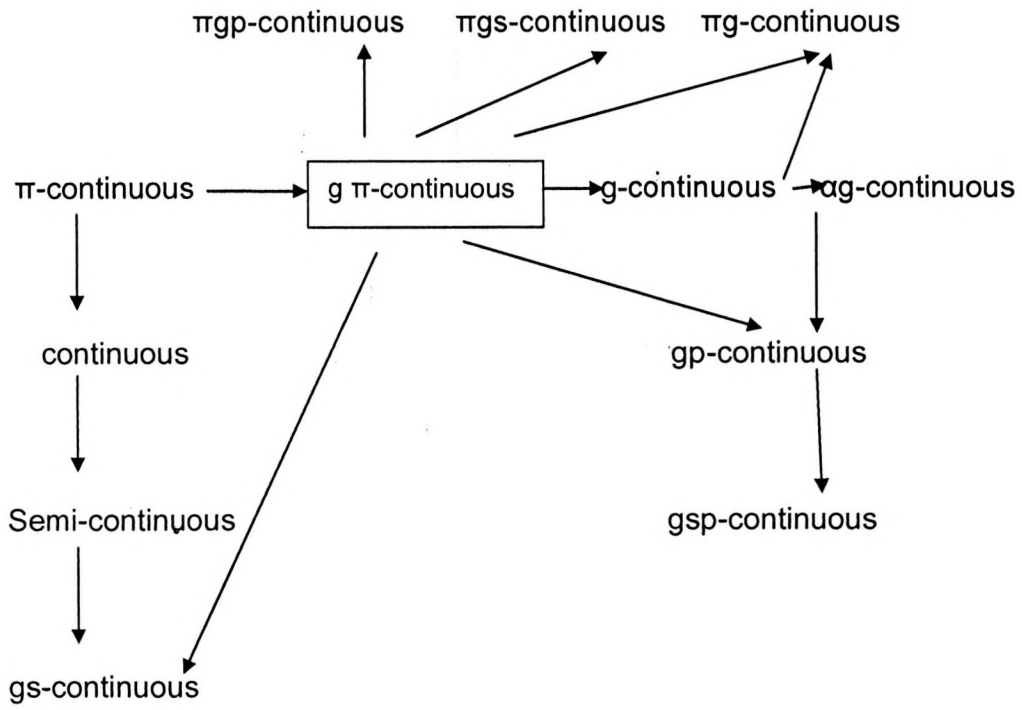
Let f : (X, $\tau$ ) $\rightarrow$ (Y, $\sigma$ ) and g : (Y, $\sigma$ ) $\rightarrow$ (Z, $\mu$ ) be any two functions. Then

a). g $\circ$ f is gπ-continuous, if g is continuous and f is gπ-continuous.

b). g $\circ$ f is gπ-irresolute, if g is gπ-irresolute and f is gπ-irresolute.

c). g $\circ$ f is gπ-continuous, if g is gπ-continuous and f is gπ-irresolute.

The following diagram shows the relationships between gπ -continuous functions with various continuous functions.




where A $\longrightarrow$ B represents A implies B but not conversely.

The concept of contra gπ-continuity parallel to contra continuity is defined and some of their properties are proved.

A function f : X$\rightarrow$ Y is called **contra gπ-continuous** if $f^{-1}(V)$ is gπ-closed in X for every open set V of Y.

1. The following are equivalent for a function $f : X \to Y$

a). f is contra-gπ continuous;

b). for every closed subset F of Y, $f^{-1}(F) \in g\pi O(X)$

c). for each $x \in X$ and each $F \in C(Y, f(x))$, there exists $U \in g\pi O(X, x)$ such that $f(U) \subseteq F$

d). $f(g\pi Cl(A)) \subseteq ker(f(A))$ for every subset A of X

e). $g\pi Cl(f^{-1}(B)) \subseteq f^{-1}(ker(B))$ for every subset B of Y.

2. If a function $f : X \to Y$ is contra-gπ-continuous and Y is regular, then f is gπ-continuous.

3. A contra- gπ -continuous function $f : X \to Y$ is continuous when X is locally gπ -indiscrete, where every gπ-open set is closed.

Similar types of gπ-continuous functions are defined and studied.

1. A function $f : X \to Y$ is called **almost-gπ-continuous** if for each $x \in X$ and each open set V of Y containing f (x), there exists $U \in g\pi O(X, x)$ such that $f(U) \subseteq g\pi\text{- int}(cl(V))$.

2. A function $f : X \to Y$ is said to be **pre-gπ-open** if the image of each gπ-open set is gπ-open.

3. A function $f : X \to Y$ is said to be **almost weakly gπ-continuous** if for each $x \in X$ and each open set V of f (x) there exists $U \in g\pi O(X,x)$ such that $f(U) \subseteq cl(V)$.

Many separation axioms in terms of gπ-closed sets are defined and studied in detail.

Finally, contra gπ-closed graph is defined as follows:

The graph G( f ) of a function f : X→Y is said to be **contra-gπ-closed** if for each (x, y) ∈ (X×Y) − G( f ), there exist U ∈ gπO(X,x) and V ∈ C(Y, y) such that (U ×V)∩G( f ) = φ.

Characterization of this concept is obtained and many properties are discussed.

Chapter 3 is devoted to the study of gπ-closed map, gπ-open map, gπ* -closed map, gπ*-open map, gπ-homeomorphism, gπ*-homeomorphism, gπ compact spaces, gπ-connected spaces, gπ-US spaces, Sequentially gπ-closed set and Sequentially GπO-compact set.

Various maps with respect to gπ-closed sets are defined and studied.

1. A map f : $(X,\tau) \to (Y,\sigma)$ is called **gπ-closed** if the image of every closed set in $(X, \tau)$ is gπ-closed in $(Y, \sigma)$.

2. A map f : $(X, \tau) \to (Y,\sigma)$ is said to be a **gπ-open** map if the image f(A) is gπ-open in $(Y, \sigma)$ for each open set A in $(X,\tau)$.

3. A map f : $(X, \tau) \to (Y, \sigma)$ is said to be a **gπ*-closed** map if the image f(A) is gπ-closed in $(Y,\sigma)$ for every gπ-closed set A in $(X, \tau)$.

4. A map f : $(X, \tau) \to (Y, \sigma)$ is said to be a **gπ*-open** map if the image f(A) is gπ-open in $(Y,\sigma)$ for every gπ-open set A in $(X,\tau)$.

5. Let A be a subset of X. A mapping r : X → A is called **gπ-continuous retraction** if r is gπ-continuous and the restriction $r_A$ is the identity mapping on A.

6. A topological space $(X, \tau)$ is called **gπ-Hausdorff** if for each pair x, y of distinct points of X, there exist gπ-neighborhoods $U_1$ and $U_2$ of x and y, respectively, that are disjoint.

7. Let A be a subset of X and r : X → A be a gπ-continuous retraction. If X is gπ-Hausdorff, then A is a gπ-closed set of X.

8. A bijection f : (X, $\tau$ ) $\rightarrow$ ( y, $\sigma$ ) is called **generalized π-homeomorphism (gπ -homeomorphism)** if f and $f^{-1}$ are gπ -continuous.

9. A bijection f : (X, $\tau$ ) $\rightarrow$ ( y, $\sigma$ ) is said to be **gπ*-homeomorphism** if both f and $f^{-1}$ are gπ -irresolute.

The concept of gπ-compactness and gπ-connectedness are studied.

A collection $\{A_i : i \in \Lambda\}$ of gπ-open sets in a topological space X is called **gπ-open cover** of a subset B of X if $B \subset \cup\{A_i : i \in \Lambda\}$ holds.

A topological space X is **gπ-compact** if every gπ-open cover of X has a finite sub-cover.

1. A topological space X is said to be **gπ-connected** if X cannot be expressed as a disjoint union of two non-empty gπ-open sets. A subset of X is gπ-connected if it is gπ-connected as a subspace.

Following are the results of convergence obtained with respect to gπ-closed sets.

1. A sequence $\{ x_n \}$ in a space X, **gπ-converges** to a point $x \in X$ if $\{ x_n \}$ is eventually in every gπ-open set containing x.

2. A space X is said to be **gπ-US** if every sequence in X, gπ - converges to a point of X.

3. A subset A of a space X is said to be **Sequentially gπ-closed** if every sequence in A, gπ-converges to a point in A.

4. A subset A of a space X is said to be **Sequentially GπO-compact** if every sequence in A has a subsequence which gπ- converges to a point in A.

In chapter 4, we have introduced the concept of quasi gπ-closed and quasi gπ-open functions, gπ-quasi irresolute functions and gπ-quasi closed graphs. The properties of gπ-Hausdorff spaces and gπ-normal spaces are discussed.

A function f : X → Y is said to be **quasi gπ-open** if the image of every gπ-open set in X is open in Y .

Some characterization theorems obtained are as follows:

1. A function f : X → Y is quasi gπ-open iff for every subset U of X, $f(g\pi int(U)) \subset^{\bullet} int(f(U))$.

2. A function f : X → Y is quasi gπ-open if and only if for any subset B of Y and for any gπ-closed set F of X, containing $f^{-1}(B)$, there exists a closed set G of Y containing B such that $f^{-1}(G) \subset F$.

3. A function f: X → Y is quasi gπ-open if and only if $f^{-1}(cl(B)) \subset g\pi Cl(f^{-1}(B))$ for every subset B of Y.

4. Let f : X → Y and g : Y → Z be two functions such that g ∘ f : X →Z is quasi gπ-open. If g is continuous injective, then f is quasi gπ-open.

A function f : X → Y is said to **quasi gπ-closed** if the image of each gπ-closed set in X is closed in Y.

Some important properties obtained are as follows:

1. If a function f : X → Y is quasi gπ-closed, then $f^{-1}(int(B)) \subset g\pi int(f^{-1}(B))$ for every subset B of Y .

2. If f : X → Y and g : Y → Z are two quasi gπ-closed functions, then g ∘ f is a quasi gπ-closed function.

3. Let f : X → Y and g : Y → Z be any two functions. Then

i). If f is gπ-closed and g is quasi gπ-closed, then g ∘ f is closed.

ii). If f is quasi gπ-closed and g is gπ-closed, then g ∘ f is $g\pi^{*}$-closed.

iii) If f is $g\pi^{*}$-closed and g is quasi gπ-closed, then g ∘ f is quasi gπ-closed.

4. Let $f : X \to Y$ and $g : Y \to Z$ be two functions such that $g \circ f : X \to Z$ is quasi $g\pi$-closed.

i). If f is $g\pi$-irresolute surjective, then g is closed.

ii). If g is $g\pi$-continuous injective, then f is $g\pi^*$-closed.

A function $f : X \to Y$ is called **$g\pi$-quasi -irresolute** if for each $x \in X$ and each $V \in SO(Y, f(x))$ there exists a $g\pi$-open set U in X containing x such that $f(U) \subset cl(V)$.

Some of the important theorems proved here are as follows:

Suppose that $g\pi O(X)$ is closed under arbitrary union, then the following statements are equivalent for a function $f : X \to Y$

1. f is $g\pi$- quasi-irresolute.

2. $f^{-1}(V) \subset g\pi\ int(f^{-1}(cl(V)))$ for every $V \in SO(Y)$.

3. The inverse image of a regular closed set of Y is $g\pi$-open.

4. The inverse image of a regular open set of Y is $g\pi$-closed.

5. The inverse image of a $\theta$-semi open set of Y is $g\pi$-open.

6. $f^{-1}(int(cl(G)))$ is $g\pi$-closed. for every open subset G of Y.

7. $f^{-1}(cl(int(F)))$ is $g\pi$-open for every closed subset F of Y.

The graph G(f) of a function $f : X \to Y$ is said to be **$g\pi$-quasi-closed** if for each $(x, y) \in X \times Y - G(f)$, there exist $U \in g\pi O(X, x)$ and $V \in SO(Y, y)$ such that $(U \times cl(V)) \cap G(f) = \varphi$

The following properties are equivalent for a graph G( f) of a function f : X → Y

1. The graph G (f) is gπ-quasi-closed in X × Y

2. For each point ( x, y ) ∈ X× Y − G(f) , there exist U ∈ gπO(X, x) and V ∈ SO( Y, y) such that f(U) ∩cl(V) = φ.

3. For each point (x, y) ∈ X ×Y − G(f), there exist U ∈ gπO(X, x) and F ∈ RC(Y, y) such that f(U) ∩F = φ.

In chapter 5, we have introduced the notion of gπ-closed sets in Bi- ˇ Cech closure space. Properties of $(k_1, k_2)$-gπ-open sets, $(k_1, k_2)$-gπ-continuous functions, $(k_1, k_2)$-gπ- irresolute functions, $_{g\pi}$ $C_0$ bi-ˇCech spaces, $_{g\pi}$ $C_1$ bi-ˇCech spaces, $_{g\pi}T^{*}_{1/2}$ bi-ˇCech spaces and $_{g\pi}T^{**}_{1/2}$ bi-ˇCech spaces are studied.

A subset A of a bi- ˇ Cech closure space $(X, k_1, k_2)$ is said to be **$(k_1, k_2)$- gπ closed** if $k_2$ π cl (A) ⊆ U whenever A ⊆ U and U is $k_1$- open in X.

Some of the properties proved in this chapter are listed here:

1. If A and B are $(k_1, k_2)$- gπ closed sets then so is A ∪ B.

2. If A is $(k_1, k_2)$- gπ closed set, then $k_2$ π cl (A) − A contains no nonempty $k_1$ - closed sets.

3. If A is a $(k_1, k_2)$- gπ closed set, then $k_1$ π cl (x)∩A≠ φ holds for each x ∈ $k_2$ π cl (A).

4. Let $(X, k_1, k_2)$ be a bi- ˇ Cech closure space. For each x in X, {x} is either $k_1$ - closed or $\{x\}^c$ is $(k_1, k_2)$- gπ closed.

5. Let A be a $(k_1, k_2)$- gπ closed set and if A is $k_1$ - open then A = $k_2$ π cl (A).

6. Let A ⊆ Y ⊆ X and suppose that A is $(k_1, k_2)$- gπ closed in $(X, k_1, k_2)$. Then A is $(k_1, k_2)$- gπ closed relative to Y.

Similar properties are proved for $(k_1, k_2)$- gπ open sets.

In Chapter 6, we have introduced the concepts of $m_X^{(i,j)}$-gπ -closed sets in biminimal structure spaces and tried to unify certain types of modifications of gπ-closed sets. The concepts of $m_X^{(i,j)}$-gπ-continuous functions, $m_X^{(i,j)}$- gπ-irresolute functions and $m^{(i,j)}$ - gπ $T_{1/2}$ spaces are introduced and their properties are studied.

A subset A of a biminimal structure space $(X, m_X^1, m_X^2)$ is said to be a **$m_X^{(i,j)}$-gπ closed set** if $m_X^j$πCl(A) ⊆ U, whenever A ⊆ U and U is $m_X^i$- open in X, where i, j =1, 2 and i ≠ j. The complement of $m_X^{(i,j)}$- gπ -closed is said to be $m_X^{(i,j)}$-gπ open.

A subset A of a biminimal structure space $(X, m_X^1, m_X^2)$ is called **pairwise-m-gπ -closed** if A is $m_X^{(1,2)}$-gπ-closed and $m_X^{(2,1)}$-gπ -closed. The complement of pairwise m-gπ -closed is called pairwise m-gπ -open.

Some of the properties proved in this chapter are

1. If subsets A and B of a biminmal structure space $(X, m_X^1, m_X^2)$ are $m_X^{(i,j)}$-gπ -closed subsets of X , then A ∪ B is $m_X^{(i,j)}$-gπ -closed, where i, j = 1, 2 and i ≠ j.

2. If A is a $m_X^{(i,j)}$-gπ-closed set of $(X, m_X^1, m_X^2)$ such that A ⊆B ⊆ $m_X^j$ -πCl(A), then B is a $m_X^{(i,j)}$gπ-closed set, where i, j = 1, 2 and i ≠ j.

3. If a subset A of a biminimal structure space $(X, m_X^1, m_X^2)$ is both $m_X^i$-open and $m_X^{(i,j)}$-gπ-closed, then A is $m_X^j$ -π closed, where i, j = 1, 2 and i ≠ j.

A function f : $(X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ is said to be **$m_X^{(i,j)}$-gπ continuous** if and only if $f^{-1}$ (U) is $m_X^{(i,j)}$-gπ- open in X for every $m_Y^{(i,j)}$ -open U of Y , where i, j = 1, 2 and i ≠ j.

A biminimal structure space $(X, m_X^1, m_X^2)$ is said to be **$m^{(i,j)}$ - gπ $T_{1/2}$** space if for every $m_X^{(i,j)}$-gπ-closed set is a $m_X^{(i,j)}$ -closed set, where i, j = 1, 2 and i ≠ j.

The properties of $m_X^{(i,j)}$-gπ continuous functions are studied analogous to gπ-continuous functions. Finally $m_X^{(i,j)}$-gπ-dense sets are defined and characterizations are obtained.

Let $(X, m_X^1, m_X^2)$ be a biminimal structure space, A be a subset of X. A is called $\boldsymbol{m_X^{(i,j)}}$ **- gπ- dense** set in X if X = $m_X^i$ π Cl ($m_X^j$ πCl(A)), where i, j = 1, 2 and i ≠ j.

1. Let $(X, m_X^1, m_X^2)$ be a biminimal structure space and A be a subset of X. If for any non empty $m_X^{(i,j)}$- π -closed subset F of X such that A ⊆ F, then F = X if and only if G ∩ A ≠ ∅ for any non-empty $m_X^{(i,j)}$ - π-open subset G of X, where i, j = 1, 2 and i ≠ j.

2. Let $(X, m_X^1, m_X^2)$ be a biminimal structure and A be a subset of X. Then $m_X^i$ π int ($m_X^j$ πint(X\ A)) = ∅ If and only if A is $m_X^{(i,j)}$-gπ-dense set in X, where i, j = 1, 2 and i ≠ j.

In chapter 7, we have introduced and developed the notion of gπ -closed sets in terms of grills. The concepts of gπ($\theta$)-convergence, gπ($\theta$)-adherence, gπ($\theta$)-linked and gπ($\theta$)-conjoint are introduced and their properties are discussed.

Let (X, $\tau$ ) be a topological space and $\mathcal{G}$ be a grill on X. Then the mapping $\pi\Phi_{\mathcal{G}}$: $\mathcal{P}$(X) → $\mathcal{P}$(X), with respect to the grill $\mathcal{G}$ and $\tau$, is defined by $\pi\Phi_{\mathcal{G}}$(A)= {x ∈ X : U∩A∈ $\mathcal{G}$ for every π-open set U containing x } for A∈ $\mathcal{P}$(X)

Let $\mathcal{G}$ be a grill on a space X. Then the map $\pi\psi$ : $\mathcal{P}$(X) → $\mathcal{P}$(X) is defined by $\pi\psi$ **(A)** = A ∪ $\pi\Phi_{\mathcal{G}}$(A), for all A ∈ $\mathcal{P}$(X).

Using the above functions, the topology with respect to grill is defined as follows:

Let (X, $\tau$) be a topological space and $\mathcal{G}$ be a grill on a space X. There exists a **unique topology** $\tau_{\mathcal{G}}\pi$ on X given by $\tau_{\mathcal{G}}\pi = \{U \subset X: \pi\psi\ (X \setminus U) = X \setminus U\}$

Now, gπ-closed set with respect to grill $\mathcal{G}$ is defined and many characteraizations are obtained.

Let (X, $\tau$) be a topological space and $\mathcal{G}$ be a grill on X. Then a subset A of X is said to be **gπ -closed with respect to the grill $\mathcal{G}$ ($\mathcal{G}$- gπ -closed)** if $\pi\Phi_{\mathcal{G}}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

1. Let (X, $\tau$) be a topological space and $\mathcal{G}$ be a grill on X .If a subset A of X is $\mathcal{G}$- gπ-closed, then $\tau_{\mathcal{G}}\pi\ cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

2. Let (X, $\tau$) be a topological space and $\mathcal{G}$ be a grill on X .If a subset A of X is $\mathcal{G}$- gπ –closed, then for all $x \in \tau_{\mathcal{G}}\pi\ cl(A)$ , $\pi cl(\{x\}) \cap A \neq \varphi$.

3. Let (X, $\tau$) be a topological space and $\mathcal{G}$ be a grill on X .If a subset A of X is $\mathcal{G}$- gπ-closed, then $\tau_{\mathcal{G}}\pi\ cl(A) \setminus A$ contains no non-empty closed set of (X, $\tau$). Moreover $\pi\Phi_{\mathcal{G}}(A) \setminus A$ contains no non-empty closed set of (X, $\tau$).

4. Let $\mathcal{G}$ be grill on a space (X, $\tau$) and A be a $\mathcal{G}$- gπ-closed set. Then the following are equivalent

a) A is $\tau_{\mathcal{G}}\pi$ -closed.

b) $\tau_{\mathcal{G}}\pi\ cl(A) \setminus A$ is closed in (X, $\tau$)

c) $\pi\Phi_{\mathcal{G}}(A) \setminus A$ is closed in (X, $\tau$)

5. For any subset A of a space (X, $\tau$) and a grill $\mathcal{G}$ on X. If A is $\mathcal{G}$- gπ -closed then $A \cup (X \setminus \pi\Phi_{\mathcal{G}}(A))$ is $\mathcal{G}$- gπ closed.

6. For any subset A of a space (X, $\tau$) and a grill $\mathcal{G}$ on X, the following are equivalent

a) $A \cup (X \setminus \pi\Phi_{\mathcal{G}}(A))$ is $\mathcal{G}$- gπ –closed

b) $\pi\Phi_{\mathcal{G}}(A) \setminus A$ is $\mathcal{G}$- gπ -open.

The concepts of gπ($\theta$)-convergence, gπ($\theta$)-adherence are introduced here and some of their properties are studied.

1. A grill $\mathcal{G}$ on a topological space X is said to:

a) **gπ($\theta$)-adhere** at $x \in X$ if for each $U \in g\pi O(x)$ and each $G \in \mathcal{G}$, $g\pi cl U \cap G \neq \varphi$,

b) **gπ($\theta$)-converge** to a point $x \in X$ if for each $U \in g\pi O(x)$, there is some $G \in \mathcal{G}$ such that $G \subseteq g\pi cl(U)$ (in this case we shall also say that $\mathcal{G}$ is gπ($\theta$)-convergent to x).

2. A filter $\mathcal{F}$ on a topological space X is said to gπ($\theta$)-adhere at $x \in X$ (gπ($\theta$)-converge to $x \in X$) if for each $F \in \mathcal{F}$ and each $U \in g\pi O(x)$, $F \cap g\pi cl(U) \neq \varphi$ (resp. to each $U \in g\pi O(x)$, there corresponds $F \in \mathcal{F}$ such that $F \subseteq g\pi cl(U)$.

The properties obtained in this chapter are listed here

1. If $\mathcal{G}$ is a grill (or a filter) on a space X, then the section of $\mathcal{G}$, denoted by sec$\mathcal{G}$, is given by $sec\mathcal{G} = \{A \subseteq X : A \cap G \neq \varphi$; for all $G \in \mathcal{G}\}$

2. If a grill $\mathcal{G}$ on a topological space X, gπ($\theta$)-adheres at some point $x \in X$, then $\mathcal{G}$ is gπ($\theta$)-convergent to x.

3. A grill $\mathcal{G}$ on a space X, gπ($\theta$)-adheres to a point $x \in X$ iff $\mathcal{G} \subseteq \mathcal{G}(g\pi(\theta), x)$.

4. A grill $\mathcal{G}$ on a topological space X is $g\pi(\theta)$-convergent to a point x of X iff $sec_{\mathcal{G}}(g\pi(\theta), x) \subseteq \mathcal{G}$.

Finally the concept of $g\pi(\theta)$-linked and $g\pi(\theta)$-conjoint are introduced and their properties are discussed.

A grill $\mathcal{G}$ on a space X is said to be:

a) $g\pi(\theta)$-linked if for any two members A, B $\in \mathcal{G}$, $g\pi(\theta)$-cl(A) $\cap$ $g\pi(\theta)$-cl(B) $\neq \varphi$.

b) $g\pi(\theta)$-conjoint if for every finite subfamily $A_1, A_2, \ldots . A_n$ of $\mathcal{G}$, $g\pi int [\bigcap_{i=1}^{n} g\pi(\theta)\text{-cl}(A_i)] \neq \varphi$.