

**Avinashilingam Institute for Home Science and Higher Education for Women
(Deemed to be University), Coimbatore-641 043
Bachelor's Degree Examination- November 2018
I-Semester**

Class : I UG

Major: Mathematics

Time: 3 hours

Max. Marks: 100

18MAI01 Allied-I (Mathematical Statistics -I)

Part-A

1x10=10

Circle the correct answer

1. Probability can take values
a) $-\infty$ to ∞ b) $-\infty$ to 1 c) -1 to 1 d) 0 to 1
2. If A and B are two events the probability of occurrence of either A or B is observed as
a) $P(A) + P(B)$ b) $P(A \cup B)$ c) $P(A \cap B)$ d) $P(A) P(B)$
3. From a pack of 52 cards, two cards are drawn at random. The probability that one is an ace and other one is a king is evaluated as
a) $\frac{2}{13}$ b) $\frac{1}{169}$ c) $\frac{16}{169}$ d) $\frac{8}{663}$
4. An Urn A contains 5 white and 3 black balls and B contains 4 white and 4 black balls An Urn is selected and a ball is drawn from it, the probability, that the ball is white is calculated as
a) $\frac{9}{8}$ b) $\frac{9}{16}$ c) $\frac{5}{32}$ d) $\frac{5}{16}$
5. The height of persons in country is a random variable of the type
a) Continuous Random Variable c) Neither Discrete nor Continuous Random Variable
b) Discrete Random Variable d) Continuous as well as Discrete Random Variable
6. The Probability density function $f(x)$ cannot exceed
a) ∞ b) Unity c) Zero d) $-\infty$
7. Joint distribution function of (X, Y) is equivalent to the probability which is observed to be
a) $P(X = x, Y = y)$ b) $P(X \leq x, Y \leq y)$ c) $P(X \leq x, Y = y)$ d) $P(X \geq x, Y \geq y)$
8. For the joint p.d.f $f(x, y)$, the marginal distribution of Y given $X = x$ is given as
a) $\sum_{all x} f(x, y)$ b) $\int_{-\infty}^{\infty} f(x, y) dx$ c) $\int_{-\infty}^{\infty} f(x, y) dx dy$ d) $\int_{-\infty}^{\infty} f(x, y) dx$
9. If X and Y are two independent random variables, then Cov (X, Y) is
a) Zero b) one c) ∞ d) $-\infty$
10. If two variables X and Y are independent then $E(X, Y)$ is
a) $E(X) + E(Y)$ b) $E(X) \cdot E(Y)$ c) $E(X/Y)$ d) $V(Y/X)$

Part B

(5 X 6=30)

Answer all the questions

11. a) A and B alternatively throw a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, Show that his chance of winning is $\frac{30}{61}$.
(or)
b) Show that $P\left(\bigcup_{i=1}^n A_i\right) \leq \sum_{i=1}^n P(A_i)$
12. a) A bag contains 5 balls and it is not known how many of them are white. Two balls are drawn at random from the bag and they are noted to be white. What is the chance that all the balls in the bag are white?
(or)
b) For any two events A and B, relate that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
13. a) If X is a discrete random variable with the following probability distribution

x	1	2	3	4
P(x)	a	2a	3a	4a

 Find $P(2 < X < 4)$.
(or)
b) The diameter of an electric cable, say X is assumed to be a continuous random variable with pdf given by $f(x) = Kx(1-x)$, $0 \leq x \leq 1$. Determine K and $P(x \leq 1/3)$.
14. a) The bivariate random variable x and y have the pdf $f(x, y) = Kx^2(8-y)$, $x < y < 2x$, $0 \leq x \leq 2$. Compute the value of k.
(or)
b) The Joint pdf of (x, y) is given by $e^{-(x+y)}$, $0 < x, y < \infty$ are X and Y independent? why?

15. a) Calculate the MGF of the random variable 'X' whose probability function

$$P(X=x) = \frac{1}{2^x}, x=1,2,3.$$

(or)

b) The number of hardware failures of a computer system in a week of operations has the following pmf

No of Failures	0	1	2	3	4	5	6
Probability	0.18	0.28	0.25	0.18	0.16	0.04	0.01

Find the mean of the number of failures in a week.

Part-C

5x12=60

16. a) Prove that

i) For any two A and B $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

ii) Define the following

a. Equally likely Events

b. Independent Events

(or)

b) i) Give the Mathematical and Statistical definition of probability

ii) From a bag containing 4 white and 6 black balls, 2 balls are drawn at random. If the balls are drawn one after the other without replacement find the probability that

a. Both balls are white

b. Both balls are black

c. The first ball is white and second ball is black

17. a) State and Prove Baye's Theorem

(or)

b) A box contains 5 red and 4 white balls. Two balls are drawn successively from the box without replacement and it is noted that the second one is white. What is the probability that the first is also white?

18. a) If X is a continuous random variable whose pdf is given by $f(x) = \begin{cases} c(4x - 2x^2) & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$

i) Find the value of c and

ii) $P(X > 1)$

(or)

b) A random variable X has following the probability function

x	0	1	2	3	4
P(x)	k	3k	5k	7k	9k

Find

i) The value of k

ii) $P(x < 3)$ and $P(0 < x < 4)$ and

iii) The distribution function of 'x'

19. a) Express that the function

$$f(x,y) = \begin{cases} \frac{2}{5}(2x + 3y), & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

is a joint pdf of x and y

(or)

b) If the joint pdf of a random variable (x,y) is given by

$$f(x,y) = \begin{cases} x^2 + \frac{xy}{3} & 0 \leq x \leq 1; 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Estimate the conditional densities of x given y and y given x

20. a) Prove that $E(xy) = E(x) \cdot E(y)$

(or)

b) Find the MGF of a random variable having the density function

$$f(x) = \begin{cases} \frac{x}{2}, & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$
