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Appendix L2

**(Item No 5 of
Check List) Details of Research
Publications**

S.No	Article	Journal	Other Details Vol/No/Page No/ Year	Published in UGC- CARE / Scopus Indexed/ Web of Science
1	Neutrosophic Spherical sets in MCDM	Neutrosophic sets and systems	vol. 68 pp. 136-153	UGC/CARE Scopus Indexed
2	Distance Measure for Fuzzy Neutrosophic sets: A comparative Framework for Qualitative Evaluation	Neutrosophic sets and system	vol. 80, 2025/	Scopus Indexed
3	Neutrosophic over Topological spaces	Boletim da Sociedade Paranaense de matemática	Vol. 43 2025. PP. 1-11 ISSN: 0037 -8712	Web of Science

*Proof of list of Journals from Internet to be attached along with copies of reprints.

Scholar : Bhavaneshwari

Supervisor : A. Jeyaraj
19/02/25

The scholar Miss. Bhavaneshwari, S(21PHMAF009)
has published her research articles in the
following journals:

1. Neutrosophic sets and Systems - indexed and active in Scopus from
2018 to present.

This may be considered.

Checked By: [Signature]
19/2/25

HoD/Dean of Respective School

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Neutrosophic Spherical Sets in MCDM

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Abstract:

Decision-making plays a crucial role in achieving success across various scenarios, especially when confronted with complex issues inundated with abundant facts and information. Employing multi-criteria decision-making (MCDM) methods and techniques becomes particularly indispensable in tackling such formidable challenges. This study introduces novel neutrosophic SWGM and SWAM accuracy functions, which enhance traditional aggregation operators. Furthermore, it introduces the CODAS technique tailored for addressing Multiple Attribute Group Decision Making problems utilizing the newly defined operators. To exemplify the proposed methodology, a supplier selection problem is examined.

Keywords: Neutrosophic Spherical Set (NSS); Decision Matrix (D-Mx); Negative Ideal Solution (NIS); Positive Ideal Solution (PIS); Spherical Weighted Arithmetic Mean (SWAM); Spherical Weighted Geometric Mean (SWGM); Score function (SF); Accuracy Function (AF).

1. Introduction

Multi-Criteria Decision-Making (MCDM) is a structured approach that considers multiple criteria and attributes to assess and pinpoint the best option or resolution among a set of competing alternatives. Decision-makers are often tasked with navigating conflicting objectives or standards when selecting from available options in diverse real-life contexts. MCDM serves as a tool to assist decision-makers in achieving the most advantageous decision by carefully weighing and addressing these considerations.

The MCDM consists of criteria, a set of alternatives, and expert evaluations of the alternatives for each criterion. These sections evaluate the specialized knowledge and score the options based on suitability. These days, a vast array of MCDM approaches have been developed and applied in many different kinds of industries [1], the transportation industry [2], economics [3], health [4], energy planning [5], manufacturing [6], construction [7], supplier selection [8], and more.

A recent development in MCDM is the distance-based methodology known as Combinative Distance-based Assessment (CODAS). This methodology compares the Euclidean distance (ED) with the Taxicab distance (TD) to determine which alternatives are preferred.

Uncertainty stands as one of the pivotal factors impacting the decision-making process. Employing a Fuzzy Set (FS) offers a means to surmount this uncertainty. As a development of classical set theory, Lotfi A. Zadeh invented FSs [9] in 1965. The idea behind FSs is to represent and manipulate uncertainty more flexibly and realistically, especially in situations where traditional binary logic may not be suitable. Fuzzy MCDM techniques aim to resolve the uncertainty associated with decision-making problems [10].

Atanassov presented a broader version of fuzzy sets called intuitionistic fuzzy sets, which provide a more comprehensive treatment of ambiguity and uncertainty [11], and are formally known as Intuitionistic Fuzzy Sets (IFS). IFSs extend the capabilities of traditional fuzzy sets by encompassing notions of non-membership and hesitancy. Employing an intuitionistic fuzzy Multi-Criteria Decision-Making (MCDM) approach, Karagoz S, Deveci M, Simic V, Aydin N, and Bolukbas [12] evaluated various choices for the selection of a designated dismantling center location.

An approach utilizing CODAS technique, grounded on intuitionistic fuzzy [13] Multiple Criteria Decision Making (MCDM), is proposed to aid in waste management. The method involves employing the intuitionistic fuzzy weighted averaging operator to amalgamate the diverse viewpoints of decision-makers into a unified consensus.

Expanding on the notion of Intuitionistic Fuzzy Sets (IFS), a mathematical concept called Interval-Valued Intuitionistic Fuzzy Sets (IVIFS) incorporates intervals to represent degrees of membership and non-membership. Roy, Das, Kar, and Pamučar (2019) extended the CODAS approach with IVIFS, offering a framework for assessing Multiple Criteria Decision Making (MCDM) challenges where only partial weight information is available. Additionally, Peng and Garg [15] introduced methodologies for addressing emergency decision-making using similarity measures, CODAS, and weighted distance-based approximation within interval-valued fuzzy soft sets.

The Pythagorean Fuzzy Set (PFS) is the foundation of a recently introduced novel structure intended to handle uncertainty in practical decision-making situations. When awarding membership, nonstandard FSs, IFSs, and IVIFSs permit a degree of commitment smaller than one. A class of nonstandard Pythagorean fuzzy subsets is introduced in [16], where the membership grades are pairs (a, b) that meet the condition that $a^2 + b^2 \leq 1$. PFS is far more effective at modeling such uncertainty than an IFS.

Peng, Xindong, and Ma, Xueling investigated an algorithm for resolving MCDM problems based on CODAS and created a novel approach for handling MCDM difficulties in a Pythagorean fuzzy environment [17]. Zhang, X., Xu, Z outline some new Pythagorean Fuzzy Set (PFS) operating regulations and discuss their beneficial characteristics [18]. To successfully address the MCDM difficulties involving PFSs, it is also suggested that an enhanced strategy for order preference be similar to the optimum solution method.

The Neutrosophic Set (NS) theory extends classical sets, FSs, and IFSs that aim to manage unclear, incomplete, and contrasting facts. This approach, which resolved indeterminacy using a new type of set and allowed for a more refined representation of uncertain particulars, was given by Florentin Smarandache [19-20].

Smarandache illustrates in [21-22] that offsets and off-uninorms have applicability within digital image processing, particularly for tasks like image segmentation and edge detection. Furthermore, the paper offers algorithms and examples to elucidate these concepts.

One specific type of NS is a single-valued set which has been proposed to manage with incomplete information. [23] offers a novel method for solving multi-attribute group decision-making issues by applying the order choose by similarity technique to a single-valued neutrosophic environment. Additionally, create the TOPSIS technique for MADM in a streamlined neutrosophic setting.

Broumi, Je, and Smarandache are set to enhance the TOPSIS method [24] to accommodate interval neutrosophic uncertain linguistic information. They will introduce an extended version of the TOPSIS method tailored for resolving multiple attribute decision-making dilemmas where attribute values are expressed as interval neutrosophic uncertain linguistic variables and attribute weights remain unspecified. Broumi introduced the innovative concept of the Neutrosophic Inverse Soft Expert Set (NISES) in [25], which finds application within the Failure Mode and Effect Analysis (FMEA) framework.

H. Garg presents novel applications for combining Single-Valued Neutrosophic (SVN) data, which are applied to solve problems related to MCDM [26]. Gundogdu, F. Kutlu, and Kahraman, C. presented the idea of generalized three-dimensional Spherical Fuzzy Sets (SFSs) with a few critical distinctions from previous FSs [27]. The spherical vague distances, established with examples, provide the basis of the new kind of FS. An illustrated example of spherical SF TOPSIS, a MCDM approach, is shown.

To evaluate the obstacles to the growth of clean energy, a proposed technique based on MCDM approaches in a SFS has been mentioned in [28]. Additionally, CODAS outperformed the other approaches when the outcomes of the MOORA, COPRAS, and CODAS procedures were compared. Biswas, Chatterjee, and Majumder [29] apply a SFS to rank the statements. After calculating scores, they utilize an MCDA based on the SFS to determine the statements' relative ranking according to the judgments of a selection panel. The LOPCOW (modified SF Logarithmic Percentage Change-driven Objective Weighting) approach is employed.

Smarandache introduced the concepts of Neutrosophic Two-Fold Algebra [30-31] along with its corresponding Neutrosophic Two-Fold Law, and explored their extensions into Fuzzy Two-Fold Algebras and Laws. Additionally, they discovered nine novel topologies while enhancing and revisiting seven previously established ones [32]. Smarandache demonstrated that the Super Hyper Function [33] serves as a broader framework encompassing classical Function, Super Function, and Hyper Function. They also pioneered the Super Hyper Soft Set and its variations, including the Fuzzy and Fuzzy Extension Super Hyper Soft Set, [34] while establishing that the Super Hyper Soft Set comprises multiple Hyper Soft Sets.

In this study, we create a novel notion, the Neutrosophic Spherical Set (NSS), by fusing the ideas of spherical measure and neutrosophic logic. The spherical fuzzy distances established in the literature are the foundation for the new class of Neutrosophic sets. The presentation includes the proofs for addition, subtraction, and multiplication arithmetic operations. Accuracy functions, scoring, and aggregation operations are constructed. An exemplary example of Spherical Neutrosophic CODAS, a MCDM process, is shown.

2. Preliminaries

Definition 2.1 [19]

Consider M be the universe. A NS \tilde{K} in M is characterized by a truth $T_{\tilde{K}}$, indeterminacy $I_{\tilde{K}}$ and a falsity $F_{\tilde{K}}$ membership functions

$$\tilde{K} = \left\{ \left\langle \tilde{k}, \left(T_{\tilde{K}}(\tilde{k}), I_{\tilde{K}}(\tilde{k}), F_{\tilde{K}}(\tilde{k}) \right) \right\rangle : \tilde{k} \in M, T_{\tilde{K}}, I_{\tilde{K}}, F_{\tilde{K}} \in]0, 1[\right\}$$

then

$$0^- \leq \left(T_{\tilde{K}}(\tilde{k}) \right) + \left(I_{\tilde{K}}(\tilde{k}) \right) + \left(F_{\tilde{K}}(\tilde{k}) \right) \leq 3^+$$

Definition 2.2 [27]

A SFS \tilde{S} of the universe of discourse Z is given by $\tilde{S} = \{ \langle s, (T_{\tilde{S}}(s), I_{\tilde{S}}(s), F_{\tilde{S}}(s)) \rangle | s \in Z \}$

Where $T_{\tilde{S}}(s) : Z \rightarrow [0, 1]$, $I_{\tilde{S}}(s) : Z \rightarrow [0, 1]$, $F_{\tilde{S}}(s) : Z \rightarrow [0, 1]$ and

$$0 \leq T_{\tilde{S}}^2(s) + I_{\tilde{S}}^2(s) + F_{\tilde{S}}^2(s) \leq 1 \quad \forall s \in Z$$

For each s , the numbers $T_{\tilde{S}}(s)$, $I_{\tilde{S}}(s)$ and $F_{\tilde{S}}(s)$ are membership, non-membership and hesitancy of s to \tilde{A} , respectively.

3. Neutrosophic spherical set

The squared sum of the parameters in NSSs can range 0 and $\sqrt{3}$, it is possible to define each of them individually between 0 and 1 independently. In this section, the explanation of NSS and overview of spherical distance measurement, arithmetic operation and aggregation and de-neutrosophication processes are provided.

Definition 3.1. NSS \tilde{S} of the universe of discourse Z is given by

$$\tilde{S} = \{ \langle s, (T_{\tilde{S}}(s), I_{\tilde{S}}(s), F_{\tilde{S}}(s)) \rangle | s \in Z \} \tag{1}$$

Where,

$$T_{\tilde{S}}(s) : Z \rightarrow [0, 1], \quad I_{\tilde{S}}(s) : Z \rightarrow [0, 1], \quad F_{\tilde{S}}(s) : Z \rightarrow [0, 1]$$

and

$$0 \leq T_{\tilde{S}}^2(s) + I_{\tilde{S}}^2(s) + F_{\tilde{S}}^2(s) \leq \sqrt{3} \quad \forall s \in Z \tag{2}$$

For each s , the numbers $T_{\tilde{S}}(s)$, $I_{\tilde{S}}(s)$ and $F_{\tilde{S}}(s)$ are the degree of Membership, Non-Membership, and Hesitant Membership of s to \tilde{S} , respectively. **Error! Reference source not found.**

Definition 3.2. Basic Operators

$$\bar{A} \oplus \bar{B} = \left\{ \left(T_A^2 + T_B^2 - T_A^2 T_B^2 \right)^{\frac{1}{2}}, \left(I_A^2 + I_B^2 - I_A^2 I_B^2 \right)^{\frac{1}{2}}, \left(F_A^2 + F_B^2 - F_A^2 F_B^2 \right)^{\frac{1}{2}} \right\} \quad (3)$$

$$\bar{A} \otimes \bar{B} = \left\{ (T_A T_B), (I_A I_B), (F_A F_B) \right\} \quad (4)$$

$$\lambda \bullet \bar{A} = \left\{ \left(1 - (1 - T_A^2)^\lambda \right)^{\frac{1}{2}}, \left(1 - (1 - I_A^2)^\lambda \right)^{\frac{1}{2}}, \left(1 - (1 - F_A^2)^\lambda \right)^{\frac{1}{2}} \right\} \quad (5)$$

$$\bar{A}^\lambda = \{ T_A^\lambda, I_A^\lambda, F_A^\lambda \} \lambda > 0 \quad (6)$$

Definition 3.3. For these NSS $\bar{M} = (T_M, I_M, F_M)$ and $\bar{N} = (T_N, I_N, F_N)$, the following applies to

$\lambda, \lambda_1, \lambda_2 > 0$.

1. $\bar{M} \oplus \bar{N} = \bar{N} \oplus \bar{M}$ (7)

2. $\bar{M} \otimes \bar{N} = \bar{M} \otimes \bar{N}$ (8)

3. $\lambda (\bar{M} \oplus \bar{N}) = \lambda \bar{M} \oplus \lambda \bar{N}$ (9)

4. $\lambda_1 \bar{M} \oplus \lambda_2 \bar{M} = (\lambda_1 + \lambda_2) \bar{M}$ (10)

5. $(\bar{M} \otimes \bar{N})^\lambda = \bar{M}^\lambda \otimes \bar{N}^\lambda$ (11)

6. $\bar{M}^{\lambda_1} \otimes \bar{M}^{\lambda_2} = \bar{M}^{\lambda_1 + \lambda_2}$ (12)

Proof:

According to Definition 3.2, we will prove equations (7-9 and 11) since equation (10 and 12) are comparable to the corresponding proofs of equations (9 and 11),

1. $\bar{M} \oplus \bar{N} = \bar{N} \oplus \bar{M}$

$$\bar{M} \oplus \bar{N} = \left\{ \left(T_M^2 + T_N^2 - T_M^2 T_N^2 \right)^{\frac{1}{2}}, \left(I_M^2 + I_N^2 - I_M^2 I_N^2 \right)^{\frac{1}{2}}, \left(F_M^2 + F_N^2 - F_M^2 F_N^2 \right)^{\frac{1}{2}} \right\}$$

$$\bar{N} \oplus \bar{M} = \left\{ \left(T_N^2 + T_M^2 - T_N^2 T_M^2 \right)^{\frac{1}{2}}, \left(I_N^2 + I_M^2 - I_N^2 I_M^2 \right)^{\frac{1}{2}}, \left(F_N^2 + F_M^2 - F_N^2 F_M^2 \right)^{\frac{1}{2}} \right\}$$

Hence 1 is proved.

2. $\bar{M} \otimes \bar{N} = \bar{M} \otimes \bar{N}$

$$\bar{M} \otimes \bar{N} = \left\{ (T_M T_N), (I_M I_N), (F_M F_N) \right\}$$

$$\bar{N} \otimes \bar{M} = \left\{ (T_N T_M), (I_N I_M), (F_N F_M) \right\}$$

Hence 2 is proved.

3. $\lambda(\tilde{M} \oplus \tilde{N}) = \lambda\tilde{M} \oplus \lambda\tilde{N}$

$$\begin{aligned} \lambda(\tilde{M} \oplus \tilde{N}) &= \lambda \left\{ (T_M^2 + T_N^2 - T_M^2 T_N^2)^{\frac{1}{2}}, (I_M^2 + I_N^2 - I_M^2 I_N^2)^{\frac{1}{2}}, (F_M^2 + F_N^2 - F_M^2 F_N^2)^{\frac{1}{2}} \right\} \\ &= \left\{ \left(1 - (1 - (T_M^2 + T_N^2 - T_M^2 T_N^2))^{\lambda} \right)^{\frac{1}{2}}, \left(1 - (1 - (I_M^2 + I_N^2 - I_M^2 I_N^2))^{\lambda} \right)^{\frac{1}{2}}, \right. \\ &\quad \left. \left(1 - (1 - (F_M^2 + F_N^2 - F_M^2 F_N^2))^{\lambda} \right)^{\frac{1}{2}} \right\} \\ \lambda\tilde{M} \oplus \lambda\tilde{N} &= \left\{ \left(1 - (1 - T_M^2)^{\lambda} \right)^{\frac{1}{2}}, \left(1 - (1 - I_M^2)^{\lambda} \right)^{\frac{1}{2}}, \left(1 - (1 - F_M^2)^{\lambda} \right)^{\frac{1}{2}} \right\} \\ &\quad \oplus \left\{ \left(1 - (1 - T_N^2)^{\lambda} \right)^{\frac{1}{2}}, \left(1 - (1 - I_N^2)^{\lambda} \right)^{\frac{1}{2}}, \left(1 - (1 - F_N^2)^{\lambda} \right)^{\frac{1}{2}} \right\} \\ &= \left\{ \left(1 - (1 - T_M^2)^{\lambda} + 1 - (1 - T_N^2)^{\lambda} - (1 - (1 - T_M^2)^{\lambda})(1 - (1 - T_N^2)^{\lambda}) \right)^{\frac{1}{2}}, \right. \\ &\quad \left. \left(1 - (1 - I_M^2)^{\lambda} + 1 - (1 - I_N^2)^{\lambda} - (1 - (1 - I_M^2)^{\lambda})(1 - (1 - I_N^2)^{\lambda}) \right)^{\frac{1}{2}}, \right. \\ &\quad \left. \left(1 - (1 - F_M^2)^{\lambda} + 1 - (1 - F_N^2)^{\lambda} - (1 - (1 - F_M^2)^{\lambda})(1 - (1 - F_N^2)^{\lambda}) \right)^{\frac{1}{2}} \right\} \\ &= \left\{ \left(1 - (1 - T_M^2)^{\lambda} (1 - T_N^2)^{\lambda} \right)^{\frac{1}{2}}, \left(1 - (1 - I_M^2)^{\lambda} (1 - I_N^2)^{\lambda} \right)^{\frac{1}{2}}, \left(1 - (1 - F_M^2)^{\lambda} (1 - F_N^2)^{\lambda} \right)^{\frac{1}{2}} \right\} \\ &= \left\{ \left(1 - (1 - (T_M^2 + T_N^2 - T_M^2 T_N^2))^{\lambda} \right)^{\frac{1}{2}}, \left(1 - (1 - (I_M^2 + I_N^2 - I_M^2 I_N^2))^{\lambda} \right)^{\frac{1}{2}}, \right. \\ &\quad \left. \left(1 - (1 - (F_M^2 + F_N^2 - F_M^2 F_N^2))^{\lambda} \right)^{\frac{1}{2}} \right\} \end{aligned}$$

Hence 3 is proved.

Since

4. $\lambda_1 \tilde{M} \oplus \lambda_2 \tilde{M} = (\lambda_1 + \lambda_2) \tilde{M}$

5. $(\tilde{M} \otimes \tilde{N})^{\lambda} = \tilde{M}^{\lambda} \otimes \tilde{N}^{\lambda}$

$$\begin{aligned}
 (\tilde{M} \otimes \tilde{N})^\lambda &= \left\{ (T_M T_N, I_M I_N, F_M F_N)^\lambda \right\} \\
 &= \left\{ T_M^\lambda T_N^\lambda, I_M^\lambda I_N^\lambda, F_M^\lambda F_N^\lambda \right\} \\
 \tilde{M}^\lambda \otimes \tilde{N}^\lambda &= \left\{ T_M^\lambda, I_M^\lambda, F_M^\lambda \right\} \otimes \left\{ T_N^\lambda, I_N^\lambda, F_N^\lambda \right\} \\
 &= \left\{ T_M^\lambda T_N^\lambda, I_M^\lambda I_N^\lambda, F_M^\lambda F_N^\lambda \right\}
 \end{aligned}$$

Hence 5 is proved.

Definition 3.4. SWAM as, $z = (z_1, z_2, z_3, \dots, z_n)$; $z_j \in [0, 1]$; $\sum_{j=1}^n z_j \leq \sqrt{3}$ SWAM is defined as;

$$\begin{aligned}
 SWAM_z(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) &= z_1 \tilde{A}_1 + z_2 \tilde{A}_2 + z_3 \tilde{A}_3 + \dots + z_n \tilde{A}_n \\
 &\left\{ \left[1 - \prod_{j=1}^n (1 - T_A^2)^{\tilde{z}_j} \right]^{\frac{1}{2}}, \left[1 - \prod_{j=1}^n (1 - I_A^2)^{\tilde{z}_j} \right]^{\frac{1}{2}}, \left[1 - \prod_{j=1}^n (1 - F_A^2)^{\tilde{z}_j} \right]^{\frac{1}{2}} \right\}
 \end{aligned} \tag{13}$$

Definition 3.5. SWGM as, $z = (z_1, z_2, z_3, \dots, z_n)$; $z_j \in [0, 1]$; $\sum_{j=1}^n z_j \leq \sqrt{3}$

SWGM is defined as; $SWGM_z(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) = \tilde{A}_1^{\tilde{z}_1} + \tilde{A}_2^{\tilde{z}_2} + \tilde{A}_3^{\tilde{z}_3} + \dots + \tilde{A}_n^{\tilde{z}_n}$

$$\left\{ \prod_{j=1}^n T_A^{\tilde{z}_j}, \prod_{j=1}^n I_A^{\tilde{z}_j}, \prod_{j=1}^n F_A^{\tilde{z}_j} \right\} \tag{14}$$

Definition 3.6. The SF and AF for NSS classification are defined by;

$$Score(\tilde{S}) = (T_{\tilde{y}w} - F_{\tilde{y}w})^2 - (I_{\tilde{y}w} - F_{\tilde{y}w})^2 \tag{15}$$

$$Accuracy(\tilde{S}) = T_S^2 + I_S^2 + F_S^2 \tag{16}$$

Note that: $\tilde{S} < \tilde{T}$ iff

1. $Score(\tilde{S}) < Score(\tilde{T})$ or
2. $Score(\tilde{S}) = Score(\tilde{T})$ and $Accuracy(\tilde{S}) < Accuracy(\tilde{T})$ (17)

4. Neutrosophic Spherical CODAS

A D-Mx with entries that represent the assessment scores of every choice in relation to every criterion in a neutrosophic environment can be used to represent an MCDM problem. Suppose that

$S = \{s_1, s_2, s_3, \dots, s_m\}$ ($m \geq 2$) represents distinct collection of m possible options and

$K = \{K_1, K_2, K_3, \dots, K_n\}$ be the weight vector derived from every requirement that meet

$$0 \leq z_j \leq 1 \text{ and } \sum_{j=1}^n z_j \leq \sqrt{3}.$$

Step 1. Let DMs use the linguistic terms (LT) listed in Table 1 to complete the assessment matrices for decisions and criteria.

Table 1. Terms used in linguistics and their associated Spherical Neutrosophic Number

LT		(T, I, F)
Probably More Significant	PMS	(.9, .6, .2)
Extremely Significant	ES	(.8, .7, .2)
High Priority	HP	(.7, .6, .5)
Relatively Greater Significance	RGS	(.6, .7, .4)
Equally Important	EI	(.5, .8, .4)
Very Minimal Significance	VMS	(.4, .6, .7)
Low Priority	LP	(.5, .7, .6)
extremely low significant	ELS	(.5, .6, .6)
Definitely Not Important	DNI	(.2, .9, .6)

Step 2. Aggregate the outcomes reached by DM.

Aggregate the outcomes reached by DM using SWAM. Aggregate the DMs' Neutrosophic Spherical linguistic judgements of the selection criteria. Assemble and neutrosophic D-Mx based on DMs' views. Indicate the Alternative's evaluation value.

$$S_i (i = 1, 2, \dots, m) \text{ with respect to criterion } K_j (j = 1, 2, \dots, n) \text{ by } K_j(\tilde{S}_i) = (T_{ij}, I_{ij}, F_{ij})$$

and $D = (K_j(\tilde{S}_i))_{m \times n}$ is a Neutrosophic Spherical Decision Matrix (NS D-Mx). D-Mx for MCDM

problem using NSS, $D = (K_j(\tilde{S}_i))_{m \times n}$ must be put together as shown in equation (18).

$$D = (K_j(\tilde{S}_i))_{m \times n} = \begin{pmatrix} (\tilde{T}_{11}, \tilde{I}_{11}, \tilde{F}_{11}) & (\tilde{T}_{12}, \tilde{I}_{12}, \tilde{F}_{12}) & \dots & (\tilde{T}_{1n}, \tilde{I}_{1n}, \tilde{F}_{1n}) \\ (\tilde{T}_{21}, \tilde{I}_{21}, \tilde{F}_{21}) & (\tilde{T}_{22}, \tilde{I}_{22}, \tilde{F}_{22}) & \dots & (\tilde{T}_{2n}, \tilde{I}_{2n}, \tilde{F}_{2n}) \\ \vdots & \vdots & \dots & \vdots \\ (\tilde{T}_{m1}, \tilde{I}_{m1}, \tilde{F}_{m1}) & (\tilde{T}_{m2}, \tilde{I}_{m2}, \tilde{F}_{m2}) & \dots & (\tilde{T}_{mn}, \tilde{I}_{mn}, \tilde{F}_{mn}) \end{pmatrix} \tag{18}$$

Step 3. Build the weighted aggregated NS D-Mx. Following the determination of the alternative ratings and the weights assigned to the criteria, the aggregated weighted NS D-Mx is built using multiplication equation and then the aggregated weighted NS D-Mx can be defined as follows:

$$D = \left(K_j \left(\tilde{S}_{ix} \right) \right)_{m \times n} = \begin{pmatrix} (T_{11x}, I_{11x}, F_{11x}) & (T_{12x}, I_{12x}, F_{12x}) & \dots & (T_{1nx}, I_{1nx}, F_{1nx}) \\ (T_{21x}, I_{21x}, F_{21x}) & (T_{22x}, I_{22x}, F_{22x}) & \dots & (T_{2nx}, I_{2nx}, F_{2nx}) \\ \vdots & \vdots & \dots & \vdots \\ (T_{m1x}, I_{m1x}, F_{m1x}) & (T_{m2x}, I_{m2x}, F_{m2x}) & \dots & (T_{mnx}, I_{mnx}, F_{mnx}) \end{pmatrix} \quad (19)$$

Step 4. Utilising Eq. (20), deneutrosophic the aggregated weighted D-Mx.

$$Score \left(K_j \left(\tilde{S}_{ix} \right) \right) = (T_{ix} - F_{ix})^2 - (I_{ix} - F_{ix})^2 \quad (20)$$

Step 5. Find the NSPIS and NSNIS according to the SF acquired in Step 4.

Regarding the NS-PIS:

$$S^* = \left\{ K_j, \max_i \left(Score \left(K_j \left(S_{ix} \right) \right) \mid j = 1, 2, \dots, n \right) \right\} \quad (21)$$

$$S^* = \left\{ \left\langle K_1, (T_1^*, I_1^*, F_1^*) \right\rangle, \left\langle K_2, (T_2^*, I_2^*, F_2^*) \right\rangle, \dots, \left\langle K_n, (T_n^*, I_n^*, F_n^*) \right\rangle \right\}$$

Regarding the NS -NIS:

$$S^- = \left\{ K_j, \min_i \left(Score \left(K_j \left(S_{ix} \right) \right) \mid j = 1, 2, \dots, n \right) \right\} \quad (22)$$

$$S^- = \left\{ \left\langle K_1, (T_1^-, I_1^-, F_1^-) \right\rangle, \left\langle K_2, (T_2^-, I_2^-, F_2^-) \right\rangle, \dots, \left\langle K_n, (T_n^-, I_n^-, F_n^-) \right\rangle \right\}$$

Step 6. The distances between alternative S_i , NS-PIS, and NS-NIS should be calculated, accordingly.

For the NS-NIS:

$$D(S_i, S^-) = \sqrt{\frac{1}{2} \sum_{i=1}^n \left((T_{S_i} - T_{S^-})^2 + (I_{S_i} - I_{S^-})^2 + (F_{S_i} - F_{S^-})^2 \right)} \quad (23)$$

For the NS-PIS:

$$D(S_i, S^*) = \sqrt{\frac{1}{2} \sum_{i=1}^n \left((T_{S_i} - T_{S^*})^2 + (I_{S_i} - I_{S^*})^2 + (F_{S_i} - F_{S^*})^2 \right)} \quad (24)$$

Step 7 Calculate the minimum and maximum distances to the NS-NIS and NS-PIS, respectively.

$$D_{\max} (S_i, S^-) = \max_{i \leq i \leq m} (S_i, S^-) \quad (25)$$

$$D_{\min} (S_i, S^*) = \min_{i \leq i \leq m} (S_i, S^*) \quad (26)$$

Step 8 Compute the revised proximity ratio in Equation (27).

$$\xi(S_i) = \frac{D(S_i, S^-)}{D_{\max}(S_i, S^-)} - \frac{D(S_i, S^*)}{D_{\min}(S_i, S^*)} \quad (27)$$

Equation (27) because the subtraction's second element is at least equal to its first element, the result is zero or negative. We altered this equality from Equation (28) so that we might get zero or a result.

$$\xi(S_i) = \frac{D(S_i, S^*)}{D_{\min}(S_i, S^*)} - \frac{D(S_i, S^-)}{D_{\max}(S_i, S^-)} \tag{28}$$

Step 9. Determine the best solution by rating the alternatives in the best possible order. We organize the alternatives according to the rising closeness ratio values since we wish to use Equation (28).

5. Illustrative Example

A supplier selection issue is devised and solved by employing our recommended technique. Four vendors of air conditioners were considered count (S_1, S_2, S_3, S_4) and evaluated for their efficacy. The number of qualitative and quantitative aspects considered will determine how many different criteria are used to pick suppliers. In accordance with on the number of qualitative and quantitative factors are considered, the decision-making criteria for supplier selection may change. Several criteria and sub-criteria have been established using a comprehensive literature assessment. Four of these criteria are used in this exemplary example: price (K_1) , quality (K_2) , delivery (K_3) and performance (K_4) . Three decision makers with experience in supply chain and logistics management (DM1, DM2, and DM3) take part in the procedure for evaluation. The weights of these DMs, which are, respectively, 0.4, 0.5 and 0.3, represent their various levels of experience.

First, the judgements made by the decision-makers are compiled using the language phrases listed in Table 1 with regard to the objective. A decision is rendered in Tables 2-4.

Table 2. Decisions of DM1

DM1	(K_1)	(K_2)	(K_3)	(K_4)
S_1	ES	HP	EI	RGS
S_2	PMS	EI	HP	EI
S_3	LP	RGS	ES	ELS
S_4	ELS	ES	LP	HP

Table 3. Decisions of DM2

DM2	(K_1)	(K_2)	(K_3)	(K_4)
S_1	PMS	HP	ES	PMS

S_2	VMS	ES	HP	EI
S_3	HP	RGS	RGS	RGS
S_4	ELS	EI	LP	LP

Table 4. Decisions of DM3

DM3	(K_1)	(K_2)	(K_3)	(K_4)
S_1	HP	ES	PMS	RGS
S_2	VMS	PMS	ES	VMS
S_3	VMS	ELS	HP	HP
S_4	LP	EI	ES	RGS

The significance levels of the DMs are considered when combining these judgements utilizing the SWAM and SWGM operators. The decision matrices shown in Tables 5 and 6 are obtained.

Table 5. NS D-Mx by using SWAM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S_1	(0.873,0.682,0.340)	(0.773,0.673,0.487)	(0.821,0.764,0.311)	(0.825,0.707,0.364)
S_2	(0.743, 0.643,0.652)	(0.821,0.764,0.311)	(0.773,0.673,0.487)	(0.517,0.806,0.549)
S_3	(0.629,0.682,0.638)	(0.621,0.723,0.502)	(0.752,0.723,0.415)	(0.646,0.690,0.544)
S_4	(0.540,0.673,0.643)	(0.687,0.814,0.379)	(0.657,0.744,0.582)	(0.649,0.715,0.568)

Table 6. NS D-Mx by using SWGM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S_1	(0.779,0.576,0.190)	(0.678,0.567,0.330)	(0.656,0.656,0.191)	(0.66,0.603,0.235)

S_2	(0.460,0.541,0.394)	(0.656,0.656,0.191)	(0.678,0.567,0.330)	(0.407,0.701,0.393)
S_3	(0.481,0.576,0.517)	(0.512,0.622,0.376)	(0.636,0.622,0.269)	(0.527,0.585,0.418)
S_4	(0.435,0.567,0.541)	(0.525,0.725,0.252)	(0.501,0.651,0.389)	(0.525,0.612,0.445)

Table 7 displays the important weights of the language phrases used to express the criteria determined by DMs.

Table 7. The weights assigned to each criterion

Criteria	DM1	DM2	DM3
(K_1)	LP	VMS	HP
(K_2)	RGS	EI	RGS
(K_3)	PMS	RGS	ES
(K_4)	HP	HP	VMS

The weight of each criterion is determined by the decision-makers' strategies for the criteria aggregated by the SWAM operator provided in Equation (13), which are shown in Table 8.

Table 8. Aggregation of Criteria weights according to SWAM operator

Criteria	Weights of each criterion
(K_1)	(0.576,0.682,0.672)
(K_2)	(0.605,0.791,0.434)
(K_3)	(0.834,0.715,0.330)
(K_4)	(0.751,0.691,0.535)

The aggregated weighted neutrosophic spherical choice matrices are constructed using Equation (4) once the weights assigned to the criteria and evaluations of the substitutions have been determined, as illustrated in Tables 9 and 10.

Table 9. Weighted NS D-Mx according to SWAM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S_1	(0.503,0.465,0.228)	(0.468,0.532,0.211)	(0.685,0.547,0.103)	(0.620,0.488,0.195)
S_2	(0.428,0.439,0.438)	(0.497,0.604,0.135)	(0.645,0.481,0.161)	(0.388,0.557,0.293)
S_3	(0.362,0.465,0.429)	(0.376,0.571,0.218)	(0.627,0.517,0.137)	(0.485,0.477,0.291)
S_4	(0.311,0.459,0.432)	(0.415,0.643,0.165)	(0.548,0.532,0.192)	(0.487,0.494,0.304)

Table 10. Weighted NS D-Mx according to SWGM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S_1	(0.449,0.393,0.128)	(0.410,0.448,0.143)	(0.548,0.469,0.063)	(0.498,0.416,0.126)
S_2	(0.265,0.369,0.265)	(0.397,0.519,0.083)	(0.566,0.405,0.109)	(0.306,0.484,0.210)
S_3	(0.277,0.393,0.348)	(0.310,0.492,0.163)	(0.531,0.445,0.089)	(0.396,0.404,0.224)
S_4	(0.251,0.387,0.364)	(0.317,0.573,0.109)	(0.418,0.466,0.128)	(0.395,0.423,0.238)

SF are calculated using Equation (19) and Tables 11 and 12, which are based on Tables 9 and 10. PIS are represented by blue values, while NIS values are represented by yellow values.

Table 11. SF according to SWAM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S_1	0.0196191	-0.0370441	0.1424804	0.0945117
S_2	0.0001038	-0.0893889	0.1322302	-0.0605558
S_3	0.0031443	-0.1001252	0.0961946	0.0030986
S_4	0.0140321	-0.1663112	0.0111351	-0.0024082

Table 12. SF according to SWGM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S_1	0.033096	-0.0217872	0.0700743	0.0542487
S_2	-0.0108362	-0.0913385	0.1209456	-0.0660912
S_3	0.0029492	-0.0864905	0.0687862	-0.002763
S_4	0.0122932	-0.1718773	-0.0300535	-0.0095923

The NS-PIS and NS-NIS corresponding to the highest and worst scores are shown in Tables 13 and 14.

Table 13. NS-PIS and NS-NIS according to SWAM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S^* (Best)	(0.503,0.465,0.228)	(0.468,0.532,0.211)	(0.685,0.547,0.103)	(0.620,0.488,0.195)
S^- (Worst)	(0.428,0.439,0.438)	(0.415,0.643,0.165)	(0.548,0.532,0.192)	(0.388,0.557,0.293)

Table 14. NS-PIS and NS-NIS according to SWGM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S^* (Great)	(0.449,0.393,0.128)	(0.410,0.448,0.143)	(0.566,0.405,0.109)	(0.498,0.416,0.126)
S^- (Poor)	(0.265,0.369,0.265)	(0.317,0.573,0.109)	(0.418,0.466,0.128)	(0.306,0.484,0.210)

Based on Equations (23 and 24), the next step we can figure out how far apart option S_i is from both the NS-PIS and NS-NIS, respectively. Tables 15 and 16 provide their information.

Table 15. Distance to PIS and NIS according to SWAM operator

Alternatives	$D(S_i, S^*)$	$D(S_i, S^-)$
S_1	1.06252	0.142724324
S_2	0.132168332	0.052504021
S_3	0.113622049	0.070743093

S_4	0.138092728	0.059237447
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Table 16. Distance to PIS and NIS according to SWGM operator

Alternatives	$D(S_i, S^+)$	$D(S_i, S^-)$
S_1	0.028542681	0.136333827
S_2	0.117780274	0.067011546
S_3	0.119173652	0.07645776
S_4	0.145694319	0.053394324

We calculate the maximum and minimum distances to the NS-NIS and NS-PIS, respectively, from Tables 15 and 16. The closeness ratios are computed using Equation (28), and they are shown in Tables 17 and 18.

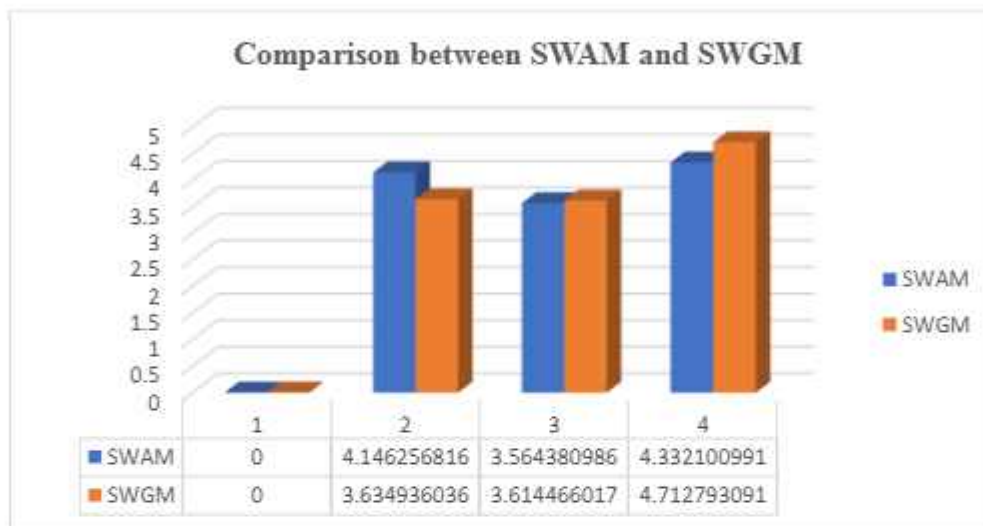
Table 17. Every alternative's closeness ratio according to the SWAM operator

Alternatives	Closeness Ratio	Rank
S_1	0	1
S_2	12438.77	3
S_3	10693.143	2
S_4	12996.303	4

Table 18. Closeness ratio of each alternative according to SWGM operator

Alternatives	Closeness Ratio	Rank
S_1	0	1
S_2	3.634936	3
S_3	3.614466	2
S_4	4.7127931	4

According to the SWAM operator, the closeness ratio for each alternative show that the best option is S_1 , and over all ranking is $S_1 > S_3 > S_2 > S_4$. The closest alternative, according to the proximity ratios based on the SWGM operator, is S_1 , and overall ranking is $S_1 > S_3 > S_2 > S_4$. The aggregation operators determine how the ranks differ. However, in both strategies, the best and worst options are the same.



6. Conclusions

This study introduces two novel accuracy functions, neutrosophic SWGM and SWAM, which represent significant advancements over conventional aggregation operators by integrating neutrosophic spherical sets. Through the development and application of an algorithm for the CODAS technique, we have effectively addressed the supplier selection problem. Our approach prioritizes alternatives based on distance measurements, utilizing the neutrosophic spherical CODAS approach to compute closeness ratios between criteria. Significantly, our comparison between SWAM and SWGM operators demonstrates comparable rankings and their efficacy in assessing alternatives. This research contributes to the advancement of decision-making methodologies, particularly in complex scenarios where traditional methods may fall short.

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Distance Measure for Fermatean Neutrosophic Sets: A Comparative Framework for Qualitative Evaluation

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Abstract: This work introduces a novel distance measure specifically designed for Fermatean Neutrosophic sets (FNS), extending upon existing measures such as the intuitionistic fuzzy distance and single-valued Neutrosophy distance measures. The study proposes a Hypothetical Framework for Fermatean Neutrosophic Sets to evaluate minimum distances and compares six different distance measures. These measures are applied within the contexts of both Fermatean Neutrosophic Sets and Neutrosophic Sets. The primary objective is to identify the shortest distance, enabling precise and appropriate decision-making. The study highlights the importance of false membership in FNS, which alters the ranking of FNS and establishes the Fermatean Neutrosophic set as a more accurate tool for evaluation in specific cases.

Keywords: Fermatean Neutrosophic set, Euclidean Distance (ED), Normalized Euclidean Distance (N-ED), Hamming Distance (HD), Normalized Hamming Distance (N-HD), Sine Metric Single-Valued Neutrosophic Distance Measure (SMSVNDM), Tangent Matric Fermatean Neutrosophic Distance Measure (TMFNDM).

1. Introduction

Distance measures are vital for assessing similarity or dissimilarity between elements, enabling accurate decision-making, ranking, and analysis in uncertain or complex scenarios. They are widely used in fields like medical diagnosis, pattern recognition, and risk analysis to ensure precise evaluations and optimal solutions [1-4].

Zadeh [5] introduced fuzzy set theory, which models vagueness through membership degrees, laying the foundation for advanced uncertainty modelling. Atanassov [6] introduced intuitionistic fuzzy sets, which account for both membership and non-membership degrees, enabling better handling of uncertainty. Distance measures play a key role in assessing similarity or dissimilarity within fuzzy, intuitionistic, and neutrosophic frameworks, which have been widely applied in areas like site selection (Rouyendegh et al. [7]) and education (Citil [8]).

Smarandache [9] introduced neutrosophy, extending intuitionistic fuzzy sets by incorporating indeterminacy, offering greater flexibility in managing incomplete and inconsistent information (Smarandache) [10]. Gong et al. [11] further enhanced decision analysis with spherical distance measures for intuitionistic fuzzy sets, improving similarity assessments. Building upon existing methodologies, this work introduces a novel distance measure tailored specifically to Fermatean Neutrosophic sets [12].

Fermatean Neutrosophic sets extend the framework of Neutrosophic sets [13], introducing additional parameters to capture uncertainty, indeterminacy, and falsity inherent in medical data. By integrating insights from intuitionistic fuzzy distance measures [14] and single-valued Neutrosophy (SVN) distance measures [15], this new distance measure offers a comprehensive approach to evaluating medical conditions within the Fermatean Neutrosophic framework. Recent works on Fermatean neutrosophic sets include foundational matrix operations [21], a framework for green biomedical waste management [22], and a correlation-based measure for electric vehicle selection [23], showcasing their real-world applicability.

Here, we defined distance measures for the Fermatean neutrosophic set to handle higher membership values in a set. Also, we can use this distance measure to handle all the lower case of neutrosophic sets and their extensions like fuzzy, intuitionistic fuzzy, Pythagorean fuzzy, spherical fuzzy, and Fermatean fuzzy.

Throughout this exploration, we have scrutinize six distinct distance measures, including Euclidean Distance, Normalized Euclidean Distance, Hamming Distance, Normalized Hamming Distance, SMSVNDM Distance, and TMFNDM Distance. By comparing and contrasting these measures within the context of Fermatean Neutrosophic sets and traditional Neutrosophic sets, we seek to elucidate the advantages and limitations of each approaches.

2. Preliminaries

Definition 2.1. [19]

The HD between two SVN sets I and J is defined as

$$d_H(I, J) = \frac{1}{3} \sum_{j=1}^n \left(\left| \mu_I(x_j) - \mu_J(x_j) \right| + \left| \sigma_I(x_j) - \sigma_J(x_j) \right| + \left| \gamma_I(x_j) - \gamma_J(x_j) \right| \right) \quad (1)$$

Definition 2.2. [19]

The ED between two SVN sets I and J is defined as

$$d_E(I, J) = \left\{ \frac{1}{3} \sum_{j=1}^n \left(\left(\mu_I(x_j) - \mu_J(x_j) \right)^2 + \left(\sigma_I(x_j) - \sigma_J(x_j) \right)^2 + \left(\gamma_I(x_j) - \gamma_J(x_j) \right)^2 \right) \right\}^{\frac{1}{2}} \quad (2)$$

Definition 2.3. [19]

The N-HD between two SVN sets I and J is defined as

$$d_{n-H}(I, J) = \frac{1}{3n} \sum_{j=1}^n \left(\left| \mu_I(x_j) - \mu_J(x_j) \right| + \left| \sigma_I(x_j) - \sigma_J(x_j) \right| + \left| \gamma_I(x_j) - \gamma_J(x_j) \right| \right) \quad (3)$$

Definition 2.4. [19]

The N-ED between two SVN sets I and J is defined as

$$d_{n-E}(I, J) = \left\{ \frac{1}{3n} \sum_{j=1}^n \left((\mu_I(x_j) - \mu_J(x_j))^2 + (\sigma_I(x_j) - \sigma_J(x_j))^2 + (\gamma_I(x_j) - \gamma_J(x_j))^2 \right) \right\}^{\frac{1}{2}} \quad (4)$$

Definition 2.5. [15]

Let $A = \{x_1, x_2, \dots, x_n\}$ be a universal set. Let $I = \{x_i, \mu_I(x_i), \sigma_I(x_i), \gamma_I(x_i) : x_i \in A\}$ and $J = \{x_i, \mu_J(x_i), \sigma_J(x_i), \gamma_J(x_i) : x_i \in A\}$ be two SVN sets on A . Then define a mapping

$$d : SVN(A) \times SVN(A) \rightarrow [0, 1] \text{ as: } d_{SMN}(I, J) = \frac{5 \sum_{i=1}^n \left\{ \sin \left\{ \frac{\pi}{6} |\mu_I(x_i) - \mu_J(x_i)| \right\} + \sin \left\{ \frac{\pi}{6} |\sigma_I(x_i) - \sigma_J(x_i)| \right\} + \sin \left\{ \frac{\pi}{6} |\gamma_I(x_i) - \gamma_J(x_i)| \right\} \right\}}{3n \sum_{i=1}^n \left\{ 1 + \sin \left\{ \frac{\pi}{6} |\mu_I(x_i) - \mu_J(x_i)| \right\} + \sin \left\{ \frac{\pi}{6} |\sigma_I(x_i) - \sigma_J(x_i)| \right\} + \sin \left\{ \frac{\pi}{6} |\gamma_I(x_i) - \gamma_J(x_i)| \right\} \right\}} \quad (5)$$

Definition 2.6 [6]

Let X be a non-empty set (universe). A FNS \tilde{S} on X is an object of the form:

$$\tilde{S} = \{ \langle s, (T_{\tilde{S}}(s), I_{\tilde{S}}(s), F_{\tilde{S}}(s)) \rangle \mid s \in X \}$$

where, $T_{\tilde{S}}(s), I_{\tilde{S}}(s), F_{\tilde{S}}(s) : X \rightarrow [0, 1]$, $0 \leq T_{\tilde{S}}^3(s) + F_{\tilde{S}}^3(s) \leq 1, 0 \leq I_{\tilde{S}}^3(s) \leq 1$ then

$$0 \leq T_{\tilde{S}}^3(s) + I_{\tilde{S}}^3(s) + F_{\tilde{S}}^3(s) \leq 2 \quad \forall s \in X$$

$T_{\tilde{S}}(s)$ is the degree of membership, $I_{\tilde{S}}(s)$ is the degree of indeterminacy and $F_{\tilde{S}}(s)$ is the degree of non-membership. Here $T_{\tilde{S}}(s)$ and $F_{\tilde{S}}(s)$ are dependent components and $I_{\tilde{S}}(s)$ is an independent component.

3. Distance Measure on Fermatean Neutrosophic Sets

The distance measure on Fermatean Neutrosophic Sets defined in this section is a generalization of the single-valued neutrosophic distance measure and intuitionistic fuzzy distance measure. It includes several derived properties.

Definition 3.1.

Let $A = \{x_1, x_2, \dots, x_n\}$ be a universal set. Let $I = \{x_i, T_I(x_i), I_I(x_i), F_I(x_i) : x_i \in A\}$

and $J = \{x_i, T_J(x_i), I_J(x_i), F_J(x_i) : x_i \in A\}$ be two Fermatean Neutrosophic Sets on A . Then define a mapping $d : FNS(A) \times FNS(A) \rightarrow [0,1]$ as:

$$d_{TMFN}(I, J) = \frac{2}{\sqrt{3}n} \sum_{i=1}^n \left[\begin{aligned} & \tan \left\{ \frac{\pi}{6} |T_I(x_i) - T_J(x_i)| \right\} \\ & + \tan \left\{ \frac{\pi}{6} |I_I(x_i) - I_J(x_i)| \right\} \\ & + \tan \left\{ \frac{\pi}{6} |F_I(x_i) - F_J(x_i)| \right\} \end{aligned} \right] \tag{6}$$

Theorem 3.2.

The FNSs hold the following properties. I, J, K .

- i. $d_{TMFN}(I, J) \geq 0$ for all $I, J \in FNS(A)$.
- ii. $d_{TMFN}(I, J) = 0$ if and only if $I = J$ for all $I, J \in FNS(A)$.
- iii. $d_{TMFN}(I, J) = d_{TMFN}(J, I)$ for all $I, J \in FNS(X)$.
- iv. If $I \subseteq J \subseteq K$ for all $I, J, K \in FNS(A)$, then $d_{TMFN}(I, K) \geq d_{TMFN}(I, J)$ and $d_{TMFN}(I, K) \geq d_{TMFN}(J, K)$.

Proof:

Part (i):

If $I, J \in FNS(A)$, then $0 \leq T_I(x_i) \leq 1, 0 \leq I_I(x_i) \leq 1, 0 \leq F_I(x_i) \leq 1, \forall x_i \in A$

$$\Rightarrow 0 \leq |T_I(x_i) - T_J(x_i)| \leq 1, 0 \leq |I_I(x_i) - I_J(x_i)| \leq 1, \text{ and } 0 \leq |F_I(x_i) - F_J(x_i)| \leq 1.$$

$$\Rightarrow 0 \leq \tan \left\{ \frac{\pi}{6} |T_I(x_i) - T_J(x_i)| \right\} \leq \frac{1}{\sqrt{3}},$$

$$0 \leq \tan \left\{ \frac{\pi}{6} |I_I(x_i) - I_J(x_i)| \right\} \leq \frac{1}{\sqrt{3}},$$

$$0 \leq \tan \left\{ \frac{\pi}{6} |F_I(x_i) - F_J(x_i)| \right\} \leq \frac{1}{\sqrt{3}}$$

Then

$$0 \leq \tan \left\{ \frac{\pi}{6} |T_I(x_i) - T_J(x_i)| \right\} + \tan \left\{ \frac{\pi}{6} |I_I(x_i) - I_J(x_i)| \right\} + \tan \left\{ \frac{\pi}{6} |F_I(x_i) - F_J(x_i)| \right\} \leq \sqrt{3}$$

$$\forall x_i \in X$$

Multiple by 2

$$0 \leq \frac{2}{\sqrt{3}} \left[\begin{array}{l} \tan \left\{ \frac{\pi}{6} |T_I(x_i) - T_J(x_i)| \right\} \\ + \tan \left\{ \frac{\pi}{6} |I_I(x_i) - I_J(x_i)| \right\} \\ + \tan \left\{ \frac{\pi}{6} |F_I(x_i) - F_J(x_i)| \right\} \end{array} \right] \leq 2, \forall x_i \in X.$$

This implies

$$0 \leq \frac{2}{\sqrt{3n}} \sum_{i=1}^n \left[\begin{array}{l} \tan \left\{ \frac{\pi}{6} |T_I(x_i) - T_J(x_i)| \right\} \\ + \tan \left\{ \frac{\pi}{6} |I_I(x_i) - I_J(x_i)| \right\} \\ + \tan \left\{ \frac{\pi}{6} |F_I(x_i) - F_J(x_i)| \right\} \end{array} \right] \leq 2$$

Thus, $0 \leq d_{TM\Delta S}(I, J) \leq 2$.

Part (ii):

$$d_{TM\Delta S}(I, J) = 0$$

$$\Leftrightarrow 0 \leq \frac{2}{\sqrt{3n}} \sum_{i=1}^n \left[\begin{array}{l} \tan \left\{ \frac{\pi}{6} |T_I(x_i) - T_J(x_i)| \right\} \\ + \tan \left\{ \frac{\pi}{6} |I_I(x_i) - I_J(x_i)| \right\} \\ + \tan \left\{ \frac{\pi}{6} |F_I(x_i) - F_J(x_i)| \right\} \end{array} \right] \leq 2$$

$$\Leftrightarrow 0 \leq \frac{2}{\sqrt{3}} \left[\begin{array}{l} \tan \left\{ \frac{\pi}{6} |T_I(x_i) - T_J(x_i)| \right\} \\ + \tan \left\{ \frac{\pi}{6} |I_I(x_i) - I_J(x_i)| \right\} \\ + \tan \left\{ \frac{\pi}{6} |F_I(x_i) - F_J(x_i)| \right\} \end{array} \right] = 0, \forall x_i \in X$$

$$\Leftrightarrow \left[\begin{array}{l} \tan \left\{ \frac{\pi}{6} |T_I(x_i) - T_J(x_i)| \right\} \\ + \tan \left\{ \frac{\pi}{6} |I_I(x_i) - I_J(x_i)| \right\} \\ + \tan \left\{ \frac{\pi}{6} |F_I(x_i) - F_J(x_i)| \right\} \end{array} \right] = 0, \forall x_i \in X$$

$$\Leftrightarrow \tan \left\{ \frac{\pi}{6} |T_I(x_i) - T_J(x_i)| \right\} = 0,$$

$$\tan \left\{ \frac{\pi}{6} |I_I(x_i) - I_J(x_i)| \right\} = 0, \forall x_i \in X$$

$$\tan \left\{ \frac{\pi}{6} |F_I(x_i) - F_J(x_i)| \right\} = 0$$

$$\Leftrightarrow \left\{ \frac{\pi}{6} |T_I(x_i) - T_J(x_i)| \right\} = 0,$$

$$\left\{ \frac{\pi}{6} |I_I(x_i) - I_J(x_i)| \right\} = 0, \quad \forall x_i \in X$$

$$\left\{ \frac{\pi}{6} |F_I(x_i) - F_J(x_i)| \right\} = 0$$

$$\Leftrightarrow |T_I(x_i) - T_J(x_i)| = 0, |I_I(x_i) - I_J(x_i)| = 0, |F_I(x_i) - F_J(x_i)| = 0, \forall x_i \in A$$

$$\Leftrightarrow T_I(x_i) = T_J(x_i), I_I(x_i) = I_J(x_i), F_I(x_i) = F_J(x_i), \forall x_i \in A$$

$$\Leftrightarrow A = B.$$

Part (iii):

$$d_{TN\&S}(I, J) = \frac{2}{\sqrt{3n}} \sum_{i=1}^n \left[\begin{array}{l} \tan \left\{ \frac{\pi}{6} |T_I(x_i) - T_J(x_i)| \right\} \\ + \tan \left\{ \frac{\pi}{6} |I_I(x_i) - I_J(x_i)| \right\} \\ + \tan \left\{ \frac{\pi}{6} |F_I(x_i) - F_J(x_i)| \right\} \end{array} \right]$$

$$= \frac{2}{\sqrt{3n}} \sum_{i=1}^n \left[\tan \left\{ \frac{\pi}{6} |T_I(x_i) - T_J(x_i)| \right\} + \tan \left\{ \frac{\pi}{6} |I_I(x_i) - I_J(x_i)| \right\} + \tan \left\{ \frac{\pi}{6} |F_I(x_i) - F_J(x_i)| \right\} \right]$$

$$d_{TN\&S}(J, I).$$

Part (iv):

If $I \subseteq J \subseteq K$ then $T_I(x_i) \leq T_J(x_i) \leq T_K(x_i)$, $I_I(x_i) \leq I_J(x_i) \leq I_K(x_i)$ and

$F_I(x_i) \geq F_J(x_i) \geq F_K(x_i)$, $\forall x_i \in X$. This implies to the following inequalities

$$|T_I(x_i) - T_K(x_i)| \geq |T_I(x_i) - T_J(x_i)|, |T_I(x_i) - T_K(x_i)| \geq |T_J(x_i) - T_K(x_i)|$$

$$|I_I(x_i) - I_K(x_i)| \geq |I_I(x_i) - I_J(x_i)|, |I_I(x_i) - I_K(x_i)| \geq |I_J(x_i) - I_K(x_i)|$$

and $|F_I(x_i) - F_K(x_i)| \geq |F_I(x_i) - F_J(x_i)|$, $|F_I(x_i) - F_K(x_i)| \geq |F_J(x_i) - F_K(x_i)|$. From these

inequalities we have

$$\tan \left\{ \frac{\pi}{6} |T_I(x_i) - T_K(x_i)| \right\} + \tan \left\{ \frac{\pi}{6} |I_I(x_i) - I_K(x_i)| \right\} + \tan \left\{ \frac{\pi}{6} |F_I(x_i) - F_K(x_i)| \right\}$$

$$\geq \tan \left\{ \frac{\pi}{6} |T_I(x_i) - T_J(x_i)| \right\} + \tan \left\{ \frac{\pi}{6} |I_I(x_i) - I_J(x_i)| \right\} + \tan \left\{ \frac{\pi}{6} |F_I(x_i) - F_J(x_i)| \right\}$$

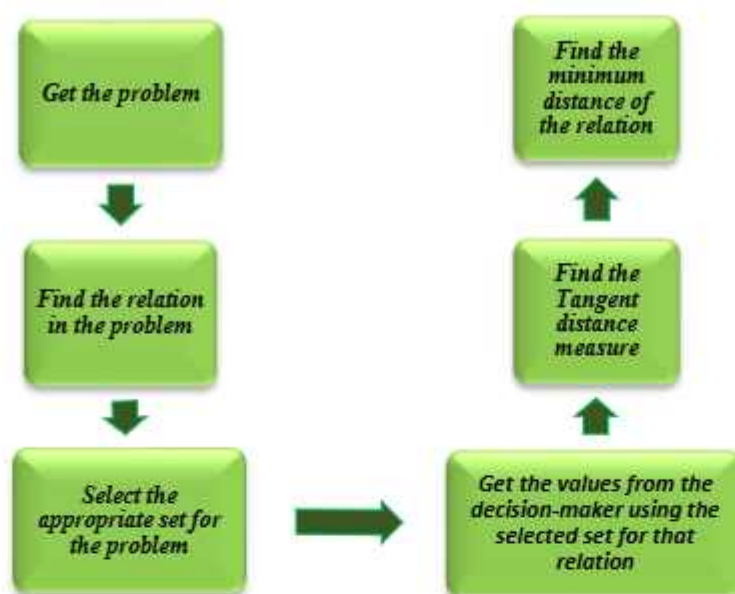
$$\frac{2}{\sqrt{3n}} \sum_{i=1}^n \tan \left\{ \frac{\pi}{6} |T_I(x_i) - T_K(x_i)| \right\} + \tan \left\{ \frac{\pi}{6} |I_I(x_i) - I_K(x_i)| \right\} + \tan \left\{ \frac{\pi}{6} |F_I(x_i) - F_K(x_i)| \right\}$$

$$\geq \frac{2}{\sqrt{3n}} \sum_{i=1}^n \tan \left\{ \frac{\pi}{6} |T_I(x_i) - T_J(x_i)| \right\} + \tan \left\{ \frac{\pi}{6} |I_I(x_i) - I_J(x_i)| \right\} + \tan \left\{ \frac{\pi}{6} |F_I(x_i) - F_J(x_i)| \right\}$$

Hence $d_{TM\Delta S}(I, K) \geq d_{TM\Delta S}(I, J)$ is proven. Likewise $d_{TM\Delta S}(I, K) \geq d_{TM\Delta S}(J, K)$ can prove, and hence the theorem.

4. A Comparative Analysis of Distance Measures

In this section, we first construct Distance Measures using Fermatean Neutrosophic sets and Neutrosophic sets. Further, we have given an illustrative example for defined Distance Measures. So, we consider an example of disease, and their associated symptoms and treatment. We examine six different Distance Measures in this context, which include Fermatean Neutrosophic Set and Neutrosophic sets: Euclidean Distance, Normalised Euclidean Distance, Hamming Distance, Normalised Hamming Distance, SMSVNDM Distance, and TMFNNDM Distance. We aim to find the minimum distance to ensure that patients receive the right treatment. The flowchart for the Distance Measures is presented below:



Step 1: Problem:

Let $S = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}\}$ be the set of patients, $C = \{C_1, C_2, C_3, C_4, C_5\}$

be the set of Treatment Strategies and $R = \{R_1, R_2, R_3, R_4, R_5, R_6\}$ be the set of Symptoms.

Table 4.1. a. shows the information about Treatment Strategies and the Symptoms of patient using Fermatean Neutrosophic Set

	R_1	R_2	R_3	R_4	R_5	R_6
C_1	(0.4,0.8,0.7)	(0.5,0.5,0.9)	(0.3,0.7,0.4)	(0.65,0.65,0.7)	(0.35,0.95,0.9)	(0.7,0.7,0.65)
C_2	(0.3,0.8,0.9)	(0.55,0.55,0.95)	(0.7,0.8,0.3)	(0.6,0.75,0.75)	(0.9,0.8,0.45)	(0.65,0.65,0.7)
C_3	(0.4,0.7,0.7)	(0.4,0.8,0.7)	(0.9,0.7,0.3)	(0.55,0.8,0.8)	(0.85,0.75,0.5)	(0.6,0.75,0.75)
C_4	(0.4,0.6,0.9)	(0.3,0.8,0.9)	(0.7,0.9,0.2)	(0.5,0.85,0.85)	(0.8,0.7,0.55)	(0.55,0.8,0.8)
C_5	(0.3,0.7,0.4)	(0.4,0.7,0.7)	(0.5,0.5,0.9)	(0.45,0.9,0.9)	(0.75,0.75,0.6)	(0.5,0.85,0.85)

Table 4.1. b. shows the information about Treatment Strategies and the Symptoms of patient using Neutrosophic Set

	R_1	R_2	R_3	R_4	R_5	R_6
C_1	(0.4,0.8,0.6)	(0.5,0.5,0.5)	(0.3,0.7,0.7)	(0.65,0.65,0.35)	(0.35,0.95,0.65)	(0.7,0.7,0.3)
C_2	(0.3,0.8,0.7)	(0.55,0.55,0.45)	(0.7,0.8,0.3)	(0.6,0.75,0.4)	(0.9,0.8,0.1)	(0.65,0.65,0.35)
C_3	(0.4,0.7,0.6)	(0.4,0.8,0.6)	(0.9,0.7,0.1)	(0.55,0.8,0.45)	(0.85,0.75,0.15)	(0.6,0.75,0.4)
C_4	(0.4,0.6,0.6)	(0.3,0.8,0.7)	(0.7,0.9,0.3)	(0.5,0.85,0.5)	(0.8,0.7,0.2)	(0.55,0.8,0.45)
C_5	(0.3,0.7,0.7)	(0.4,0.7,0.6)	(0.5,0.5,0.5)	(0.45,0.9,0.55)	(0.75,0.75,0.25)	(0.5,0.85,0.5)

Table 4.2.a. Shows the information about the patient’s Symptoms using Fermatean Neutrosophic Set

	R_1	R_2	R_3	R_4	R_5	R_6
S_1	(0.8,0.7,0.55)	(0.7,0.9,0.65)	(0.6,0.75,0.75)	(0.7,0.8,0.6)	(0.95,0.85,0.4)	(0.4,0.95,0.95)
S_2	(0.75,0.75,0.6)	(0.65,0.9,0.7)	(0.55,0.8,0.8)	(0.85,0.9,0.7)	(0.9,0.8,0.45)	(0.35,0.95,0.9)
S_3	(0.7,0.7,0.65)	(0.6,0.9,0.75)	(0.5,0.85,0.85)	(0.9,0.9,0.3)	(0.85,0.75,0.5)	(0.9,0.8,0.45)
S_4	(0.65,0.65,0.7)	(0.55,0.8,0.8)	(0.45,0.9,0.9)	(0.95,0.95,0.4)	(0.8,0.7,0.55)	(0.85,0.75,0.5)
S_5	(0.6,0.75,0.75)	(0.5,0.85,0.85)	(0.4,0.95,0.95)	(0.9,0.9,0.45)	(0.75,0.65,0.6)	(0.8,0.7,0.55)
S_6	(0.55,0.8,0.8)	(0.45,0.9,0.9)	(0.95,0.85,0.4)	(0.85,0.85,0.5)	(0.7,0.9,0.65)	(0.75,0.75,0.6)
S_7	(0.5,0.85,0.85)	(0.4,0.95,0.95)	(0.9,0.8,0.45)	(0.8,0.8,0.55)	(0.65,0.9,0.7)	(0.7,0.7,0.65)
S_8	(0.45,0.9,0.9)	(0.35,0.95,0.9)	(0.85,0.75,0.5)	(0.75,0.75,0.6)	(0.6,0.9,0.75)	(0.65,0.65,0.7)
S_9	(0.4,0.95,0.95)	(0.9,0.8,0.45)	(0.8,0.7,0.55)	(0.7,0.7,0.65)	(0.55,0.8,0.8)	(0.6,0.75,0.75)
S_{10}	(0.95,0.85,0.4)	(0.85,0.75,0.5)	(0.75,0.65,0.6)	(0.65,0.65,0.7)	(0.5,0.85,0.85)	(0.55,0.8,0.8)

Table 4.2.b. Shows the information about the patient’s Symptoms using Neutrosophic Set

	R_1	R_2	R_3	R_4	R_5	R_6
S_1	(0.8,0.7,0.2)	(0.7,0.9,0.3)	(0.6,0.75,0.4)	(0.7,0.8,0.3)	(0.95,0.85,0.05)	(0.4,0.95,0.6)
S_2	(0.75,0.75,0.25)	(0.65,0.9,0.35)	(0.55,0.8,0.45)	(0.85,0.9,0.15)	(0.9,0.8,0.1)	(0.35,0.95,0.65)
S_3	(0.7,0.7,0.3)	(0.6,0.9,0.4)	(0.5,0.85,0.5)	(0.9,0.9,0.2)	(0.85,0.75,0.15)	(0.9,0.8,0.1)
S_4	(0.65,0.65,0.35)	(0.55,0.8,0.45)	(0.45,0.9,0.55)	(0.95,0.95,0.05)	(0.8,0.7,0.2)	(0.85,0.75,0.15)
S_5	(0.6,0.75,0.4)	(0.5,0.85,0.5)	(0.4,0.95,0.6)	(0.9,0.9,0.1)	(0.75,0.65,0.25)	(0.8,0.7,0.2)
S_6	(0.55,0.8,0.45)	(0.45,0.9,0.55)	(0.95,0.85,0.05)	(0.85,0.85,0.15)	(0.7,0.9,0.3)	(0.75,0.75,0.25)
S_7	(0.5,0.85,0.5)	(0.4,0.95,0.6)	(0.9,0.8,0.1)	(0.8,0.8,0.2)	(0.65,0.9,0.35)	(0.7,0.7,0.3)
S_8	(0.45,0.9,0.55)	(0.35,0.95,0.65)	(0.85,0.75,0.15)	(0.75,0.75,0.25)	(0.6,0.9,0.4)	(0.65,0.65,0.35)
S_9	(0.4,0.95,0.6)	(0.9,0.8,0.1)	(0.8,0.7,0.2)	(0.7,0.7,0.3)	(0.55,0.8,0.45)	(0.6,0.75,0.4)
S_{10}	(0.95,0.85,0.05)	(0.85,0.75,0.15)	(0.75,0.65,0.25)	(0.65,0.65,0.35)	(0.5,0.85,0.5)	(0.55,0.8,0.45)

Step 2: Distance measure:

The ED measure $d_E(I, J)$ determines the shortest distance between each patient's symptoms (Tables 4.2.a and 4.2.b) and treatment strategies (Tables 4.1.a and 4.1.b), with the results presented in Tables 4.3.a and 4.3.b. Here, 'a' represents Fermatean Neutrosophic Sets and 'b' represents Neutrosophic Sets throughout the paper.

Table 4.3.a. $d_E(I, J)$					
	C_1	C_2	C_3	C_4	C_5
S_1	.747774	.543906	.513971	1.814754	.510718
S_2	.736546	.61101	.53929	.6	.500833

S_3	.715309	.653835	.65192	.720532	.691616
S_4	.654472	.607591	.625833	.665207	.62849
S_5	.578072	.597216	.627827	.645497	.606218
S_6	.581664	.423281	.376386	.446281	.677003
S_7	.53929	.414327	.375278	.426224	.643558
S_8	.5058	.422295	.403113	.432049	.623832
S_9	.560506	.527573	.499166	.606905	.639661
S_{10}	.604152	.707696	.6	.728583	.619139

Table 4.3.b. $d_E(I, J)$

	C_1	C_2	C_3	C_4	C_5
S_1	.764308	.558271	.543906	1.624038	.593717
S_2	.744983	.580948	.57735	.564948	.602771
S_3	.685565	.588076	.635085	.63705	.70946
S_4	.608961	.538516	.603462	.570088	.640963
S_5	.498331	.51559	.587367	.526783	.595819
S_6	.695821	.465475	.361709	.467262	.630476
S_7	.63901	.44441	.32914	.423281	.570088
S_8	.587367	.438748	.32914	.403113	.521217
S_9	.595819	.469929	.522813	.618466	.602771

S_{10}	.691014	.701189	.680074	.737111	.754431
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Tables 4.4.a and 4.4.b present the shortest distances between each patient (Tables 4.2.a and 4.2.b) and each treatment strategy (Tables 4.1.a and 4.1.b) calculated using the N-ED Measure $d_{n-E}(I, J)$.

Table 4.4.a. $d_{n-E}(I, J)$					
	C_1	C_2	C_3	C_4	C_5
S_1	.431728	.314024	.296742	1.047749	.294863
S_2	.425245	.352767	.311359	.34641	.289156
S_3	.412984	.377492	.376386	.415999	.399305
S_4	.377859	.350793	.361325	.384057	.362859
S_5	.33375	.344803	.362476	.372678	.35
S_6	.335824	.244381	.217307	.25766	.390868
S_7	.311359	.239212	.216667	.24608	.371558
S_8	.292024	.243812	.232737	.249444	.36017
S_9	.323608	.304594	.288194	.350397	.369309
S_{10}	.348807	.408588	.34641	.420648	.35746

Table 4.4.b. $d_{n-E}(I, J)$					
	C_1	C_2	C_3	C_4	C_5
S_1	.441273	.322318	.314024	.937639	.342783
S_2	.430116	.33541	.333333	.326173	.34801

S_3	.395811	.339526	.366667	.367801	.409607
S_4	.351584	.310913	.348409	.32914	.37006
S_5	.287711	.297676	.339116	.304138	.343996
S_6	.401732	.268742	.208833	.269774	.364005
S_7	.368932	.25658	.190029	.244381	.32914
S_8	.339116	.253311	.190029	.232737	.300925
S_9	.343996	.271314	.301846	.357071	.34801
S_{10}	.398957	.404832	.392641	.425572	.435571

HD Measure $d_H(I, J)$ is applied in Tables 4.5.a and 4.5.b to calculate the shortest distance between each patient (Tables 4.2.a and 4.2.b) and each treatment strategy (Tables 4.1.a and 4.1.b).

	C_1	C_2	C_3	C_4	C_5
S_1	1.55	1.2	1.016667	4.333333	1.05
S_2	1.55	1.166667	1.05	4.466667	0.983333
S_3	1.533333	1.316667	1.166667	1.35	1.233333
S_4	1.366667	1.25	1.133333	1.15	1.166667
S_5	1.216667	1.2	1.183333	1.166667	1.15
S_6	1	0.883333	0.8	0.916667	1.4
S_7	0.916667	0.8	0.783333	0.933333	1.383333

S_8	0.916667	0.766667	0.85	0.933333	1.383333
S_9	1.05	1	0.883333	1.133333	1.35
S_{10}	1.133333	1.35	1.166667	1.416667	1.2

Table 4.5.b $d_H(I, J)$					
	C_1	C_2	C_3	C_4	C_5
S_1	1.583333	1.066667	1.116667	3.65	1.216667
S_2	1.633333	1.116667	1.233333	3.7	1.2
S_3	1.5	1.183333	1.233333	1.25	1.3
S_4	1.316667	1.133333	1.183333	1.066667	1.216667
S_5	1.1	1.116667	1.166667	1.083333	1.133333
S_6	1.25	1	0.75	1	1.283333
S_7	1.1	0.916667	0.633333	0.95	1.2
S_8	1.033333	0.85	0.666667	0.883333	1.166667
S_9	1.066667	0.916667	0.833333	1.083333	1.266667
S_{10}	1.283333	1.266667	1.283333	1.366667	1.483333

Tables 4.6.a and 4.6.b use the N-HD Measure $d_{n-H}(I, J)$ to calculate the shortest distance between each patient (Tables 4.2.a and 4.2.b) and each treatment strategy (Tables 4.1.a and 4.1.b).

Table 4.6.a $d_{n-H}(I, J)$					
	C_1	C_2	C_3	C_4	C_5

S_1	.516667	.4	.338889	1.444444	.35
S_2	.516667	.388889	.35	1.488889	.327778
S_3	.511111	.438889	.388889	.45	.411111
S_4	.455556	.416667	.377778	.383333	.388889
S_5	.405556	.4	.394444	.388889	.383333
S_6	.333333	.294444	.266667	.305556	.466667
S_7	.305556	.266667	.261111	.311111	.461111
S_8	.305556	.255556	.283333	.311111	.461111
S_9	.35	.333333	.294444	.377778	.45
S_{10}	.377778	.45	.388889	.472222	.4

Table 4.6.b $d_{n-H}(I, J)$

	C_1	C_2	C_3	C_4	C_5
S_1	.527778	.355556	.372222	1.216667	.405556
S_2	.544444	.372222	.411111	1.233333	.4
S_3	.5	.394444	.411111	.416667	.433333
S_4	.438889	.377778	.394444	.355556	.405556
S_5	.366667	.372222	.388889	.361111	.377778
S_6	.416667	.333333	.25	.333333	.427778
S_7	.366667	.305556	.211111	.316667	.4

S_8	.344444	.283333	.222222	.294444	.388889
S_9	.355556	.305556	.277778	.361111	.422222
S_{10}	.427778	.422222	.427778	.455556	.494444

SMSVNDM $d_{SMSVN}(I, J)$ is applied in Tables 4.7.a and 4.7.b to calculate the shortest distance between each patient (Tables 4.2.a and 4.2.b) and each treatment strategy (Tables 4.1.a and 4.1.b).

Table 4.7.a $d_{SMSVN}(I, J)$					
	C_1	C_2	C_3	C_4	C_5
S_1	.464307	.379313	.343275	.874075	.355207
S_2	.467474	.367576	.344268	.885766	.337049
S_3	.464007	.411326	.355037	.415623	.385918
S_4	.421581	.397855	.348495	.356958	.369734
S_5	.379576	.385099	.364244	.367209	.368729
S_6	.330478	.309958	.284596	.317735	.426397
S_7	.304578	.281881	.279889	.323697	.429843
S_8	.310954	.262432	.297704	.322444	.435095
S_9	.3364	.324272	.29669	.364477	.423775
S_{10}	.353776	.410054	.371496	.4165	.386503

Table 4.7.b $d_{SMSVN}(I, J)$					
	C_1	C_2	C_3	C_4	C_5

S_1	.469595	.337518	.367908	.803274	.390976
S_2	.490022	.350467	.394929	.808717	.386716
S_3	.458565	.376773	.380913	.393788	.402805
S_4	.411555	.367563	.370281	.342259	.383257
S_5	.354198	.370483	.366593	.356914	.363875
S_6	.378302	.342953	.267461	.342778	.396847
S_7	.33707	.314587	.228695	.329424	.381951
S_8	.327754	.284782	.239657	.308627	.381117
S_9	.330455	.305328	.271538	.345515	.400498
S_{10}	.383377	.382179	.388731	.396423	.437752

The TMFNDM $d_{TMFN}(I, J)$ is used in Tables 4.8.a and 4.8.b to calculate the shortest distance between each patient (Tables 4.2.a and 4.2.b) and each treatment strategy (Tables 4.1.a and 4.1.b).

	C_1	C_2	C_3	C_4	C_5
S_1	.95103	.73298	.61939	2.77556	.63946
S_2	.9496	.71299	.64042	2.87073	.59897
S_3	.93827	.80546	.71558	.8302	.75801
S_4	.83509	.76368	.69464	.70746	.71434
S_5	.74384	.73386	.72561	.71758	.70299

S_6	.6133	.53647	.48535	.55705	.8562
S_7	.56139	.48621	.47516	.56657	.84436
S_8	.56002	.46632	.51589	.56665	.84355
S_9	.64143	.61013	.53876	.6938	.82489
S_{10}	.69333	.82934	.71326	.86955	.7348

Table 4.8.b $d_{TMFN}(I, J)$					
	C_1	C_2	C_3	C_4	C_5
S_1	.48593	.32588	.34009	1.16465	.37139
S_2	.49996	.34084	.37563	1.18268	.36643
S_3	.4583	.36104	.37696	.38222	.3992
S_4	.40142	.34494	.36139	.32555	.37245
S_5	.33515	.33955	.35635	.32983	.3463
S_6	.3849	.30378	.2275	.30384	.39198
S_7	.33817	.27852	.19204	.28827	.36557
S_8	.3168	.25841	.20213	.26798	.35468
S_9	.3265	.27896	.25502	.33261	.38657
S_{10}	.39393	.39009	.3938	.42044	.45638

Comparison:

Six different distance measures have been employed to calculate the distance between each patient and each treatment strategy. The comparison of these distance measures is shown in the following table (Table 4.9.a and Table 4.9.b). In this table, the minimum distance for each measure is

highlighted with a yellow shade, while the Tangent Metric Fermatean Neutrosophic Distance Measure is highlighted with a red shade.

Table 4.9.a Comparison of distance measures					
	C_1	C_2	C_3	C_4	C_5
S_1	0.747774476	0.543906	0.513971	1.814754	0.510718
	0.294863434	0.314024	0.296742	1.047749	0.294863
	1.016666667	1.2	1.016667	4.333333	1.05
	0.338888889	0.4	0.338889	1.444444	0.35
	0.343275464	0.379313	0.343275	0.874075	0.355207
	0.95103165	0.732979	0.619392	2.775565	0.639462
S_2	0.736545993	0.61101	0.53929	0.6	0.500833
	0.28915586	0.352767	0.311359	0.34641	0.289156
	1.166666667	1.166667	1.05	4.466667	0.983333
	0.516666667	0.388889	0.35	1.488889	0.327778
	0.467474217	0.367576	0.344268	0.885766	0.337049
	0.94959805	0.712986	0.640418	2.870727	0.598968
S_3	0.715308791	0.653835	0.65192	0.720532	0.691616
	0.412983723	0.377492	0.376386	0.415999	0.399305
	1.533333333	1.316667	1.166667	1.35	1.233333
	0.511111111	0.438889	0.388889	0.45	0.411111
	0.4640068	0.411326	0.355037	0.415623	0.385918
	0.938269576	0.805462	0.71558	0.830203	0.758013
S_4	0.654471797	0.607591	0.625833	0.665207	0.62849
	0.377859468	0.350793	0.361325	0.384057	0.362859
	1.366666667	1.25	1.133333	1.15	1.166667
	0.455555556	0.416667	0.377778	0.383333	0.388889
	0.421580569	0.397855	0.348495	0.356958	0.369734
	0.835094884	0.763681	0.69464	0.707462	0.714335
S_5	0.578071507	0.597216	0.627827	0.645497	0.606218
	0.33374974	0.344803	0.362476	0.372678	0.35
	1.216666667	1.2	1.183333	1.166667	1.15
	0.405555556	0.4	0.394444	0.388889	0.383333
	0.379576186	0.385099	0.364244	0.367209	0.368729

	0.743837083	0.733858	0.725612	0.717584	0.702993
S_6	0.581664279	0.423281	0.376386	0.446281	0.677003
	0.335824028	0.244381	0.217307	0.25766	0.390868
	1	0.883333	0.8	0.916667	1.4
	0.333333333	0.294444	0.266667	0.305556	0.466667
	0.330477572	0.309958	0.284596	0.317735	0.426397
	0.613302537	0.536471	0.485349	0.55705	0.856196
S_7	0.539289656	0.414327	0.375278	0.426224	0.643558
	0.311359028	0.239212	0.216667	0.24608	0.371558
	0.916666667	0.8	0.783333	0.933333	1.383333
	0.305555556	0.266667	0.261111	0.311111	0.461111
	0.304578182	0.281881	0.279889	0.323697	0.429843
	0.561392765	0.486208	0.475156	0.566565	0.844364
S_8	0.505799697	0.422295	0.403113	0.432049	0.623832
	0.292023591	0.243812	0.232737	0.249444	0.36017
	0.916666667	0.766667	0.85	0.933333	1.383333
	0.305555556	0.255556	0.283333	0.311111	0.461111
	0.310954344	0.262432	0.297704	0.322444	0.435095
	0.560016442	0.466317	0.515891	0.566649	0.843551
S_9	0.560505724	0.527573	0.499166	0.606905	0.639661
	0.323608131	0.304594	0.288194	0.350397	0.369309
	1.05	1	0.883333	1.133333	1.35
	0.35	0.333333	0.294444	0.377778	0.45
	0.336400426	0.324272	0.29669	0.364477	0.423775
	0.64143395	0.61013	0.538757	0.693804	0.824894
S_{10}	0.604152299	0.707696	0.6	0.728583	0.619139
	0.348807492	0.408588	0.34641	0.420648	0.35746
	1.133333333	1.35	1.166667	1.416667	1.2
	0.377777778	0.45	0.388889	0.472222	0.4
	0.353775568	0.410054	0.371496	0.4165	0.386503
	0.693329647	0.82934	0.713259	0.869545	0.734797

Table 4.9.b Comparison of distance measures

	C_1	C_2	C_3	C_4	C_5

S_1	0.764307966	0.558271	0.543906	1.624038	0.593717
	0.441273	0.322318	0.314024	0.937639	0.342783
	1.583333	1.066667	1.116667	3.65	1.216667
	0.527778	0.355556	0.372222	1.216667	0.405556
	0.469595	0.337518	0.367908	0.803274	0.390976
	0.48593	0.32588	0.34009	1.16465	0.37139
S_2	0.744983	0.580948	0.57735	0.564948	0.602771
	0.430116	0.33541	0.333333	0.326173	0.34801
	1.633333	1.116667	1.233333	3.7	1.2
	0.544444	0.372222	0.411111	1.233333	0.4
	0.490022	0.350467	0.394929	0.808717	0.386716
	0.49996	0.34084	0.37563	1.18268	0.36643
S_3	0.685565	0.588076	0.635085	0.63705	0.70946
	0.395811	0.339526	0.366667	0.367801	0.409607
	1.5	1.183333	1.233333	1.25	1.3
	0.5	0.394444	0.411111	0.416667	0.433333
	0.458565	0.376773	0.380913	0.393788	0.402805
	0.4583	0.36104	0.37696	0.38222	0.3992
S_4	0.608961	0.538516	0.603462	0.570088	0.640963
	0.351584	0.310913	0.348409	0.32914	0.37006
	1.316667	1.133333	1.183333	1.066667	1.216667
	0.438889	0.377778	0.394444	0.355556	0.405556
	0.411555	0.367563	0.370281	0.342259	0.383257
	0.40142	0.34494	0.36139	0.32555	0.37245
S_5	0.498331	0.51559	0.587367	0.526783	0.595819
	0.287711	0.297676	0.339116	0.304138	0.343996
	1.1	1.116667	1.166667	1.083333	1.133333
	0.366667	0.372222	0.388889	0.361111	0.377778
	0.354198	0.370483	0.366593	0.356914	0.363875
	0.33515	0.33955	0.35635	0.32983	0.3463
S_6	0.695821	0.465475	0.361709	0.467262	0.630476
	0.401732	0.268742	0.208833	0.269774	0.364005
	1.25	1	0.75	1	1.283333
	0.416667	0.333333	0.25	0.333333	0.427778
	0.378302	0.342953	0.267461	0.342778	0.396847

	0.3849	0.30378	0.2275	0.30384	0.39198
S_7	0.63901	0.44441	0.32914	0.423281	0.570088
	0.368932	0.25658	0.190029	0.244381	0.32914
	1.1	0.916667	0.633333	0.95	1.2
	0.366667	0.305556	0.211111	0.316667	0.4
	0.33707	0.314587	0.228695	0.329424	0.381951
	0.33817	0.27852	0.19204	0.28827	0.36557
S_8	0.587367	0.438748	0.32914	0.403113	0.521217
	0.339116	0.253311	0.190029	0.232737	0.300925
	1.033333	0.85	0.666667	0.883333	1.166667
	0.344444	0.283333	0.222222	0.294444	0.388889
	0.327754	0.284782	0.239657	0.308627	0.381117
	0.3168	0.25841	0.20213	0.26798	0.35468
S_9	0.595819	0.469929	0.522813	0.618466	0.602771
	0.343996	0.271314	0.301846	0.357071	0.34801
	1.066667	0.916667	0.833333	1.083333	1.266667
	0.355556	0.305556	0.277778	0.361111	0.422222
	0.330455	0.305328	0.271538	0.345515	0.400498
	0.3265	0.27896	0.25502	0.33261	0.38657
S_{10}	0.691014	0.701189	0.680074	0.737111	0.754431
	0.398957	0.404832	0.392641	0.425572	0.435571
	1.283333	1.266667	1.283333	1.366667	1.483333
	0.427778	0.422222	0.427778	0.455556	0.494444
	0.383377	0.382179	0.388731	0.396423	0.437752
	0.39393	0.39009	0.3938	0.42044	0.45638

Step 3: Comparative result

The consolidated results are presented in Table 4.9.a and Table 4.9.b, and further summarized in Table 4.10.a and Table 4.10.b. The shortest distance values for each patient and treatment strategy are shown in Table 4.10 below. Therefore, the $d_{TMFN}(I, J)$ offers the most consistent and straightforward solution for measuring distance using Fermatean neutrosophic set values and neutrosophic set values.

Table 4.10.a Comparison (Fermatean neutrosophic set)
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	$d_E(I, J)$	$d_{n-E}(I, J)$	$d_H(I, J)$	$d_{n-H}(I, J)$	$d_{SMSTN}(I, J)$	$d_{TMFN}(I, J)$
S_1	C5	C5	C3	C1&C3	C1&C3	C3
S_2	C5	C1&C5	C5	C5	C5	C5
S_3	C3	C3	C3	C3	C3	C3
S_4	C2	C2	C3	C3	C3	C3
S_5	C1	C1	C5	C5	C3	C5
S_6	C3	C3	C3	C3	C3	C3
S_7	C3	C3	C3	C3	C3	C3
S_8	C3	C3	C2	C2	C2	C2
S_9	C3	C3	C3	C3	C3	C3
S_{10}	C3	C3	C5	C1	C1	C1

Table 4.10.b Comparison (Neutrosophic set)

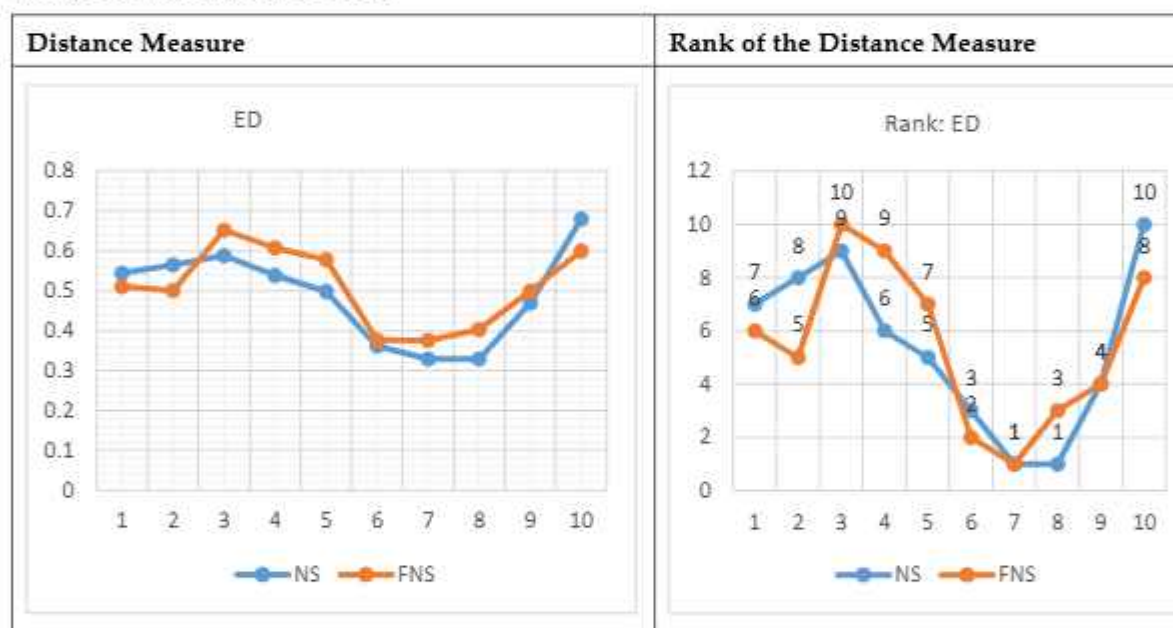
	$d_E(I, J)$	$d_{n-E}(I, J)$	$d_H(I, J)$	$d_{n-H}(I, J)$	$d_{SMSTN}(I, J)$	$d_{TMFN}(I, J)$
S_1	C3	C3	C2	C2	C2	C2
S_2	C4	C4	C2	C2	C2	C2
S_3	C2	C2	C2	C2	C2	C2
S_4	C2	C2	C4	C4	C4	C4
S_5	C2	C1	C4	C4	C1	C4
S_6	C3	C3	C3	C3	C3	C3
S_7	C3	C3	C3	C3	C3	C3

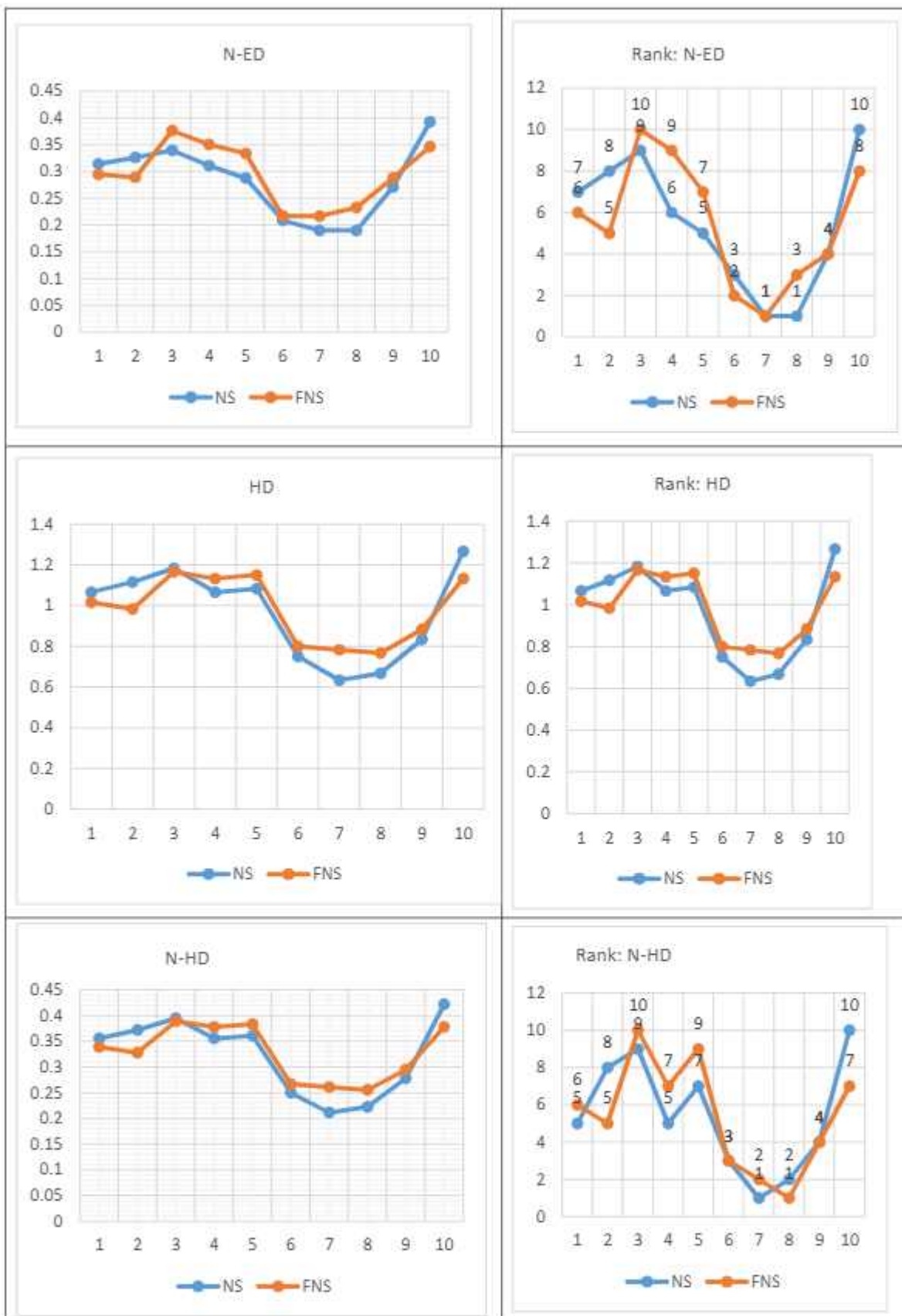
S_8	C3	C3	C3	C3	C3	C3
S_9	C2	C2	C3	C3	C3	C3
S_{10}	C3	C3	C2	C2	C2	C2

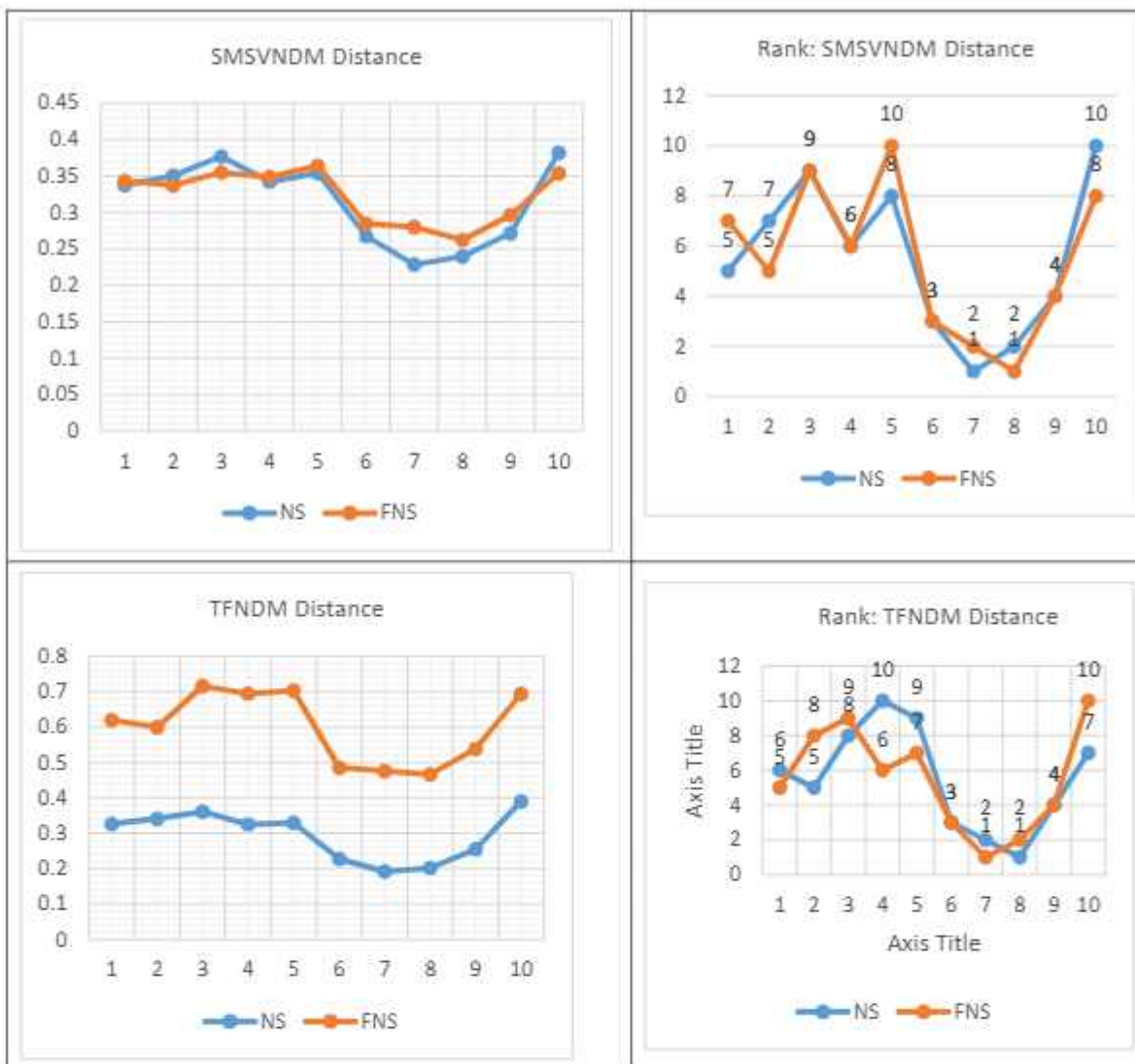
The above table shows that the most frequently occurring shortest distance is found in the TMFNMDM. Hence TMFNMDM generalized case of other six distance measure.

Step 4: Graphical result

In this step, we compare the results using the FNS and NS values. Also, we present graphical representations of six distinct distance measurements comparing each patient to each Treatment Strategy given in the model example. Additionally, we provide graphs below illustrating rankings using FNS and NS. Top of Form







In the graphical representation, we observed a difference in ranking between the Neutrosophic and Fermatean Neutrosophic Sets. The Fermatean Neutrosophic Set permits higher membership degree values, resulting in comparatively higher false values, which influence the outcome.

Conclusion

This study presents a novel distance measure tailored for FNSs, with the TMFNDM serving as a generalized case. A comparative analysis showed that the graphical rankings differ between neutrosophic and Fermatean neutrosophic sets, primarily due to the latter's allowance for higher membership degree values. Fermatean Neutrosophic Set values lead to comparatively higher false values, significantly influencing the results. The findings emphasize that selecting an appropriate set is crucial for achieving optimal solutions in scenarios requiring high membership values. The Tangent Metric Fermatean Neutrosophic Distance Measure provides superior outcomes in such cases. Fields like medicine, research, and disaster management often involve high membership values, making this distance measure particularly relevant and effective.

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Neutrosophic Over Topological Spaces

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ABSTRACT: The concepts of Pythagorean and Fermatean sets are studied in a Neutrosophic environment and their topological structures are developed. The article also presents the idea of neutrosophic gradation of openness on neutrosophic subsets as well as the definition of Topological settings of Neutrosophic variants with examples.

Key Words: Pythagorean Neutrosophic Set (PNS); Fermatean Neutrosophic Set (FNS); Pythagorean Neutrosophic Topological Space (PNTS); Fermatean Neutrosophic Topological Space (FNTS); Neutrosophic gradation of openness ($N - go$);

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1. Introduction

Zadeh proposed the concept of a Fuzzy Set (FS) [18] in 1965 and Chang proposed fuzzy topology [3] in 1968. Atanassov [2] proposed Intuitionistic Fuzzy Sets (IFSs) as a generalization of FSs. Coker proposed the concept of intuitionistic fuzzy topological spaces (IFTS) [4] in 1997 which opened an important area of research in IFSs. Coker has investigated its different properties and defined the concept of IFTS with reference to Chang's concept.

Lowen developed the structure of fuzzy topological spaces [13]. He modified Chang's definition of fuzzy topology. He also introduced two functions, that will allow us to see the connection between fuzzy topological spaces and general topological spaces more clearly. After classifying FSs and topological spaces, I. Zahan and R. Nasrin [19] created relationships between the elements that compose them and also developed the relation between fuzzy sets and topological spaces.

An IFTS is defined by Tapas Kumar Mondal and S.K. Samanta [17] together with a knowledge of the intuitionistic grading of openness on nonempty fuzzy subsets. Lee, S. J. and Lee, E. P. (2000) [12] explored intuitionistic fuzzy topological spaces

Smarandache added indeterminacy to the notion of intuitionistic fuzzy and introduced the concept of Neutrosophic Set (NS) as a new tool to the world of mathematics in [15], [8]. Also, Smarandache (2024) [9] expanded on revolutionary topologies and neutrosophic systems, offering key insights into handling uncertainty. Salama and Alblowi [14] explored the topological structure of the family of NSs and introduced the idea of Neutrosophic Topological Space (NTS) by using non-membership, indeterminacy, and membership functions all of which have a one-to-one correspondence between the members of a set and $-|0, 1|+$. As a more sophisticated tool than generic topological spaces, this new kind of topological space

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has been embraced. Scientists have been drawn to the concept of NT and a lot has been developed, still there are many subareas to be investigated.

Pythagorean Fuzzy sets(PFSs) and NSs are used in Pythagorean neutrosophic topology [11] to address vagueness and uncertainty in topological spaces. This approach offers a more thorough framework for defining and analyzing incomplete or uncertain information inside topological structures [1]. Fermatean neutrosophic topology's [10] main aim is to expand on classical topology by using the concepts of neutrosophic sets to examine spaces where unclear or indeterminacy is usual. It means developing new concepts, tools, and methods to handle unclear or incomplete data within the topological domain [16].

In many situations in our real world, the degree of membership is both more than and less than 1 [5]. A NS, which contains elements with a membership of more than 1 is called an Overset, introduced and studied by Smarandache. The set which contains elements whose membership concerning a set is less than 0, is known as Underset. A set that has elements with memberships above 1 and below 0 is referred to as being offset and underset, [6]- [7] that is, certain components have memberships that fall outside of the range $[0, 1]$.

In this framework, the concepts of Pythagorean, and Fermatean Neutrosophic Sets have been studied. The ideas of PNTSSs, and FNTSSs are investigated and their basic properties are examined. The notion of a NTS and the concept of $(N - go)$ on neutrosophic subsets of a nonempty set are presented.

2. Preliminaries

Definition 2.1 [15] *Each element in a NS has degrees of truth, indeterminacy, and falsity associated with it. These degrees can be represented by one of three membership functions: the truth(T), indeterminacy(I), and falsity(F) membership functions. T , I , and F added up equals 3. These functions indicate, respectively, how much an element is part of the set, how ambiguous its membership is, or how much it does not belong to the set.*

Definition 2.2 [8] *If $G = \{\langle n, T_G(n), I_G(n), F_G(n) \rangle | n \in N\}$ and $H = \{\langle n, T_H(n), I_H(n), F_H(n) \rangle | n \in X\}$ are any two NSs of N then*

1. $(G) \subseteq (H)$ iff $T_G(n) \leq T_H(n)$; $I_G(n) \leq I_H(n)$; and $F_G(n) \geq F_H(n)$
2. $(G) = (B)$ iff $T_G(n) = T_H(n)$; $I_G(n) = I_H(n)$; and $F_G(n) = F_H(n) \forall n \in N$
3. $G = \{\langle n, F_G(n), 1 - I_G(n), T_G(n) \rangle | n \in N\}$
4. $G \cap H = \{\langle n, T_{G \cap H}(n), I_{G \cap H}(n), F_{G \cap H}(n) \rangle | n \in N\}$ where,
 - $T_{G \cap H}(n) = \min\{T_G(n), T_H(n)\}$
 - $I_{G \cap H}(n) = \min\{I_G(n), I_H(n)\}$
 - $F_{G \cap H}(n) = \max\{F_G(n), F_H(n)\}$
5. $G \cup H = \{\langle n, T_{G \cup H}(n), I_{G \cup H}(n), F_{G \cup H}(n) \rangle | n \in N\}$ where,
 - (a) $T_{G \cup H}(n) = \max\{T_G(n), T_H(n)\}$
 - (b) $I_{G \cup H}(n) = \max\{I_G(n), I_H(n)\}$
 - (c) $F_{G \cup H}(n) = \min\{F_G(n), F_H(n)\}$
6. Let $G = \{\langle n, T_G(n), I_G(n), F_G(n) \rangle | n \in N\}$ Neutrosophic Set on N , then the complement of the set G ($C(G)$ for short), defined as

$$C(G) = \{\langle n, F_G(n), 1 - I_G(n), T_G(n) \rangle | n \in N\}$$

Definition 2.3 [7] *Let E be a universe of discourse and the NS $N_1 \subset E$. Let $T_N(n), I_N(n), F_N(n)$ membership, indeterminate and non- membership function respectively, a element $n \in E$,*

$$A_1 : T_N(n), I_n(x), F_N(n) : E \rightarrow [0, \Omega]$$

where $0 < 1 < \Omega$ and Ω is called over limit.

A Single-Valued Neutrosophic Overset N_1 is defined as: $N_1 = \{(n, \langle T_N(n), I_N(n), F_N(n) \rangle), x \in E\}$ such that there exists at least one element in N_1 that includes of at least one neutrosophic element > 1 and there are no elements with neutrosophic components that are < 0 .

3. Neutrosophic Set and its Variants

Pythagorean Neutrosophic Set

Definition 3.1 Consider P be a not empty set. A PNS in A_P is defined

$$A_P = \{\langle p : T_{A_P}(p), I_{A_P}(p), F_{A_P}(p) \rangle | p \in P\}$$

where T_{A_P} is truth, I_{A_P} is indeterminate, F_{A_P} is false degree membership. where

$$T_{A_P}(p), I_{A_P}(p), F_{A_P}(p) | p \in P \rightarrow [0, 1],$$

$$0 \leq [T_{A_P}(p)]^2 + [I_{A_P}(p)]^2 + [F_{A_P}(p)]^2 \leq \sqrt{3}.$$

Definition 3.2 Consider P be a non-empty set and the PNSs A_p and B_p be in the form

$$A_p = \{\langle p, T_{A_p}(p), I_{A_p}(p), F_{A_p}(p) \rangle | p \in P\}$$

and $B_p = \{\langle p, T_{B_p}(p), I_{B_p}(p), F_{B_p}(p) \rangle | p \in P\}$

1. $(A_p) \subseteq (B_p)$ iff $T_{A_p}(p) \leq T_{B_p}(p)$; $I_{A_p}(p) \geq I_{B_p}(p)$; and $F_{A_p}(p) \geq F_{B_p}(p)$
2. $(A_p) = (B_p)$ iff $T_{A_p}(p) = T_{B_p}(p)$; $I_{A_p}(p) = I_{B_p}(p)$; and $F_{A_p}(p) = F_{B_p}(p) \forall p \in P$
3. $A_p \cap B_p = \{\langle p, T_{A_p \cap B_p}(p), I_{A_p \cap B_p}(p), F_{A_p \cap B_p}(p) \rangle | p \in P\}$ where,
 - (a) $T_{A_p \cap B_p}(p) = \min\{T_{A_p}(p), T_{B_p}(p)\}$
 - (b) $I_{A_p \cap B_p}(p) = \max\{I_{A_p}(p), I_{B_p}(p)\}$
 - (c) $F_{A_p \cap B_p}(p) = \max\{F_{A_p}(p), F_{B_p}(p)\}$
4. $A_p \cup B_p = \{\langle p, T_{A_p \cup B_p}(p), I_{A_p \cup B_p}(p), F_{A_p \cup B_p}(p) \rangle | p \in P\}$ where,
 - (a) $T_{A_p \cup B_p}(p) = \max\{T_{A_p}(p), T_{B_p}(p)\}$
 - (b) $I_{A_p \cup B_p}(p) = \min\{I_{A_p}(p), I_{B_p}(p)\}$
 - (c) $F_{A_p \cup B_p}(p) = \min\{F_{A_p}(p), F_{B_p}(p)\}$

Definition 3.3 Let $A_p = \{\langle p, T_{A_p}(p), I_{A_p}(p), F_{A_p}(p) \rangle | p \in P\}$ PNS on P , then the complement of the set A_p ($C(A_p)$), defined as

- (C1) $C(A_p) = \{\langle p, F_{A_p}(p), 1 - I_{A_p}(p), T_{A_p}(p) \rangle | p \in P\}$
- (C2) $C(A_p) = \{\langle p, F_{A_p}(p), (T_{A_p}(p) + F_{A_p}(p)) - I_{A_p}(p), T_{A_p}(p) \rangle | p \in P\}$

Definition 3.4 The PNS 0_{PN} and 1_{PN} in P are outlined below:

0_{PN} it can be described as::

$$(0_1) \quad 0_{PN} = \{\langle p, 0, 0, 1 \rangle : p \in P\}$$

$$(0_2) \quad 0_{PN} = \{\langle p, 0, 0, 0 \rangle : p \in P\}$$

1_{PN} it can be described as:

$$(1_1) \quad 1_{PN} = \{\langle p, 1, 0, 0 \rangle : p \in P\}$$

$$(1_2) \quad 1_{PN} = \{\langle p, 0, 1, 0 \rangle : p \in P\}$$

In this case, the integers 0 and 1 belong to the constant mappings that translate each element of P to 1 and 0.

Fermatean Neutrosophic set

Definition 3.5 Consider a set F that is not empty. A Fermatean Neutrosophic set (FNS) in A_F is clarified $A_F = \{\langle f, (T_{A_F}(f), I_{A_F}(f), F_{A_F}(f)) \rangle | f \in F\}$ where T_{A_F} is truth, I_{A_F} is indeterminate, F_{A_F} is false degree membership. Where, $T_{A_F}(f), I_{A_F}(f), F_{A_F}(f) | f \in F \rightarrow [0, 1]$ and $0 \leq T_{A_F}^3(f) + I_{A_F}^3(f) + F_{A_F}^3(f) \leq \sqrt{3} \forall f \in F$.

Definition 3.6 Consider a set F is a non-empty set and the FNSs A_F and B_F is clarified $A_F = \{\langle f, (T_{A_F}(f), I_{A_F}(f), F_{A_F}(f)) \rangle | f \in F\}$ and $B_F = \{\langle f, (T_{B_F}(f), I_{B_F}(f), F_{B_F}(f)) \rangle | f \in F\}$

1. $(A_F) \subseteq (B_F)$ iff $T_{A_F}(f) \leq T_{B_F}(f); I_{A_F}(f) \geq I_{B_F}(f)$; and $F_{A_F}(f) \geq F_{B_F}(f)$
2. $(A_F) = (B_F)$ iff $T_{A_F}(f) = T_{B_F}(f); I_{A_F}(f) = I_{B_F}(f)$; and $F_{A_F}(f) = F_{B_F}(f) \forall f \in F$
3. $A_F \cap B_F = \{\langle f, (T_{A_F \cap B_F}(f), I_{A_F \cap B_F}(f), F_{A_F \cap B_F}(f)) \rangle | f \in F\}$ where,
 - (a) $T_{A_F \cap B_F}(f) = \min\{T_{A_F}(f), T_{B_F}(f)\}$
 - (b) $I_{A_F \cap B_F}(f) = \max\{I_{A_F}(f), I_{B_F}(f)\}$
 - (c) $F_{A_F \cap B_F}(f) = \max\{F_{A_F}(f), F_{B_F}(f)\}$
4. $A_F \cup B_F = \{\langle f, (T_{A_F \cup B_F}(f), I_{A_F \cup B_F}(f), F_{A_F \cup B_F}(f)) \rangle | f \in F\}$ where,
 - (a) $T_{A_F \cup B_F}(f) = \max\{T_{A_F}(f), T_{B_F}(f)\}$
 - (b) $I_{A_F \cup B_F}(f) = \min\{I_{A_F}(f), I_{B_F}(f)\}$
 - (c) $F_{A_F \cup B_F}(f) = \min\{F_{A_F}(f), F_{B_F}(f)\}$

Definition 3.7 Let $A_F = \{\langle f, (T_{A_F}(f), I_{A_F}(f), F_{A_F}(f)) \rangle | f \in F\}$ Fermatean Neutrosophic Set on X , then the complement of the set A_F ($C(A_F)$), defined as

- (C₁) $C(A_F) = \{\langle f, (1 - T_{A_F}(f), I_{A_F}(f), 1 - F_{A_F}(f)) \rangle | f \in F\}$
- (C₂) $C(A_F) = \{\langle f, (F_{A_F}(f), I_{A_F}(f), T_{A_F}(f)) \rangle | f \in F\}$
- (C₃) $C(A_F) = \{\langle f, (F_{A_F}(f), 1 - I_{A_F}(f), T_{A_F}(f)) \rangle | f \in F\}$

Definition 3.8 The FNS 0_{FN} and 1_{FN} in F are outlined below:
 0_{FN} may be defined as:

- (0₁) $0_{FN} = \{\langle f, (0, 0, 1) \rangle : f \in F\}$
- (0₂) $0_{FN} = \{\langle f, (0, 0, 0) \rangle : f \in F\}$

1_{FN} may be defined as:

- (1₁) $1_{FN} = \{\langle f, (1, 0, 0) \rangle : f \in F\}$
- (1₂) $1_{FN} = \{\langle f, (0, 1, 0) \rangle : f \in F\}$

In this case, the integers 0 and 1 belong to the constant mappings that translate each element of F to 1 and 0.

4. Neutrosophic gradation of openness: Neutrosophic over set

Definition 4.1 Consider a Neutrosophic Over set (NOS) X is non-empty. A $(N - go)$ of neutrosophic over subsets of X , is a triplet $(\tau_{over}^T, \tau_{over}^I, \tau_{over}^F)$ of functions $\tau_{over}^T, \tau_{over}^I, \tau_{over}^F : I^X \rightarrow I$ such that

1. $\tau_{over}^T(\lambda) + \tau_{over}^I(\lambda) + \tau_{over}^F(\lambda) \leq 3, \forall \lambda \in I^X$
2. $\tau_{over}^T(0) = \tau_{over}^T(\Omega) = 1, \tau_{over}^I(0) = \tau_{over}^I(\Omega) = 1, \tau_{over}^F(0) = \tau_{over}^F(\Omega) = 0$
 $\tau_{over}^T(\lambda_1 \cap \lambda_2) \geq \tau_{over}^T(\lambda_1) \wedge \tau_{over}^T(\lambda_2)$,
 $\tau_{over}^I(\lambda_1 \cap \lambda_2) \geq \tau_{over}^I(\lambda_1) \wedge \tau_{over}^I(\lambda_2)$

$$3. \tau_{\text{over}}^F(\lambda_1 \cap \lambda_2) \leq \tau_{\text{over}}^F(\lambda_1) \vee \tau_{\text{over}}^F(\lambda_2), \lambda_i \in I^X, i = 1, 2$$

$$4. \tau_{\text{Over}}^T\left(\bigcup_{n \in \Delta} \lambda_i\right) \geq \bigwedge_{n \in \Delta} \tau_{\text{over}}^T(\lambda_i)$$

$$\tau_{\text{Over}}^I\left(\bigcup_{n \in \Delta} \lambda_i\right) \geq \bigwedge_{n \in \Delta} \tau_{\text{over}}^I(\lambda_i),$$

$$\tau_{\text{Over}}^F\left(\bigcup_{n \in \Delta} \lambda_i\right) \leq \bigvee_{i \in \Delta} \tau_{\text{over}}^F(\lambda_i), \lambda_i \in I^X, i \in \Delta.$$

Where $\tau_{\text{Over}}^T, \tau_{\text{Over}}^I$ and τ_{Over}^F are an independent component. The triplet $(\tau_{\text{Over}}^T, \tau_{\text{Over}}^I, \tau_{\text{Over}}^F)$ is a $(N-go)$ on X . Then the collections $(X, \tau_{\text{Over}}^T, \tau_{\text{Over}}^I, \tau_{\text{Over}}^F)$ is recognised as a NOTS.

Similarly can define the concepts of PNGO, and FNGO.

Example 4.1 Let $X = \{a, b, c\}$ be a nonempty Neutrosophic Over set. A NGO of neutrosophic over subsets of X

$$X_{N_{\text{over}}} = \left\{ \left\langle x, \left(\frac{a}{1.3}, \frac{b}{0.4}, \frac{c}{0.6} \right), \left(\frac{a}{0.4}, \frac{b}{1.2}, \frac{c}{0.5} \right), \left(\frac{a}{0.6}, \frac{b}{0.5}, \frac{c}{0.2} \right) \right\rangle, \right. \\ \left. \left\langle y, \left(\frac{a}{0.7}, \frac{b}{0.3}, \frac{c}{0.8} \right), \left(\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{0.4} \right), \left(\frac{a}{0.5}, \frac{b}{0.8}, \frac{c}{1.1} \right) \right\rangle : x, y \in [0, \Omega] \right\}$$

$$\tau_{N_{\text{over}}} = \left\{ \left\langle \left(\frac{a}{1.3}, \frac{b}{0.4}, \frac{c}{0.6} \right), \left(\frac{a}{0.4}, \frac{b}{1.2}, \frac{c}{0.5} \right), \left(\frac{a}{0.6}, \frac{b}{0.5}, \frac{c}{0.2} \right) \right\rangle \right. \\ \left. \left\langle \left(\frac{a}{0.7}, \frac{b}{0.3}, \frac{c}{0.8} \right), \left(\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{0.4} \right), \left(\frac{a}{0.5}, \frac{b}{0.8}, \frac{c}{1.1} \right) \right\rangle \right. \\ \left. \left\langle \left(\frac{a}{0.7}, \frac{b}{0.3}, \frac{c}{0.6} \right), \left(\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{0.4} \right), \left(\frac{a}{0.5}, \frac{b}{0.8}, \frac{c}{0.2} \right) \right\rangle \right. \\ \left. \left\langle \left(\frac{a}{1.3}, \frac{b}{0.4}, \frac{c}{0.8} \right), \left(\frac{a}{0.4}, \frac{b}{1.2}, \frac{c}{0.5} \right), \left(\frac{a}{0.6}, \frac{b}{0.8}, \frac{c}{1.1} \right) \right\rangle : x, y \in [0, \Omega] \right\}$$

Gradation of Openness

$$\tau_{N_{\text{over}}}^{T_1} \left(\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{1.1} \right) = 0.6$$

$$\tau_{N_{\text{over}}}^{I_1} \left(\frac{a}{1.3}, \frac{b}{0.7}, \frac{c}{0.2} \right) = 0.5$$

$$\tau_{N_{\text{over}}}^{F_1} \left(\frac{a}{0.1}, \frac{b}{1.2}, \frac{c}{0.4} \right) = 0.7$$

$$\tau_{N_{\text{over}}}^{T_2} \left(\frac{a}{0.7}, \frac{b}{0.3}, \frac{c}{0.6} \right) = 0.9$$

$$\tau_{N_{\text{over}}}^{I_2} \left(\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{0.4} \right) = 0.8$$

$$\tau_{N_{\text{over}}}^{F_2} \left(\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{1.1} \right) = 0.8$$

$$1. \tau_{N_{\text{over}}}^{T_1}(\lambda) + \tau_{N_{\text{over}}}^{I_1}(\lambda) + \tau_{N_{\text{over}}}^{F_1}(\lambda) \leq 3, \forall \lambda \in I^X : X \rightarrow [0, \Omega]$$

$$0.6 + 0.5 + 0.7 \leq 3$$

$$1.8 \leq 3$$

$$\tau_{N_{\text{over}}}^{T_2}(\lambda) + \tau_{N_{\text{over}}}^{I_2}(\lambda) + \tau_{N_{\text{over}}}^{F_2}(\lambda) \leq 3, \forall \lambda \in I^X : X \rightarrow [0, \Omega]$$

$$0.9 + 0.8 + 0.8 \leq 3$$

$$2.5 \leq 3$$

$$2. \tau_{\text{over}}^T(0) = \tau_{\text{over}}^T(\Omega) = 1, \tau_{\text{over}}^I(0) = \tau_{\text{over}}^I(\Omega) = 1, \tau_{\text{over}}^F(0) = \tau_{\text{over}}^F(\Omega) = 0$$

$$3. \tau_{N_{\text{over}}}^T(\lambda_1 \cap \lambda_2) \geq \tau_{N_{\text{over}}}^T(\lambda_1) \wedge \tau_{N_{\text{over}}}^T(\lambda_2), \lambda_i \in I^X, i = 1, 2$$

$$\tau_{N_{\text{over}}}^T \left(\frac{a}{0.7}, \frac{b}{0.3}, \frac{c}{1.1} \right) \Rightarrow 0.8 \geq 0.6$$

$$\tau_{N_{\text{over}}}^I(\lambda_1 \cap \lambda_2) \geq \tau_{N_{\text{over}}}^I(\lambda_1) \wedge \tau_{N_{\text{over}}}^I(\lambda_2), \lambda_i \in I^X, i = 1, 2$$

$$\tau_{N_{over}}^I \left(\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{0.4} \right) \Rightarrow 1 \geq 0.5$$

$$\tau_{N_{over}}^F (\lambda_1 \cap \lambda_2) \leq \tau_{N_{over}}^F (\lambda_1) \vee \tau_{N_{over}}^F (\lambda_2), \lambda_i \in I^X, i = 1, 2$$

$$\tau_{N_{over}}^F \left(\frac{a}{0.5}, \frac{b}{1.2}, \frac{c}{0.4} \right) \Rightarrow 0.5 \leq 0.7$$

4.

$$\tau_{N_{over}}^I \left(\frac{a}{1.4}, \frac{b}{0.6}, \frac{c}{0.8} \right) \Rightarrow 0.8 \geq 0.6$$

$$\tau_{N_{over}}^I \left(\bigcup_{i \in \Delta} \lambda_i \right) \geq \bigwedge_{i \in \Delta} \tau_{N_{over}}^I (\lambda_i), \lambda_i \in I^X, i \in \Delta$$

$$\tau_{N_{over}}^I \left(\frac{a}{0.8}, \frac{b}{1.1}, \frac{c}{0.4} \right) \Rightarrow 0.7 \geq 0.5$$

$$\tau_{N_{over}}^F \left(\bigcup_{i \in \Delta} \lambda_i \right) \leq \bigvee_{i \in \Delta} \tau_{N_{over}}^F (\lambda_i), \lambda_i \in I^X, i \in \Delta.$$

$$\tau_{N_{over}}^F \left(\frac{a}{0.6}, \frac{b}{0.7}, \frac{c}{1.1} \right) \Rightarrow 0.9 \geq 0.7$$

Definition 4.2 Consider Neutrosophic Over set X is nonempty and $F, F^* : I^X \rightarrow I$ be two mappings satisfying

$$1. F_{over}^T(\lambda) + F_{over}^I(\lambda) + F_{over}^F(\lambda) \leq 3, \forall \lambda \in I^X$$

$$2. F_{over}^T(0) = F_{over}^T(\Omega) = 1, F_{over}^I(0) = F_{over}^I(\Omega) = 1, F_{over}^F(0) = F_{over}^F(\Omega) = 0$$

$$3. F_{over}^T(\lambda_1 \cup \lambda_2) \geq F_{over}^T(\lambda_1) \wedge F_{over}^T(\lambda_2),$$

$$F_{over}^I(\lambda_1 \cup \lambda_2) \geq F_{over}^I(\lambda_1) \wedge F_{over}^I(\lambda_2)$$

$$F_{over}^F(\lambda_1 \cup \lambda_2) \leq F_{over}^F(\lambda_1) \vee F_{over}^F(\lambda_2), \lambda_i \in I^X, i = 1, 2$$

$$4. F_{Over}^T \left(\bigcap_{i \in \Delta} \lambda_i \right) \geq \bigwedge_{i \in \Delta} F_{Over}^T(\lambda_i),$$

$$F_{Over}^I \left(\bigcap_{i \in \Delta} \lambda_i \right) \geq \bigwedge_{i \in \Delta} F_{Over}^I(\lambda_i)$$

$$F_{Over}^F \left(\bigcap_{i \in \Delta} \lambda_i \right) \leq \bigvee_{i \in \Delta} F_{Over}^F(\lambda_i), \lambda_i \in I^X, i \in \Delta.$$

Then the triplet $(\tau_{Over}^T, \tau_{Over}^I, \tau_{Over}^F)$ is a neutrosophic gradation of closedness on X (NGC on X).

Similarly can define the concepts of PNGC, and FNGC.

Definition 4.3 For two pairs of mappings (τ^T, τ^I, τ^F) and (F^T, F^I, F^F) from $I^X \rightarrow I$ define $\tau_{F^T}^T(\lambda) = F^T(\lambda^C), \tau_{F^I}^I(\lambda) = F^I(\lambda^C), \tau_{F^F}^F(\lambda) = F^F(\lambda^C)$ $F_{\tau^T}^T(\lambda) = \tau^T(\lambda^C), F_{\tau^I}^I(\lambda) = \tau^I(\lambda^C), F_{\tau^F}^F(\lambda) = \tau^F(\lambda^C)$

Theorem 4.1 1. (τ^T, τ^I, τ^F) is an NGO on X iff $(F_{\tau^T}^T, F_{\tau^I}^I, F_{\tau^F}^F)$ is an NGC on X ,

2. (F^T, F^I, F^F) is an NGC on X iff $(\tau_{F^T}^T, \tau_{F^I}^I, \tau_{F^F}^F)$ is an NGO on X ,

$$\tau_{F^T}^T = \tau^T, \tau_{F^I}^I = \tau^I, \tau_{F^F}^F = \tau^F$$

$$3. F_{\tau_{\tau^T}}^T = F^T, F_{\tau_{\tau^I}}^I = F^I, F_{\tau_{\tau^F}}^F = F^F$$

$$\text{Proof: } F_{\tau_{\tau^T}}^T(\mu) + F_{\tau_{\tau^I}}^I(\mu) + F_{\tau_{\tau^F}}^F(\mu) = \tau^T(\mu^C) + \tau^I(\mu^C) + \tau^F(\mu^C), \forall \mu \in I^X$$

So

$$F_{\tau_{\tau^T}}^T(\mu) + F_{\tau_{\tau^I}}^I(\mu) + F_{\tau_{\tau^F}}^F(\mu) \leq 2 \text{ iff } \tau^T(\mu^C) + \tau^I(\mu^C) + \tau^F(\mu^C) \leq 2, \forall \mu \in I^X \quad (4.1)$$

Clearly,

$$F_{\tau_{\tau^T}}^T(0_X) = F_{\tau_{\tau^T}}^T(1_X) = 1 \iff \tau^T(0_X) = \tau^T(1_X) = 1$$

$$F_{\tau_{\tau^I}}^I(0_X) = F_{\tau_{\tau^I}}^I(1_X) = 1 \iff \tau^I(0_X) = \tau^I(1_X) = 1$$

$$F_{\tau_{\tau^F}}^F(0_X) = F_{\tau_{\tau^F}}^F(1_X) = 0 \iff \tau^F(0_X) = \tau^F(1_X) = 0$$

$$F_{\tau_{\tau^T}}^T(\mu_1 \cup \mu_2) = \tau^T[(\mu_1 \cup \mu_2)^c] = \tau^T(\mu_1^c \cap \mu_2^c) \quad (4.2)$$

so

$$F_{\tau_{\tau^T}}^T(\mu_1 \cup \mu_2) \geq F_{\tau_{\tau^T}}^T(\mu_1) \wedge F_{\tau_{\tau^T}}^T(\mu_2), \forall \mu_1, \mu_2 \in I^X$$

$$\iff \tau^T(\mu_1^c \cap \mu_2^c) \geq \tau^T(\mu_1^c) \wedge \tau^T(\mu_2^c), \forall \mu_1, \mu_2 \in I^X$$

$$\iff \tau^T(\mu_1 \cap \mu_2) \geq \tau^T(\mu_1) \wedge \tau^T(\mu_2), \forall \mu_1, \mu_2 \in I^X \text{ (since } (\mu^c)^c = \mu) \quad (4.3)$$

Similarly

$$F_{\tau_{\tau^I}}^I(\mu_1 \cup \mu_2) \geq F_{\tau_{\tau^I}}^I(\mu_1) \wedge F_{\tau_{\tau^I}}^I(\mu_2) \iff \tau^I(\mu_1 \cap \mu_2) \geq \tau^I(\mu_1) \wedge \tau^I(\mu_2), \forall \mu_1, \mu_2 \in I^X \quad (4.4)$$

$$F_{\tau_{\tau^F}}^F(\mu_1 \cap \mu_2) \geq F_{\tau_{\tau^F}}^F(\mu_1) \vee F_{\tau_{\tau^F}}^F(\mu_2) \iff \tau^F(\mu_1 \cap \mu_2) \leq \tau^F(\mu_1) \vee \tau^F(\mu_2), \forall \mu_1, \mu_2 \in I^X \quad (4.5)$$

$$F_{\tau_{\tau^T}}^T\left(\bigcap_{i \in \Delta} \mu_i\right) = \tau^T\left[\left(\bigcap_{i \in \Delta} \mu_i\right)^c\right] = \tau^T\left(\bigcap_{i \in \Delta} \mu_i^c\right).$$

So

$$F_{\tau_{\tau^T}}^T\left(\bigcap_{i \in \Delta} \mu_i\right) \geq \bigwedge_{i \in \Delta} F_{\tau_{\tau^T}}^T(\mu_i), \mu_i \in I^X, i \in \Delta$$

$$\iff \tau^T\left(\bigcup_{i \in \Delta} \mu_i^c\right) \geq \bigwedge_{i \in \Delta} \tau^T(\mu_i^c), \mu_i \in I^X, i \in \Delta$$

$$\iff \tau^T\left(\bigcup_{i \in \Delta} \mu_i\right) \geq \tau^T(\mu_i), \mu_i \in I^X, i \in \Delta \quad (4.6)$$

Similarly

$$F_{\tau_{\tau^I}}^I\left(\bigcap_{i \in \Delta} \mu_i\right) \geq \bigwedge_{i \in \Delta} F_{\tau_{\tau^I}}^I(\mu_i) \iff \tau^I\left(\bigcup_{i \in \Delta} \mu_i\right) \geq \bigwedge_{i \in \Delta} \tau^I(\mu_i), \mu_i \in I^X, i \in \Delta \quad (4.7)$$

$$F_{\tau_{\tau^F}}^F\left(\bigcap_{i \in \Delta} \mu_i\right) \leq \bigvee_{i \in \Delta} F_{\tau_{\tau^F}}^F(\mu_i) \iff \tau^F\left(\bigcup_{i \in \Delta} \mu_i\right) \leq \bigvee_{i \in \Delta} \tau^F(\mu_i), \mu_i \in I^X, i \in \Delta \quad (4.8)$$

Hence by 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7 and 4.8 (1) hold.

(2) The proof of (2) is similar to (1).

(3) The proof is straightforward.

Definition 4.4 Let $\{(\tau^T, \tau^I, \tau^F)\}_{i \in \Delta}$ be a family of NGOs on X . Then their intersection is defined by

$$\bigcap_{i \in \Delta} (\tau^T, \tau^I, \tau^F) = \left(\bigwedge_{i \in \Delta} \tau_i^T, \bigwedge_{i \in \Delta} \tau_i^I, \bigvee_{i \in \Delta} \tau_i^F \right), \text{ where}$$

$$\left(\bigwedge_{i \in \Delta} \tau_i^T \right) (\mu) = (\tau_i^T(\mu)),$$

$$\left(\bigwedge_{i \in \Delta} \tau_i^I \right) (\mu) = \bigwedge_{i \in \Delta} (\tau_i^I(\mu)),$$

$$\left(\bigvee_{i \in \Delta} \tau_i^F \right) (\mu) = \bigvee_{i \in \Delta} (\tau_i^F(\mu)), \quad \mu \in I^X$$

Theorem 4.2 *An arbitrary intersection of NGOs is an NGO.*

Definition 4.5 *Let $(\tau_1^T, \tau_1^I, \tau_1^F)$ and $(\tau_2^T, \tau_2^I, \tau_2^F)$ be two NGOs on X . Then we define a relation ' \leq ' by $(\tau_1^T, \tau_1^I, \tau_1^F) \leq (\tau_2^T, \tau_2^I, \tau_2^F) \iff \tau_1^T \leq \tau_2^T, \tau_1^I \leq \tau_2^I, \tau_1^F \geq \tau_2^F$*

5. Neutrosophic Topological Spaces and its Variants

Pythagorean Neutrosophic Topological Space

Definition 5.1 *Consider P is non-empty set and $\tau \subset PNTS$. Then τ is called a PNTS in the sense of Chang if the next axioms are met:*

1. $0_{PN}, 1_{PN} \in \tau$ for each $T_{AP}, I_{AP}, F_{AP} \in [0, 1]$ with $0 \leq T_{AP}^2 + I_{AP}^2 + F_{AP}^2 \leq \sqrt{2}$
2. for any $P_1, P_2 \in \tau$, implies $P_1 \cap P_2 \in \tau$.
3. for any $\{P_n | n \in I\}$, implies $\bigcup_{n \in I} P_n \in \tau$.

Example 5.1 *Let $P = \{a, b, c\}$*

$$A = \left\langle p, \left(\frac{a}{1}, \frac{b}{0.5}, \frac{c}{0.3} \right), \left(\frac{a}{0.6}, \frac{b}{1}, \frac{c}{0.7} \right), \left(\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.8} \right) \right\rangle,$$

$$B = \left\langle p, \left(\frac{a}{0.9}, \frac{b}{0.5}, \frac{c}{0.5} \right), \left(\frac{a}{0.6}, \frac{b}{0.8}, \frac{c}{0.4} \right), \left(\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{1} \right) \right\rangle,$$

$$C = \left\langle p, \left(\frac{a}{1}, \frac{b}{0.5}, \frac{c}{0.5} \right), \left(\frac{a}{0.6}, \frac{b}{1}, \frac{c}{0.7} \right), \left(\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.8} \right) \right\rangle,$$

$$D = \left\langle p, \left(\frac{a}{0.9}, \frac{b}{0.5}, \frac{c}{0.3} \right), \left(\frac{a}{0.6}, \frac{b}{0.8}, \frac{c}{0.4} \right), \left(\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{1} \right) \right\rangle$$

$\tau = \{0_{PN}, 1_{PN}, A, B, C, D\}$ *The collection τ is recognised as PNT on P , the pair (P, τ) a PNTS.*

Definition 5.2 *Consider P is non-empty set and $\tau \subset PNTS$. Then τ is called a PNTS in the sense of Lowen if the next axioms are met:*

1. $A_P \in \delta$ for each $T_{AP}, I_{AP}, F_{AP} \in \delta \subseteq [0, 1]$ with $0 \leq T_{AP}^2 + I_{AP}^2 + F_{AP}^2 \leq \sqrt{2}$
2. for any $A_{P_1}, A_{P_2} \in \tau$, implies $A_{P_1} \cap A_{P_2} \in \tau$.
3. for any $\{A_{P_n} | n \in I\}$, implies $\bigcup_{n \in I} A_{P_n} \in \tau$.

Then the pair (P, τ) is called a PNTS.

Example 5.2 *Let $P = \{a, b, c\}$*

$$A = \left\langle p, \left(\frac{a}{1}, \frac{b}{0.5}, \frac{c}{0.3} \right), \left(\frac{a}{0.6}, \frac{b}{1}, \frac{c}{0.7} \right), \left(\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.8} \right) \right\rangle,$$

$$B = \left\langle p, \left(\frac{a}{0.9}, \frac{b}{0.5}, \frac{c}{0.5} \right), \left(\frac{a}{0.6}, \frac{b}{0.8}, \frac{c}{0.4} \right), \left(\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{1} \right) \right\rangle,$$

$$C = \left\langle p, \left(\frac{a}{1}, \frac{b}{0.5}, \frac{c}{0.5} \right), \left(\frac{a}{0.6}, \frac{b}{1}, \frac{c}{0.7} \right), \left(\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.8} \right) \right\rangle,$$

$$D = \left\langle p, \left(\frac{a}{0.9}, \frac{b}{0.5}, \frac{c}{0.3} \right), \left(\frac{a}{0.6}, \frac{b}{0.8}, \frac{c}{0.4} \right), \left(\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{1} \right) \right\rangle$$

$$\tau = \{A, B, C, D\}$$

The collection τ is called a PNT on P , the pair (P, τ) a PNTS.

Definition 5.3 The complement $C(A_P)$ of an PNS A_P in an PNTS (X, τ) is called a Pythagorean Neutrosophic closed set (PNCS) in X .

Definition 5.4 Let (P, τ) be a PNTS and $A_P = \{\langle p : T_{A_P}(p), I_{A_P}(p), F_{A_P}(p) \rangle | p \in P\}$ be an PNS in X . Then the Pythagorean Neutrosophic closure of A_P are defined by

$$Cl(A_P) = \cap \{ K : K \text{ is a PNCS in } P \text{ and } A_P \subseteq K \}.$$

Definition 5.5 Let (P, τ) be a PNTS and $A_P = \{\langle p : T_{A_P}(p), I_{A_P}(p), F_{A_P}(p) \rangle | p \in P\}$ be an PNS in P . Then the Pythagorean Neutrosophic Interior of A_P are defined by

$$int(A_P) = \cup \{ G : G \text{ is a PNOS in } P \text{ and } G \subseteq A_P \}.$$

Fermatean Neutrosophic Topological Space

Definition 5.6 Consider a non-empty set F and $\tau \subset FNTS$. Then τ is known as FNTS in the sense of Chang if the next axioms are met:

1. $0_{FN}, 1_{FN} \in \tau$ for each $T_{A_F}, I_{A_F}, F_{A_F} \in [0, 1]$ with $0 \leq T_{A_F}^3 + I_{A_F}^3 + F_{A_F}^3 \leq \sqrt{3}$
2. for any $F_1, F_2 \in \tau$, implies $F_1 \cap F_2 \in \tau$.
3. for any $\{F_n | n \in I\}$, implies $\bigcup_{n \in I} F_n \in \tau$.

Example 5.3 Let $F = \{a, b, c\}$

$$A = \left\langle f, \left(\frac{a}{0.9}, \frac{b}{0.8}, \frac{c}{0.6} \right), \left(\frac{a}{0.9}, \frac{b}{1}, \frac{c}{0.9} \right), \left(\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.9} \right) \right\rangle,$$

$$B = \left\langle f, \left(\frac{a}{1}, \frac{b}{0.6}, \frac{c}{0.7} \right), \left(\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.8} \right), \left(\frac{a}{0.1}, \frac{b}{0.9}, \frac{c}{0.8} \right) \right\rangle,$$

$$C = \left\langle f, \left(\frac{a}{1.0}, \frac{b}{0.8}, \frac{c}{0.7} \right), \left(\frac{a}{0.9}, \frac{b}{1.0}, \frac{c}{0.9} \right), \left(\frac{a}{0.1}, \frac{b}{0.6}, \frac{c}{0.8} \right) \right\rangle,$$

$$D = \left\langle f, \left(\frac{a}{0.9}, \frac{b}{0.6}, \frac{c}{0.6} \right), \left(\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.8} \right), \left(\frac{a}{0.6}, \frac{b}{0.9}, \frac{c}{0.9} \right) \right\rangle$$

$$\tau = \{0_{FN}, 1_{FN}, A, B, C, D\}$$

The collection τ is called a FNT on F , the pair (F, τ) a FNTS.

Definition 5.7 Consider a non-empty set F and $\tau \subset FNTS$. Then τ is known as FNTS in the sense of Lowen if it the next axioms are met:

1. $A_F \in \delta$ for each $T_{A_F}, I_{A_F}, F_{A_F} \in \delta \subseteq [0, 1]$ with $T_{A_F}^3(x) + I_{A_F}^3(x) + F_{A_F}^3(x) \leq \sqrt{3}$
2. for any $A_{F_1}, A_{F_2} \in \tau$, implies $A_{F_1} \cap A_{F_2} \in \tau$.
3. for any $\{A_{F_n} | n \in I\}$, implies $\bigcup_{n \in I} A_{F_n} \in \tau$.

then the pair (X, τ) is called a FNTS.

Example 5.4 Let $F = \{a, b, c\}$

$$\begin{aligned} A &= \left\langle f, \left(\frac{a}{0.9}, \frac{b}{0.8}, \frac{c}{0.6} \right), \left(\frac{a}{0.9}, \frac{b}{1}, \frac{c}{0.9} \right), \left(\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.9} \right) \right\rangle, \\ B &= \left\langle f, \left(\frac{a}{1}, \frac{b}{0.6}, \frac{c}{0.7} \right), \left(\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.8} \right), \left(\frac{a}{0.1}, \frac{b}{0.9}, \frac{c}{0.8} \right) \right\rangle, \\ C &= \left\langle f, \left(\frac{a}{1.0}, \frac{b}{0.8}, \frac{c}{0.7} \right), \left(\frac{a}{0.9}, \frac{b}{1.0}, \frac{c}{0.9} \right), \left(\frac{a}{0.1}, \frac{b}{0.6}, \frac{c}{0.8} \right) \right\rangle, \\ D &= \left\langle f, \left(\frac{a}{0.9}, \frac{b}{0.6}, \frac{c}{0.6} \right), \left(\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.8} \right), \left(\frac{a}{0.6}, \frac{b}{0.9}, \frac{c}{0.9} \right) \right\rangle \end{aligned}$$

$$\tau = \{A, B, C, D\}$$

The collection τ is called a FNT on F , the pair (F, τ) a FNTS.

Definition 5.8 The complement $C(A_F)$ of a FNS A_F in an FNTS (F, τ) is known as Fermatean Neurosophic closed set(NSCS) in F

Definition 5.9 Consider (F, τ) be a FNTS and $A_F = \{ \langle f : T_{A_F}(f), I_{A_F}(f), F_{A_F}(f) \rangle | f \in F \}$ be an FNS in F . Then the Fermatean Neurosophic closure of A_F are defined by

$$Cl(A_F) = \cap \{ K : K \text{ is a FNCS in } F \text{ and } A_F \subseteq K \}.$$

Definition 5.10 Let (F, τ) be a FNTS and

$A_F = \{ \langle f : T_{A_F}(f), I_{A_F}(f), F_{A_F}(f) \rangle | f \in F \}$ be an FNS in F . Then the Fermatean Neurosophic Interior of A_F are defined by

$$int(A_F) = \cup \{ G : G \text{ is a FNOS in } F \text{ and } G \subseteq A_F \}.$$

6. Conclusion

The topological settings of Pythagorean and Fermatean neutrosophic sets are examined and their topological structures are defined. Additionally, the topologies in the sense of Chang and Lowen are derived and suitable examples are provided. The idea of neutrosophic gradation of openness on neutrosophic subsets of a nonempty set as well as the idea of neutrosophic over topological space is discussed. Further the research can be extended with underset (degrees < 0), offset (some degrees > 1 and others < 0), refined neutrosophic topology, neutrosophic superhyper topology, hypersoft set, indetermssoft Set, Indetermhyersoft set, and TreeSoft Set".

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