CHAPTER I

Preliminaries

This chapter concentrates on some basic definitions and theorems which are required for the forthcoming chapters.

All graphs considered here are finite, simple, connected and undirected graphs. Let $\mathrm{G} = (\mathrm{V}(\mathrm{G}), \mathrm{E}(\mathrm{G}))$ be a graph with the sets of vertices and edges V (G) and E (G), respectively.

1.1 Graph [Bondy, J. A et al., 1976 and Jonathan Gross, 2004]

Definition 1.1.1:

A simple graph G with n vertices and m edges consists of a vertex set $V(G) = \{v_1, v_2, ..., v_n\}$ and the edge set $E(G) = \{e_1, e_2, ..., e_n\}$ where each edge is an unordered pair of vertices. We write *uv* for the edge $\{u, v\}$. If $uv \in E(G)$, then *u* and *v* are adjacent. The vertices contained in an edge e are its endpoints. A simple graph has

- no arrows
- no loops
- Cannot have multiple edges joining vertices.

Definition 1.1.2:

If e is an edge and u, v are vertices such that $\psi(G) = uv$. Then e is said to join *u* and *v*. The vertices *u* and *v* are also called as ***end vertices*** of an edge e.

Definition 1.1.3:

The ends of an edge are to be incident with the ***edge.***

Definition 1.1.4:

Two vertices are ***adjacent*** if they are incident with common edge**.**

Definition 1.1.5:

Two edges are ***adjacent*** if they are incident with common vertex**.**

Definition 1.1.6:

An edge with identical ends is called a ***loop.***

Definition 1.1.7:

An edge with distinct ends is called a ***link.***

Definition 1.1.8:

A graph G is ***finite*** if both vertex set and edge set of G is finite.

Definition 1.1.9:

A graph with a single vertex is called ***trivial graph.*** All other graphs are non-trivial graphs.

Definition 1.1.10:

Two or more links (edges) with same ends are called ***parallel edges or multiple edges.***

Definition 1.1.11:

A simple graph in which each pair of distinct vertices are adjacent is called as a ***complete graph.***

1.2 Bi-partite Graph

Definition 1.2.1:

A ***bi-partite graph*** is one whose vertex set can be partitioned into two subsets X and Y so that each edge has one end in X and the other end in Y ; Such a partition (X, Y) is called a bi-partition of the graph. In a bi-partite graph, no two vertices in X are adjacent and no two vertices in Y are adjacent.

Definition 1.2.2:

A ***complete bi-partite graph*** is a simple bi-partite graph in which each vertex of X is joined to each vertex of Y. If $|X| = m, |Y| = n$, such a graph is denoted by $K_{m,n.}$.

Definition 1.2.3:

A ***K−partite graph*** is one whose vertex set can be partitioned into K− subsets, so that, no edge has both ends in any one subset.

Definition 1.2.4:

A ***complete K−partite*** graph is a simple graph in which each vertex is joined to every vertex that is not in the same subset.

Definition 1.2.5:

A ***graph homomorphism*** is a mapping between two graphs that respects their structure. More concretely it maps adjacent vertices to adjacent vertices.

Definition 1.2.6:

The ***graph isomorphism*** is an equivalence relation on graphs and as such it partitions the class of all graphs into equivalence classes. A set of graphs isomorphic to each other is called an isomorphism class of graphs.

Definition 1.2.7:

The graph join (or complete join) of two graphs is their graph union with all the edges that connect the vertices of the first graph with the vertices of the second graph. It is a ***commutative operation*** (for unlabelled graphs).

1.3 Sub graph

Definition 1.3.1:

A ***sub graph*** of a graph G is a graph whose vertex set is a subset of that of G, and whose adjacency relation is a subset of that of G restricted to this subset.

Definition 1.3.2:

A graph H is a ***spanning sub graph*** of G if $V(G) = V(H)$.

Definition 1.3.3:

Let V' be a non-empty subset of $V(G)$. Then the subgraph of G whose vertex set is V' and whose edge set is the subset of edges in G, that have both ends in V' is denoted by $G[V']$. This sub graph is called a ***vertex induced subgraph***.

Definition 1.3.4:

Let E' be a non-empty subset of $E(G)$. Then the sub graph of G whose edge set is E' and whose vertex set is the subset of vertices in G, that have both ends in E' is denoted by $G[E']$. This subgraph is called an ***edge induced sub graph.***

Definition 1.3.5:

Let G_1 and G_2 be two subgraphs of G. We say that G_1 and G_2 are disjoint if they have no vertex in common. i.e., $V(G_1) \cap V(G_2) = \phi$.

Definition 1.3.6:

Let G_1 and G_2 be two subgraph of G. We say that G_1 and G_2 are edge disjoint if they have no edge in common. i.e., $E(G_1) \cap E(G_2) = \phi$

Definition 1.3.7:

The union $G_1 \cup G_2$ of G_1 and G_2 are the sub graphs of G whose vertex set is $V(G_1) \cup V(G_2)$ and edge set is $E(G_1) \cup E(G_2)$.

1.4 Degree of a Graph

Definition 1.4.1:

The degree of a vertex v in G, denoted by $d(G)$, is the number of edges incident with a vertex v. Each loop is counting as twice.

- $\delta(G) = \text{Min}\{d(v) : v \in V(G)\}$
- $\Delta(G) = \text{Max}\{d(v) : v \in V(G)\}$
- $\delta \leq \Delta$
- In K_n, $d(v) = n - 1$ for all $v \in V(G)$
- $\delta(Kn) = \Delta(Kn) = n - 1$

1.5 Regular Graph [Bondy, J.A et al., 1976]

Definition 1.5.1:

A graph G is said to be ***k*-regular** if the $d(v) = k$ for every $v \in V(G)$.

Example: regular graph and non-regular graph.

(1) The Complete graph with n vertices is n−1 regular graph.

(2) If m = n then $K_{m,n}$ is n-regular; Otherwise it is a **non-regular graph.**

1.6 Connectedness

Definition 1.6.1:

A ***walk*** in G is a finite non-null sequence whose terms are alternatively of vertices and edges. A walk is closed if it has positive length with same origin and terminus. The length of a walk is the number of occurrence of arcs in it.

Definition 1.6.2:

A ***trail*** is a walk in which all edges are distinct.

Definition 1.6.3:

A Cycle is a ***closed trail*** in which the vertices are all distinct.

Definition 1.6.4:

A Path is a ***walk*** in which all the vertices are distinct.

Definition 1.6.5:

A surjective homomorphism's φ is sometimes called ***complete***. When $\varphi : G \to H$ is a complete homomorphism, H is a homomorphic image of G. Similarly, G is isomorphic to its image under φ when φ is injective and faithful.

1.7 Corona of Two Graphs

Definition 1.7.1:

The Corona is the graph obtained by taking p_1 copies of G_2 and joining each vertex of the i^{th} copy of G_2 to the i^{th} vertex of G_1 .Obviously, $|V(G)| = p_1(p_2+1)$ and $|E(G)| = q_1 + p_1(q_2 + p_2)$. According to the definition of $G_1 \circ G_2$, the degree of each vertex $u \in G_1 \circ G_2$ is given by

$$\deg_{G_1 \circ G_2}(u) = \begin{cases} \deg_{G_1}(u) + p_2 & if \quad u \in V_1 \\ \deg_{G_2}(u) + 1 & if \quad u \in V_2 \end{cases}$$

Definition 1.7.2:

A ***color class*** can be defined as a subset of vertices that are assigned with the same color.

Definition 1.7.3:

The ***chromatic number*** can be defined as the least number of colors required to color the graph. It is denoted by $\chi(G)$.

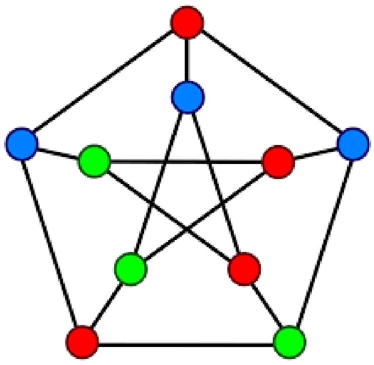

Fig 1.1: Chromatic number of G

In Figure 1.1 the color classes are blue, green and red. Then Chromatic number equal to 3.

Definition 1.7.4:

The ***achromatic number*** $\psi(G)$ of a graph G is the maximum number k for which G has a proper k-coloring.

Definition 1.7.5:

A ***clique*** in a graph V_G is a maximal subset V of G whose vertices are mutually adjacent. The clique number $\omega(G)$ of a graph G is the number of vertices in a largest clique in G.