# CHAPTER - VII

# CHAPTER VII

# FUZZY FLEXIBLE TOPSIS METHOD

Fuzzy F-TOPSIS (Fuzzy Flexible TOPSIS) is proposed with the objective of improving the Fuzzy TOPSIS ability to deal with uncertainty through the combination of the mathematical process involved in the original Fuzzy TOPSIS with the expert empirical knowledge.

Originally, Fuzzy TOPSIS is an optimization technique that uses linguistic terms to evaluate the importance of attributes and their values. In order to improve their ability to deal with vagueness, an attribute that represents the evaluation of a Fuzzy Rules Based System is added to the Fuzzy TOPSIS. The main purpose of this modification is to add the capabilities of fuzzy rules based systems to Fuzzy TOPSIS. Thus, the analysis by Fuzzy F-TOPSIS will allow the empirical knowledge of the expert, represented by fuzzy rules, also be considered in the decision making and optimization process, in addition to definitions of the membership functions of the fuzzy sets.

The use of optimization techniques associated with the empirical knowledge of experts, allows a hybrid analysis of the multi-criteria problems where the process of decision making requires the use of human sensitivity, which often can be expressed by a fuzzy rules base. Thus, the behavior of the system may have greater influence, or not, than the rules defined by the expert

In this Fuzzy F-TOPSIS method a MCDM (Multi-Criteria Decision-Making) problem with m alternatives ($A_1$, $A_2$,..., $A_m$) and n criteria ($C_1$, $C_2$,...,$C_n$) is expressed in matrix format as

$$D = \begin{matrix} & C_1 & C_2 & C_3 & \cdots & C_n \\ A_1 & \tilde{x}_{13} & \tilde{x}_{12} & \tilde{x}_{13} & \cdots & \tilde{x}_{1n} \\ A_2 & \tilde{x}_{21} & \tilde{x}_{22} & \tilde{x}_{23} & \cdots & \tilde{x}_{2n} \\ A_3 & \tilde{x}_{31} & \tilde{x}_{32} & \tilde{x}_{33} & \cdots & \tilde{x}_{3n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & \tilde{x}_{m1} & \tilde{x}_{m2} & \tilde{x}_{m3} & \cdots & \tilde{x}_{mn} \end{matrix} \quad (1)$$

$$W = [\tilde{w}_1, \tilde{w}_2, ..., \tilde{w}_n] \quad (2)$$

where $\tilde{x}_{ij}, i = 1,...,m$, and $j = 1,...,n$ are numeric data of problem and $\tilde{w}_j, j = 1,...,n$ is the importance degree of each attribute $C_1, C_2,...,C_n$, respectively.

The decision matrix are normalized by Equation (3) for the attributes that should be maximized and by Equation (4) for the attributes that should be minimized

$$r_{ij} = \frac{\tilde{x}_{ij} - \min\{\tilde{x}_{ij}\}}{[\max\{\tilde{x}_{ij}\} - \min\{\tilde{x}_{ij}\}]} \quad (3)$$

$$r_{ij} = \frac{\max\{\tilde{x}_{ij}\} - \tilde{x}_{ij}}{[\max\{\tilde{x}_{ij}\} - \min\{\tilde{x}_{ij}\}]} \quad (4)$$

$v_{ij} = r_{ij} . w_j$, where $v_{ij}$ represent the fuzzified value $r_{ij}$.

The Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS) are defined as

$$A^+ = \{\tilde{v}_1^+, ..., \tilde{v}_m^+\} = \{(\max_j \tilde{v}_{ij} | j \in \varphi_b), (\min_j \tilde{v}_{ij} | j \in \varphi_c)\} \quad (5)$$

$$A^- = \{\tilde{v}_1^-, ..., \tilde{v}_m^-\} = \{(\min_j \tilde{v}_{ij} | j \in \varphi_b), (\max_j \tilde{v}_{ij} | j \in \varphi_c)\} \quad (6)$$

Consider two fuzzy sets with pertinence for the input value $r_{ij}$. For this, two fuzzy matrices for decision making is mounted. The first matrix, $V_1$, with the fuzzy sets of greater value of membership function for each of $r_{ij}$ values and, the second, $V_2$, with the fuzzy sets of lower value of membership function.

The Euclidian distance between the fuzzy sets $v_{ij}$ of the matrix $V_1$ and $V_2$ and the FPIS and FNIS, is calculated by taking into account the Influence Level ($IL_{\tilde{v}_{ij}}$) of $r_{ij}$ in each of the fuzzy sets that has pertinence. In the set that has greater pertinence, the calculation of $IL_{\tilde{v}_{ij}}$ is made by Equation (5) and the set of lower pertinence by Equation (6).

$$IL_{\tilde{v}1_{ij}} = \frac{\mu\tilde{v}1_{ij}(r_{ij})}{\mu\tilde{v}1_{ij}(r_{ij}) + \mu\tilde{v}2_{ij}(r_{ij})} \quad (7)$$

$$IL_{\tilde{v}2_{ij}} = \frac{\mu\tilde{v}2_{ij}(r_{ij})}{\mu\tilde{v}1_{ij}(r_{ij}) + \mu\tilde{v}2_{ij}(r_{ij})} \quad (8)$$

The calculations of distances from $v_{ij}$ to the FPIS and FNIS, considering the influence level of $v_{ij}$ in the matrix, are defined as

$$d_i^+ = \sum_{j=1}^{n} d(\tilde{v}1_{ij}, \tilde{v}_j^+) \times IL_{\tilde{v}1_{ij}} + \sum_{j=1}^{n} d(\tilde{v}2_{ij}, \tilde{v}_j^+) \times IL_{\tilde{v}2_{ij}} \quad (9)$$

$$d_i^- = \sum_{j=1}^{n} d(\tilde{v}1_{ij}, \tilde{v}_j^-) \times IL_{\tilde{v}1_{ij}} + \sum_{j=1}^{n} d(\tilde{v}2_{ij}, \tilde{v}_j^-) \times IL_{\tilde{v}2_{ij}} \quad (10)$$

According to Equations (7) and (8), the similarity degree between the alternatives $A_i$ and FPIS take into account the distance of the sets $\tilde{v}_{ij}$ in $V_1$ and $V_2$ for the FPIS and FNIS.

The degree of similarity of each alternative with the FPIS is calculated as

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad (11)$$

The ranking of the alternatives is determined by the degree of similarity.