Interval-Valued Fermatean Fuzzy Logic for Optimizing 3d- Printed Denture Fabrication in Ubiquitous Healthcare Systems

By

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(23PMA021)

Supervisor

Dr. C. ANTONY CRISPIN SWEETY

Thesis submitted to

Avinashilingam Institute for Home Science and Higher Education for Women

Coimbatore-641 043

In Partial Fulfillment of the Requirements for the Degree of Master of Science in Mathematics

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Signature of the Director

20/04/25

Signature of the Supervisor

DECLARATION

I declare that the thesis **"Interval-Valued Fermatean Fuzzy Logic for Optimizing 3D-Printed Denture Fabrication in Ubiquitous Healthcare Systems"** submitted by me for the degree of **Master of Science (M.Sc.,)** is the record of work carried out during the period from December 2024 to April 2025 under the guidance of **Dr. C. Antony Crispin Sweety., B.Ed., M.Sc., M.Phil., Ph.D.,** Assistant Professor, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, and has not formed the basis for the award of any Degree, Diploma, Associateship, Fellowship, Titles in this institute or any other university or other similar institution of Higher Learning.

P. Kaviya 28/4/2025

Signature of the Candidate

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ABSTRACT

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The use of 3D printing technologies in healthcare systems has revolutionized the production of dental prosthetics by offering greater customization and efficiency. The decentralized nature of ubiquitous healthcare systems presents challenges, including uncertainties in printing times, equipment reliability, and transportation logistics. Conventional deterministic models often fail to adequately account for these variabilities. To optimize the scheduling and distribution of denture fabrication tasks among dispersed 3D printing facilities, this thesis presents an advanced decision-making framework that combines a Mixed-Integer Linear Programming model with Interval-Valued Fermatean Fuzzy Logic. The Interval-Valued Fermatean Fuzzy Logic technique effectively captures the nuances of uncertainty and expert hesitation by using interval-valued fuzzy numbers to represent imprecise parameters such as printing times, reprinting probabilities, and delivery durations. A case study involving four distinct 3D printing facilities in Taichung, Taiwan, validates the proposed model. The study demonstrates that the Interval-Valued Fermatean Fuzzy Logic - Mixed-Integer Linear Programming framework not only shortens the overall make span but also increases system resilience by accommodating potential disruptions without necessitating a complete rescheduling. This strategy outperforms traditional heuristic-based scheduling techniques, according to comparative studies, resulting in better workload distribution and shorter fulfillment times. The findings illustrate how effectively Interval-Valued Fermatean Fuzzy Logic and optimization strategies combine to manage the inherent uncertainties of distributed manufacturing systems, resulting in healthcare delivery systems that are more robust and flexible.

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CHAPTER I

Introduction

1.1 Ubiquitous Healthcare (UH)

Ubiquitous health, also known as u-health, refers to the use of interconnected technologies to provide health services anytime and anywhere. It aims to enhance accessibility, efficiency, and quality of healthcare by removing both location and time constraints. This is achieved through devices such as sensors and wearables that continuously monitor health data and transmit it for analysis and intervention.

Ubiquitous Healthcare, also referred to as smart healthcare, represents an evolution of modern medical services through the integration of pervasive computing, wireless networking, smart sensors, and real-time data analytics. This advanced healthcare paradigm allows for seamless and continuous medical monitoring and service provision, independent of location or time, through smart devices embedded within daily life environments. The primary purpose of UH systems is to ensure that healthcare services become accessible at any time and from any place, improving the efficiency of treatment and the quality of patient life, particularly for elderly or chronically ill individuals.

The concept of UH emerged alongside the development of Body Area Networks, wearable sensors, smartphones, and telemedicine systems. These technologies enable patients to transmit their real-time health data to medical professionals through wireless or cloud platforms, reducing the need for hospital visits and facilitating early detection of potential health threats. In essence, UH transforms healthcare from reactive to proactive, empowering both patients and healthcare providers to act swiftly in response to changing health conditions.

1.2 Introduction to 3D Printing in Ubiquitous Healthcare

Three-Dimensional (3D) printing, or additive manufacturing, has emerged as a transformative force in various industries, particularly healthcare. It involves layer-by-layer fabrication of physical objects from digital models, enabling the production of customized and complex geometries with exceptional precision and efficiency.

In the context of UH, 3D printing has found its niche in areas such as prosthetics, orthotics, surgical planning models, and dental applications. Denture production, in particular, has significantly benefited from this technology. Traditionally, creating dental prostheses involved labor-intensive and time-consuming manual processes, which were prone to human error and limited by the artisan's skill. 3D printing eliminates these inefficiencies by producing patient-specific dentures from digital scans, ensuring a perfect anatomical fit and superior comfort for the patient.

The traditional method for creating an impression of the patient's mouth (negative mold) involves using a rubber-like material (alginate) to capture the details of their teeth. Once the alginate sets, it is removed from the mouth. If an error occurs while making the negative model, another impression must be created, requiring the process to start over. Additionally, some quality issues are very hard to spot visually but can negatively impact the fit and feel of the finished appliance. This negative model is used to create a positive model of the patient's teeth, made from a material similar to plaster. The traditional molding process is time-consuming and complicated for both the patient and the dentist. Moreover, the materials used are costly and messy. Creating the positive model, waiting for it to harden, and cleaning up all contribute to the process's length. Furthermore, if an imperfection in the negative model is not recognized right away, the patient will need to return to the office for another alginate impression of their teeth, creating inconvenience for both the orthodontist and the patient while delaying the creation of the patient's orthodontic appliance.

This process leads to a more pleasant experience for the patient and the dental professional performing the procedure. Although the conventional molding process requires a lot of hands-on time, gathering detailed information for the 3D printing process is relatively quick and easy. With 3D printing, messy gels are unnecessary as an intraoral wand is used to create a 3D image of the patient's teeth and gums. Due to its precision, concerns related to imperfections during the molding process are essentially eliminated when utilizing 3D printing.

Furthermore, when integrated with ubiquitous healthcare systems, 3D printing offers a collaborative environment where multiple printing facilities can work simultaneously to meet urgent patient demands. This distributed production capability not only reduces order fulfillment time but also enhances resilience against individual machine failures or transportation delays.

Abbreviation:

UH	Ubiquitous Healthcare
BANs	Body Area Networks
IVFFL	Interval-Valued Fermatean Fuzzy Logic
MILP	Mixed-Integer Linear Programming
3D Printing	Three-Dimensional printing
CAD	Computer-Aided Design
CAM	Computer-Aided Manufacturing
IT2FL	Interval Type-II Fuzzy Logic
TrFNs	Interval Type-II Trapezoidal Fuzzy Numbers
FMILP	Fuzzy Mixed-Integer Linear Programming

1.3 Literature Review

The concept of Ubiquitous Healthcare (UH) and its application in denture production have progressively evolved over the years, driven by the convergence of smart technologies and medical needs. Sneha and Varshney (2006) were among the first to articulate the idea of UH, defining it as a new frontier in e-health that leverages wireless technology and pervasive computing to enable continuous medical service delivery beyond the confines of hospitals. Their work laid the foundation for later research that integrated smart devices and wearable sensors into healthcare systems.

The development of 3D printing has further accelerated UH applications, particularly in dentistry. Dawood et al. (2015) highlighted the transformative role of 3D printing in dental practices, demonstrating how it improved the production of dental prostheses by providing precise, cost-effective, and patient-specific solutions. This breakthrough in manufacturing enabled a seamless fit between the prosthetic and the patient's anatomy, which traditional manual fabrication could not easily achieve.

Tunchel et al. (2016) further validated the effectiveness of 3D printing in dentistry by conducting a three-year prospective multi-center study on 3D-printed titanium dental implants. Their results demonstrated excellent performance, biocompatibility, and longevity of 3D printed dental solutions in real-world clinical applications, strengthening the case for widespread adoption of 3D printing in dental healthcare.

Beyond these foundational works, Chen and Lin (2017) extended the discussion by exploring smart manufacturing systems, particularly focusing on the role of 3D printing in optimizing distributed production lines. Their research on dynamic resource allocation and load balancing is especially relevant for real-world ubiquitous healthcare scenarios that require on-demand production of patient-specific devices.

Jeong et al. (2018) compared the accuracy of dental models manufactured through 3D printing and CAD/CAM milling techniques. Their findings highlighted that 3D-printed models not only provided better dimensional stability but also reduced turnaround time, thereby aligning perfectly with the objectives of ubiquitous healthcare systems that prioritize speed and personalization.

Wang et al. (2019) advanced the field by proposing a collaborative and ubiquitous system in which multiple 3D printing facilities jointly fulfilled orders for dental parts. Their approach utilized Mixed Integer Quadratic Programming (MIQP) to balance workloads across facilities while minimizing delivery time, marking a significant leap in the efficiency of denture production logistics. The collaborative nature of this system enabled decentralized production, making it more adaptive to regional demands and resource availability.

Building on these advances, Chiu and Chen (2022) introduced a fuzzy optimization-based methodology to address the uncertainties inherent in the denture production and delivery process. They applied Interval Type-II Fuzzy Logic (IT2FL) combined with a Mixed Integer Linear Programming (MILP) framework to model uncertainties in both transportation and printing times. Their research demonstrated that this approach could reduce dental order fulfillment times by accounting for real-world disruptions, such as transportation delays and equipment failures, which are common in distributed healthcare systems.

Satti et al. (2020) examined interoperability challenges in healthcare data and proposed a universal health profile system to enhance collaboration and data sharing between disparate systems. Their framework emphasizes the importance of data standardization and system integration in contemporary ubiquitous healthcare systems that utilize distributed 3D printing facilities.

Pillai et al. (2021) highlighted the evolution of dental 3D printing technologies, emphasizing the critical role of cloud-based collaboration and digital workflows in bridging laboratory production and clinical practice. Their insights are particularly valuable in the context of a post-pandemic healthcare landscape where decentralized production and rapid customization are more essential than ever.

1.4 Basic Concepts

The foundation for this research lies in a UH system in which multiple 3D printing facilities are established to print dentures collaboratively. The system optimizes the assignment of print jobs to facilities by minimizing fulfillment time while accounting for uncertainties in both printing and transportation.

In their model, the authors employed interval type-II fuzzy logic to manage uncertain data. The key components include:

- **Interval Type-II Trapezoidal Fuzzy Numbers** for modeling uncertain times.
- **Fuzzy Mixed Integer Linear Programming** to optimize the allocation of denture production tasks to various facilities.

Definitions 1.4.1.

A classical fuzzy set is defined by a membership function:

$$\boldsymbol{\mu_A(x)}: \boldsymbol{X} \to [\mathbf{0}, \mathbf{1}]$$

Each element $\boldsymbol{x} \in \boldsymbol{X}$ has a degree of membership in the fuzzy set A, denoted $\mu_A(x)$, indicating the degree to which x belongs to A

Definition 1.4.2.

An interval type-II fuzzy set $\overline{A}$ Is defined as a set of ordered pairs:

$$\overline{A} = \left\{\left(x, \mu_{\overline{A}}(x)\right)\middle|\ x \in X\right\}$$

Where the membership function $\mu_{\overline{A}}(x)$ Takes interval values rather than crisp ones.

Specifically:

$$\mu_{\overline{A}}(x) = \left[\mu_{\overline{A}}^{l}(x), \mu_{\overline{A}}^{u}(x)\right], \quad 0 \le \mu_{\overline{A}}^{l}(x) \le \mu_{\overline{A}}^{u}(x) \le 1$$

Definition 1.4.3.

An IT2 Trapezoidal Fuzzy Number is characterized by two trapezoidal membership functions – A Lower Membership Function (LMF) and an Upper Membership Function (UMF). These functions are defined as:

For UMF:

$$\mu^u(x) = \begin{cases} 0, x \leq a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, \quad a_1 < x \leq a_2 \\ 1, \quad a_1 < x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, \quad a_3 < x \leq a_4 \\ 0, \quad x > a_4 \end{cases}$$

For LMF:

$$\mu^u(x) = \begin{cases} 0, x \leq b_1 \\ \dfrac{x - b_1}{b_2 - b_1}, \quad b_1 < x \leq b_2 \\ 1, \quad b_2 < x \leq b_3 \\ \dfrac{b_4 - x}{b_4 - b_3}, \quad b_3 < x \leq b_4 \\ 0, \quad x > b_4 \end{cases}$$

Where $a_1 \leq a_2 \leq a_3 \leq a_4$ and $b_1 \leq b_2 \leq b_3 \leq b_4$.

Definition 1.4.4.

Arithmetic operations on TrFNs follow the extension principle. For two fuzzy number $\overline{A}$ and $\overline{B}$, the operations are:

- Addition:

$$\overline{A} + \overline{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$$

- Subtraction:

$$\overline{A} - \overline{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$$

- Multiplication:

$$\overline{A} \times \overline{B} \approx (\min(a_1.b_1, a_1.b_4, a_4.b_1, a_4.b_4), \ldots)$$

Definition 1.4.5.

Intuitionistic Fuzzy Sets (IFS) incorporate both membership $\mu(x)$ and non-membership $\nu(x)$ functions, satisfying:

$$0 \leq \mu(x) + \nu(x) \leq 1$$

Definition 1.4.6.

An Interval-Valued Intuitionistic Fuzzy Set (IVIFS) generalizes IFS by allowing the membership and non-membership degrees to be intervals rather than precise values. For each element x in the universe X, the membership degree is an interval $[\mu^L(x),\ \mu^U(x)]$, and the non-membership degree is $[v^L(x), v^U(x)]$ with the condition:

$$0 \leq \mu^U(x) + \nu^U(x) \leq 1$$

$$0 \leq \mu^L(x) + \nu^L(x) \leq 1$$

Definition 1.4.7.

Pythagorean Fuzzy Sets (PFS) is the constraint on membership and non-membership degrees. For each element x in the universe X, a PFS is defined with membership degree μ(x) and non-membership degree ν (x), satisfying:

$$\mu^2(x) + \nu^2(x) \leq 1$$

Definition 1.4.8.

An Interval-Valued Pythagorean Fuzzy Set (IVPFS) extends PFS by representing membership and non-membership degrees as intervals. For each element x in the universe X, the membership degree is$[\mu^L(x),\ \mu^U(x)]$ and the non-membership degree is $[v^L(x), v^U(x)]$, with the condition

$$(\mu^U(x))\hat{}2 + (v^U(x))\hat{}2 \leq 1$$

$$(\mu^L(x))\hat{}2 + (v^L(x))\hat{}2 \leq 1$$

Definition 1.4.9.

Spherical Fuzzy Sets (SFS) introduce a three-dimensional approach to fuzzy logic by considering membership and non-membership degrees simultaneously. For each element x in the universe X, an SFS is defined with degrees $\mu(x)$ and $v(x)$ satisfying:

$$\boldsymbol{\mu^2\ (x) + v^2(x) \leq 1}$$

Definition 1.4.10.

Interval-Valued Spherical Fuzzy Sets (IVSFS) extend SFS by allowing each degree of membership and non-membership to be represented as intervals. For each element x in the universe X, the degrees are $[\mu^L(x),\ \mu^U(x)]\ and\ [v^L(x), v^U(x)]$, with the condition:

$$\left(\mu^U(x)\right)^2 + \left(v^U(x)\right)^2 \ \leq 1$$

$$\left(\mu^L(x)\right)^2 + \left(v^L(x)\right)^2 \leq 1$$

Definition 1.4.11.

Fermatean Fuzzy Sets, further generalize PFS using the constraint:

$$\mu^3(x) + \nu^3(x) \leq 1$$

This cubic condition allows higher degrees of membership and non-membership, thus capturing greater hesitation (the remaining part to 1).

Let:

- $\mu(x) \in [0,1]$ be the degree of membership,
- $\nu(x) \in [0,1]$ be the degree of non-membership,

Then the hesitation degree is:

$$\pi(x) = 1 - \mu^3(x) - \nu^3(x)$$

Definition 1.4.12.

An FMILP model is an optimization formulation where some or all parameters (e.g., costs, times, or constraints) are fuzzy values. It combines:

- A linear objective function
- Integer decision variables (e.g., number of denture's assigned per facility)
- Fuzzy constraints expressed through TrFNs

The general form of an FMILP model is:

$$\min Z_i = \max\left(a_i + (n_i.p_i) + d_{ic}\right)$$

Subject to the following constraints:

$$\sum_{i=1}^{n} n_i = N$$

$$n_i \in Z^+ \cup \{0\}: i = 1 \; to \; n$$

Where N is the order size.

1.5 Outline of the Thesis and Chapter Summaries

This thesis is organized to gradually introduce, develop, and implement a new decision-making methodology aimed at optimizing denture fabrication using 3D printing within a widespread healthcare context. The methodology is based on Interval-Valued Fermatean Fuzzy Logic (IVFFL) and is constructed using a Fuzzy Mixed Integer Linear Programming (FMILP) method. Each chapter is crafted to lead the reader from theoretical foundations to practical application and analysis.

Chapter 1 introduces the background and significance of ubiquitous healthcare (UH) and the role of 3D printing in dental prosthetics. It reviews relevant literature on smart healthcare systems, fuzzy logic models, and distributed manufacturing, explaining key fuzzy concepts.

Chapter 2 serves as the methodological backbone of the thesis. It begins with a comprehensive overview of the preliminaries and theoretical underpinnings of Interval-Valued Fermatean Fuzzy Logic (IVFFL), a relatively new and powerful extension of classical and Type-II fuzzy systems. IVFFL enables a more nuanced representation of uncertainty, which is particularly useful in complex decision-making environments like healthcare logistics, where both expert hesitation and data incompleteness are prevalent.

This chapter also justifies the transition from the previously used Interval Type-II Fuzzy Logic (IT2FL) to IVFFL, outlining its enhanced ability to model both membership and non-membership degrees simultaneously, while incorporating a hesitation margin that reflects real-world scenarios ambiguity.

The chapter subsequently presents a detailed mathematical formulation of the proposed IVF-FMILP model. This includes:

- The representation of uncertain transportation and printing times using IVFFL-based Trapezoidal Fuzzy Numbers (IVFF-TrFNs) is evaluated.

- The formulation of an objective function to minimize total fulfillment time (makespan).
- The inclusion of constraints for task assignment, reprinting, and resource balancing across facilities is essential.

A realistic case study follows, in which a dental clinic must fabricate a set of six customized dentures using three distributed 3D printing facilities. The problem is modeled under IVFFL logic, and the solution is obtained using mixed-integer linear programming techniques. The chapter also includes graphs, tables, and defuzzification procedures to illustrate the process and outcomes.

Chapter 3 discusses the outcomes, compares the proposed model with traditional and fuzzy logic approaches, and highlights its advantages in terms of adaptability, resilience, and decision support. It concludes with managerial implications, research contributions, limitations, and suggestions for future work. It begins by presenting the final results of the case study, which include:

- The optimal allocation of dentures to printing facilities
- The total and per-facility fulfillment times
- The make-span under best-case, worst-case, and average scenarios

The chapter then transitions into a discussion of managerial implications, where it is shown that:

- The IVFFL-based model offers improved resilience against disruptions
- It reduces emergency rescheduling needs
- It enhances resource utilization and decision transparency for clinic managers and logistics planners

The thesis concludes with a reflection on the limitations of the current study, including assumptions about static demand and limited facility interaction, and proposes future directions for research. These directions encompass real-time dynamic scheduling, integration with IoT-based monitoring, and the use of hybrid fuzzy-neural systems for adaptive healthcare logistics.

CHAPTER II

CHAPTER II

2.1. Introduction

This chapter introduces the core methodology of this research, which focuses on optimizing denture fabrication and delivery across multiple 3D printing facilities within a ubiquitous healthcare (UH) environment. It highlights the application of Interval-Valued Fermatean Fuzzy Logic (IVFFL) and Mixed-Integer Linear Programming (MILP) to address uncertainties in production and transportation times. The integration of IVFFL allows for flexible modeling of expert hesitation and the operational unpredictability challenges commonly encountered in real-time healthcare systems.

The chapter begins by presenting the system architecture of a UH-based distributed 3D printing network and explaining how this architecture manages patient orders through smart coordination and facility evaluation. It then introduces the IVFFL mathematical foundation, highlights its advantages over traditional fuzzy systems, and describes how it integrates with the optimization model to enable robust scheduling decisions in the face of uncertainty.

2.1.1. System Overview and Ubiquitous Healthcare Integration

In recent years, the integration of 3D printing technologies into ubiquitous healthcare (UH) systems has opened new opportunities for personalized and decentralized medical device manufacturing, especially in the area of dental prosthetics. However, managing production and delivery in distributed environments introduces several uncertainties, such as variable printing times, transportation delays, and equipment failures, which traditional deterministic or even Type-I fuzzy logic systems are not well equipped to handle.

To address these limitations, this study proposes using Interval-Valued Fermatean Fuzzy Logic (IVFFL) alongside Mixed-Integer Linear Programming (MILP), leading to a novel IVF-FMILP model. IVFFL was chosen for its superior ability to represent uncertain, vague, and hesitant information, which is common in expert-driven environments such as healthcare logistics.

Unlike Type-I or Type-II fuzzy sets that rely solely on membership functions—or, in the case of Type-II, a footprint of uncertainty—IVFFL allows for both interval-valued membership and non-membership degrees, subject to a cubic constraint. This constraint ensures that belief, disbelief, and hesitation are all simultaneously represented within a mathematically consistent structure. The result is a more expressive and realistic framework for modeling expert knowledge and operational variability.

The rationale for using IVFFL is based on the following advantages:

- Captures hesitation: In real-world decision-making, especially within medical systems, experts often feel uncertain about precise values. IVFFL accommodates this hesitation explicitly.
- Improves realism: Both optimistic and pessimistic estimates can be included in the model, enhancing robustness.
- Maintains tractability: Despite its expressiveness, IVFFL remains computationally manageable when integrated with MILP.
- Enhances decision support: By examining a wider range of uncertainty, IVFFL-based models offer improved insight for resource allocation and scheduling.

In this chapter, we present the theoretical foundation of IVFFL, define the core concepts, explain its integration into the FMILP framework, and demonstrate its applicability through a case study involving three 3D printing facilities and six denture components. This methodology establishes the groundwork for a robust, uncertainty-aware optimization model tailored to the needs of next-generation healthcare systems.

Definition 2.1.1.

Let X be a non-empty set. An interval-valued Fermatean Fuzzy logic $\mu \; in \; X$ is of the form

$$\mu(x) = \{x: [\mu^L(x), \mu^U(x)], [\nu^L(x), \nu^U(x)] \,|x \in X\}$$

Where $\mu^L(x), \nu^L(x) \in [0,1], 0 \leq (\mu^L(x))\hat{}3 + (\nu^L(x))\hat{}3 \leq 1$

$$\mu^U(x), \nu^U(x) \in [0,1], 0 \leq (\mu^U(x))\hat{}3 + (\nu^U(x))\hat{}3 \leq 1$$

Definition 2.1.2.

Let two IVFFSs be:

- $A = \langle x, [\mu_A^L, \mu_A^U], [\nu_A^L, \nu_A^U] \rangle$
- $B = \langle x, [\mu_B^L, \mu_B^U], [\nu_B^L, \nu_B^U] \rangle$

Then the basic operations are:

Addition (⊕):

$$\mu_{A\oplus B} = [\mu_A^L + \mu_B^L - \mu_A^L\mu_B^L, \mu_A^U + \mu_B^U - \mu_A^U\mu_B^U]$$

$$v_{A\oplus B} = [v_A^L v_B^L, v_A^U v_B^U]$$

Multiplication (⊗):

$$\mu_{A\otimes B} = [\mu_A^L\mu_B^L, \mu_A^U\mu_B^U]$$

$$v_{A\otimes B} = [v_A^L + v_B^L - v_A^L v_B^L, v_A^U + v_B^U - v_A^U v_B^U]$$

Definition 2.1.3.

To compare IVFFSs, we use:

Score Function S(A):

$$S(A) = \frac{1}{2}[(\mu^L)^3 + (\mu^U)^3 - (\nu^L)^3 - (\nu^U)^3]$$

Accuracy Function H(A):

$$H(A) = \frac{1}{2}[(\mu^L)^3 + (\mu^U)^3 + (\nu^L)^3 + (\nu^U)^3]$$

A higher score indicates a stronger membership. If the scores are equal, the accuracy function helps in breaking ties.

2.2. Methodology

In this study, a systematic methodology has been created to enhance the scheduling and production of denture manufacturing within a distributed 3D printing infrastructure under a ubiquitous healthcare framework. A key contribution of this work is the integration of Interval-Valued Fermatean Fuzzy Logic (IVFFL) with Mixed-Integer Linear Programming (MILP) to effectively address the significant uncertainties associated with both the printing and delivery phases. The proposed model prioritizes accuracy and efficiency while ensuring robustness in the face of unforeseen events such as equipment failures or transportation delays.

The system's architecture includes three primary stakeholders:

- Dental clinics or patients requesting dentures
- A centralized smart healthcare coordinator responsible for managing and scheduling workload and
- The distributed 3D printing facilities are tasked with producing the required denture components.

Once a denture request is received, the coordinator evaluates the current system state by considering previous performance data, real-time facility availability, and transportation logistics to assign tasks across multiple printing facilities. Each selected facility then independently fabricates specific parts of the denture. The completed components are delivered either through traditional logistics providers or modern home delivery systems directly to the clinic or patient.

The innovation lies in applying IVFFL to represent uncertainties in the system. Unlike traditional fuzzy logic, which typically relies on single-point or crisp membership degrees, IVFFL allows for interval-based modeling of both membership and non-membership. This is especially beneficial when expert estimations are imprecise or affected by hesitation. For instance, printing time at a facility may vary due to fluctuating workloads or machine health. Instead of committing to a single time value, IVFFL permits modeling this duration as an interval-valued Fermatean fuzzy number, incorporating optimistic and pessimistic bounds. Similarly, transportation times—subject to traffic, delivery conditions, and third-party reliability—can also be modeled with comparable fuzzy intervals.

The proposed method employs a two-tiered optimization strategy:

1. A lower-bound sub-model based on the minimum values of the fuzzy intervals represents a best-case scenario.
2. An upper-bound sub-model that is based on maximum values, representing a worst-case scenario.

This dual-model strategy allows the system to create a defined feasible region, assisting decision-makers in assessing the reliability of their schedule under both favorable and unfavorable conditions. It also improves risk management by predicting the range of possible outcomes.

A further strength of this methodology is its built-in reprinting strategy. Unlike conventional systems that require a complete rescheduling process in the event of a failure, this model assumes the possibility of a print failure in advance. If a part fails during production, the same facility automatically resumes printing without any external reassignment or global re-optimization. This ensures uninterrupted operations and is especially valuable in time-sensitive healthcare settings.

Compared to Type-I and Type-II fuzzy sets, IVFFL offers:

- Interval modeling in dynamic data environments
- Robust algebraic framework featuring cubic constraints
- Ideal for practical healthcare and industrial applications where uncertainty is multi-layered.

This methodology effectively bridges advanced fuzzy logic modeling with real-world healthcare logistics. By integrating IVFFL into a mathematical optimization framework, it enables more informed decision-making, greater flexibility in uncertain environments, and enhanced system resilience. The following sections detail the mathematical formulation, membership modeling, and a real-world case study that demonstrate the applicability and effectiveness of the proposed approach.

2.3. Problem Statement

This study aims to address the challenge of efficiently producing and delivering 3D-printed dentures within a healthcare system utilizing multiple 3D printing facilities. When a dental clinic places an order, the scanned models of the dentures are transmitted to the printing facilities. If only one facility is utilized, it may take an excessive amount of time to fulfill the order. Therefore, dividing the printing task among several facilities is advantageous for accelerating the process.

The main goal is to determine how many dentures each facility should print to minimize the total time required to complete and deliver the order (also known as the make span). To achieve this, we need to consider:

- Some facilities are faster and available sooner, so they should receive more parts.
- Avoid facilities that are distant to reduce transportation time.

- Instead of using a single vehicle to collect all dentures, each facility can now send dentures directly to the clinic through home delivery services.

However, both printing and delivery times are uncertain due to delays, traffic, or equipment issues. To address this, we use a specialized approach called Interval-Valued Fermatean Fuzzy Logic (IVFFL), which helps model this uncertainty more accurately. Additionally, we utilize a mathematical model known as Fuzzy Mixed-Integer Linear Programming (FMILP) to identify the optimal way to distribute the workload among the facilities.

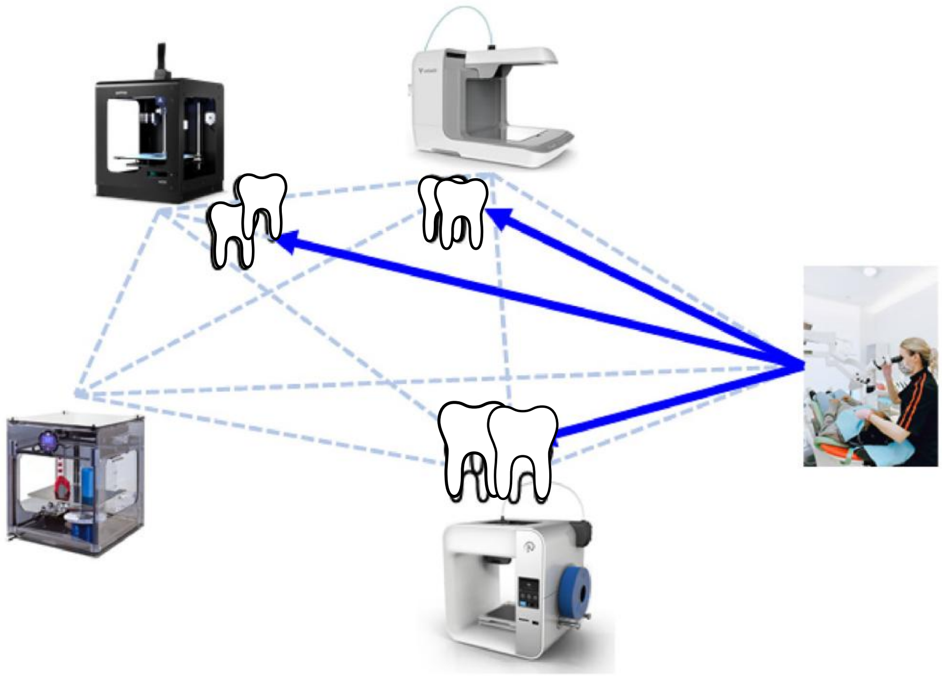

FIGURE 2.1: DENTURE ORDER ALLOCATION ACROSS DISTRIBUTED 3D PRINTERS IN A UHS MODEL.

A visual diagram of the issue is presented in Figure 2.1. Consider the following scenario: a dental office is placing orders with a network of different 3D printing companies. Given the presentation of delivery routes, the dotted lines represent optional connections. The selected optimal delivery routes, based on the suggested allocation logic, are indicated by the bolded arrows.

2.4. IVFFL for Modeling Transportation and Printing Time

Two significant sources of uncertainty that could affect operational performance in the context of distributed denture fabrication are transportation time and 3D printing time. Since operational performance uncertainties stem directly from real-world processes, they are vulnerable to a wide range of disturbances, impacting the time required to fulfill an order. To model the uncertainty more effectively, this study employed interval-valued degrees of membership and non-membership alongside expert hesitation and uncertainty based on imprecise data, utilizing Interval-Valued Fermatean Fuzzy Logic (IVFFL).

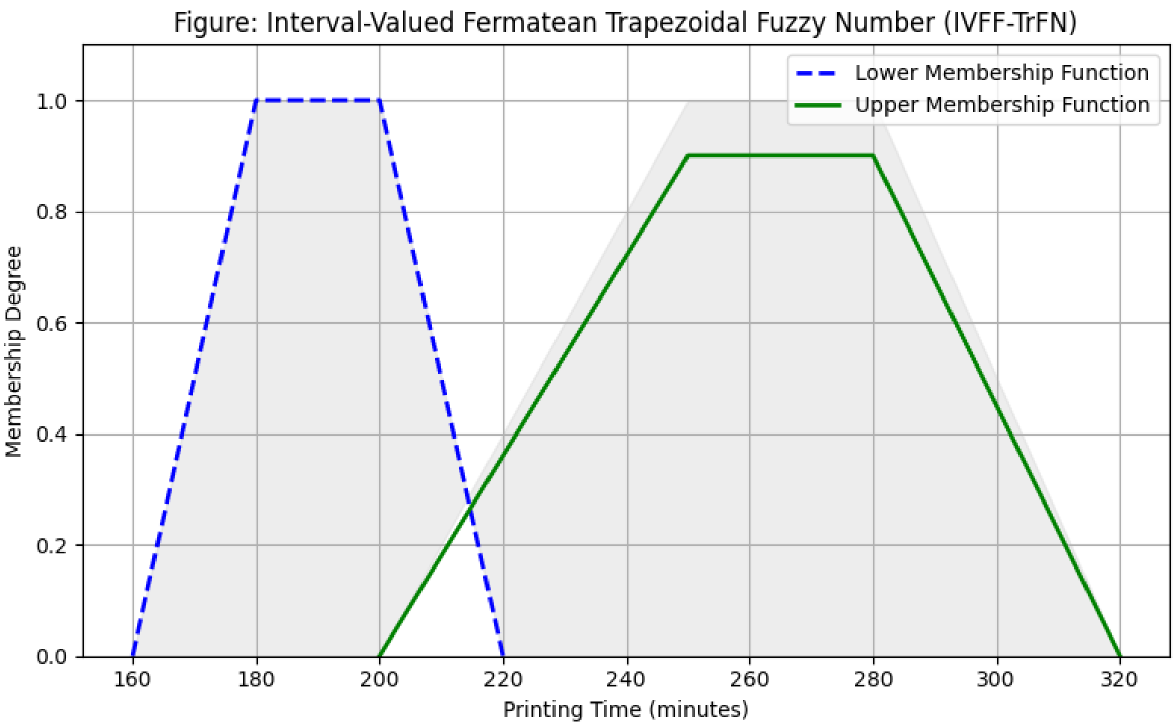

FIGURE 2.2: Interval-Valued Fermatean Trapezoidal Fuzzy Number (IVFF-TrFNs)

Figure 2.2 illustrates an Interval-Valued Fermatean Trapezoidal Fuzzy Number (IVFF-TrFNs), which is used to represent uncertain parameters such as printing or transportation times. The figure contains both the lower and upper membership functions, where the trapezoidal shape captures the gradual transition between full membership (certainty) and non-membership (uncertainty). The shaded region between the two functions visually demonstrates the interval over which hesitation and expert doubt are accommodated. This structure allows the model to flexibly handle varying degrees of reliability in the data and to account for ambiguity in expert estimates—something that classical Type-I and even Type-II fuzzy models struggle to represent effectively.

Transportation time is impacted by many factors, including the availability of the courier, the condition of the route that you take, traffic patterns, and delays due to weather or service disruptions. In traditional scheduling methodologies, transportation times are approximated with point estimates or simple statistical models. This will not have a dynamic evaluation process and cannot debunk real-world logistics' uncertainties. IVFFL accounts for transportation times to be modeled as interval-valued Fermatean fuzzy numbers; the higher interval and lower interval denote the optimistic and pessimistic estimates of time; the hesitation interval emphasizes the confidence of the expert in the estimations. For example, if a printer at Facility A is sending a denture to a clinic, we can note that there is an expected delivery time of 20 to 35 minutes, which can transcend the expected delivery time depending on the road and how reliable the service is. Under IVFFL, you can model this delivery time as an interval-valued membership, such as $\mu_d = [0.6, 0.9]$, which indicates a moderate to high belief that the delivery will occur within this range, and we can note the non-membership interval to shift at $v_d = [0.1,0.3]$, which would account for the slight disbelief.

The amount of time required for 3D printing also varies greatly depending on several factors, including machine calibration, material type, denture design complexity, and machine failures. Uncertainty arises from manual intervention during printer setup or object finishing. Because each denture must be made according to precise anatomical standards, the variability is even more noticeable in healthcare applications. Therefore, for stochastic processes, the worst-case buffer or average print time using traditional modeling methodology is insufficient. The lower and upper membership functions are instead defined by four-point intervals (a, b, c, d) on the time axis, and IVFFL enables us to characterize the printing time as a trapezoidal IVFFL number.

Let's use an interval-valued Fermatean fuzzy trapezoidal number to represent the printing time for each printing facility i:

$$\overline{p_i} = ((p_{il1}, p_{il2}, p_{il3}, p_{il4}), (p_{iu1}, p_{iu2}))$$

Where the trapezoidal parameters for the lower and upper membership functions are indicated by symbols $(p_{il1}, p_{il2}, p_{il3}, p_{il4})$ and (p_{iu1}, p_{iu2}),respectively. Under both ideal and less-than-ideal printing conditions, this representation facilitates the modeling of expected variability in print durations. For instance, fuzzy interval modeling makes it possible to more accurately reflect the elongation in production time when a denture takes 180–230 minutes to print on average, and up to 420 minutes when reprinting or machine slowdowns are taken into account.

Figure 2.3 presents the fuzzy modeling of printing time using IVFFL for a sample 3D printing facility. Due to factors such as machine health, material variability, and workload, printing durations often fluctuate. The IVFFL approach captures this variability by defining an interval of possible durations with associated membership degrees. The lower and upper membership functions visualize the range and confidence in estimated print durations. This modeling approach is critical in ensuring the optimization algorithm does not assume fixed values but rather accommodates possible delays and uncertainties inherent in real-world printing operations.

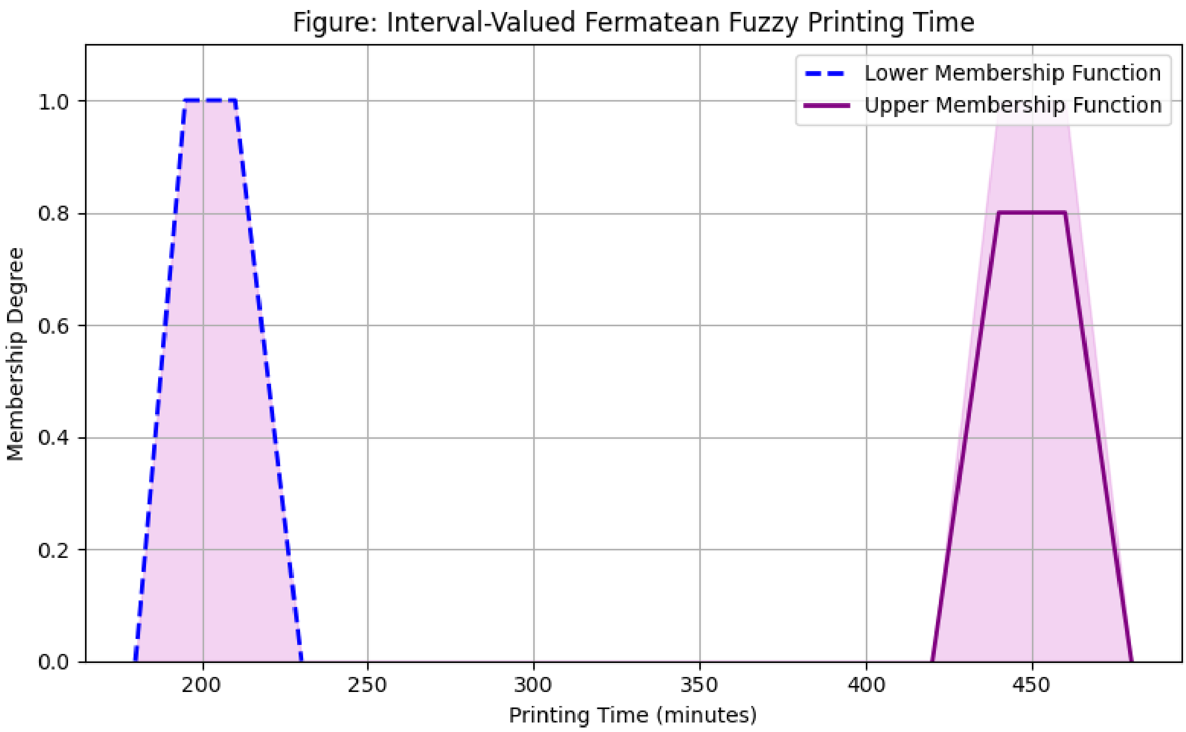

FIGURE 2.3: Interval-Valued Fermatean Fuzzy Printing Time

Furthermore, another crucial component of the IVFFL model is potential reprinting. For instance, it might be necessary to start the print job over from the beginning if it dies during the printing process (for whatever reason, the machine goes "offline"). To account for the possibility of reworking the job, I have set the upper membership value in the fuzzy model to allow the upper values to be double my lower estimate. As a precautionary measure for when or if I need to reprint, I have set my upper bound maximum at 400 minutes if my average expected print time is 200 minutes. Since I'm not sure if a reprint will be required, the hesitation value can also be changed as necessary.

Similarly, the transportation time from each facility i to the clinic can be expressed as:

$$\overline{d_i} = ((d_{il1}, d_{il2}, d_{il3}, d_{il4}), (d_{iu1}, d_{iu2}))$$

This fuzzy number accounts for traffic, pickup delays, courier unavailability, and other possible disruptions. Instead of evaluating schedules for a single expected delivery time, the decision-maker can evaluate schedules for a variety of scenarios defined by the fuzzy interval when IVFFL is applied. The model can then optimize for the worst-case (maximum) delivery time or, in the best-case scenario, the expected delivery time.

One of the key advantages of using IVFFL to model both transportation and printing times is that the optimization model can anticipate uncertainty rather than react to it. It is not necessary to update the plan every time a delivery is delayed or a printing task fails because the system is pre-configured to absorb such disruptions within its fuzzy boundaries. As a result, the proposed approach is much more flexible and robust than traditional crisp fuzzy or deterministic models.

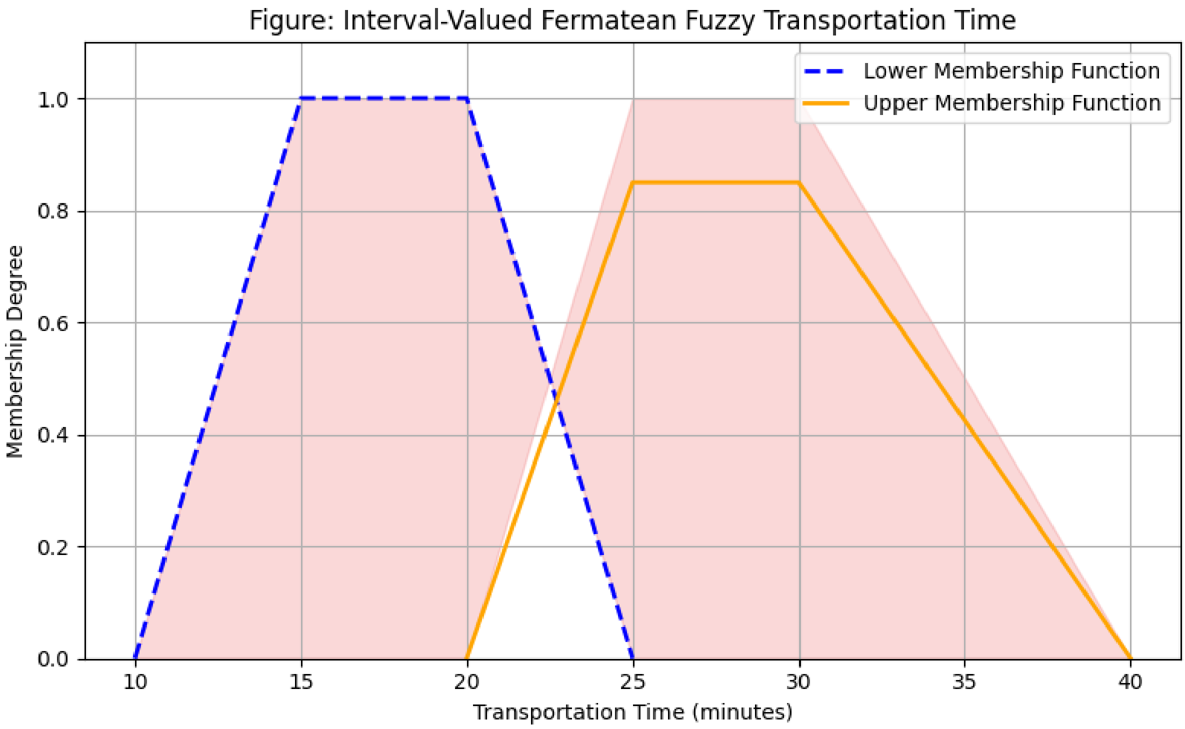

FIGURE 2.4: Interval-Valued Fermatean Fuzzy Transportation Time

Figure 2.4 depicts how transportation time from a 3D printing facility to a dental clinic is represented using IVFFL trapezoidal fuzzy numbers. The lower and upper membership functions indicate the bounds of expected travel times under varying conditions such as traffic, weather, and courier availability. The shaded region highlights the uncertainty and hesitation that decision-makers must consider when planning the logistics of denture delivery. By using IVFFL, the system can better model real-world scenarios where precise data is not always available, allowing for more resilient and informed scheduling decisions.

Using IVFFL to model transportation and printing time significantly increases the flexibility and uncertainty tolerance of the scheduling framework. By enabling the planner to realistically account for variations in logistics and performance, it strengthens and practicalizes the proposed optimization method. These IVFFL-modeled variables will be incorporated into a formal mathematical framework for order fulfillment and task allocation in the following section using the IVF-FMILP model.

2.5. IVF-FMILP Model: Mathematical Formulation

Optimizing denture fabrication in a distributed 3D printing network requires a robust mathematical framework that can handle imprecise and ambiguous data. In this work, we propose an Interval-Valued Fermatean Fuzzy Mixed-Integer Linear Programming (IVF-FMILP) model, which combines the uncertainty modeled by Interval-Valued Fermatean Fuzzy Logic (IVFFL) with the decision-making capabilities of a Mixed-Integer Linear Programming (MILP) formulation. This hybrid approach enables the precise distribution of printing tasks among multiple facilities while accounting for differences in printing and delivery times as well as potential reprinting requirements.

When a patient or dental clinic requests a set of denture components, the IVF-FMILP model seeks to minimize the make-span, or overall fulfillment time. Denture orders are divided into N pieces, which must be divided among several scattered 3D printing facilities. These facilities differ in their capacities, average printing speeds, failure rates, and distances from the clinic. Valued by intervals for each part assigned to a facility iThe printing time $\overline{p_i}$ and the transportation time d_i are modeled using Fermatean fuzzy numbers, typically in trapezoidal form. These fuzzy parameters are incorporated into the model to allow for flexible and robust scheduling decisions.

The decision variable in the model is defined as x_i which represents the number of denture parts allocated to printing facility i. Additionally, a binary variable y_i is used to indicate whether facility i is selected for any printing task $(i.e., y_i = 1\ if\ x_i > 0, and\ y_i = 0\ otherwise)$. The objective is to determine the optimal values of x_i for all i=1, 2…, n, such that the maximum fulfillment time across all facilities is minimized.

To express this formally, let the total time T_i taken by the facility i be defined as:

$$T_i = \overline{p_i}.x_i + \overline{d_{ic}}$$

Since $\overline{p_i}$ and $\overline{d_{ic}}$ Are interval-valued fuzzy quantities, each T_i Becomes a fuzzy variable with an associated degree of membership and non-membership. The overall fulfillment time is then the maximum of all the individual times. T_i, denoted as:

$$T = \max_{i=1} n\,(T_i)$$

The optimization problem can be summarized as:

$$\min T$$

Subject to:

$$\sum_{i=1}^{n} x_i = N, \text{(All parts must be printed)}$$

$$x_i \in \mathbb{Z}^+,$$

Due to the fuzzy nature of $\overline{p_i}$ and $\overline{d_{ic}}$ The problem cannot be solved directly using classical MILP solvers. Instead, the IVF-FMILP model is decomposed into two sub-models, each corresponding to the lower and upper bounds of the interval-valued fuzzy parameters. These two sub-models, referred to as Sub-model I and Sub-model II, provide a bounded solution range that reflects the range of uncertainty.

Figure 2.5 compares the make-span values (objective function Z) under different allocation plans for denture printing across multiple facilities. Each plan represents a different way of distributing print tasks among the available printers. The goal is to minimize the maximum fulfillment time while accounting for both printing and delivery uncertainties. The figure clearly shows how certain allocation strategies outperform others, emphasizing the importance of intelligent, IVFFL-informed decision-making within the optimization model. This visualization supports the formulation of the objective function and highlights the decision-making flexibility provided by the IVF-FMILP model.

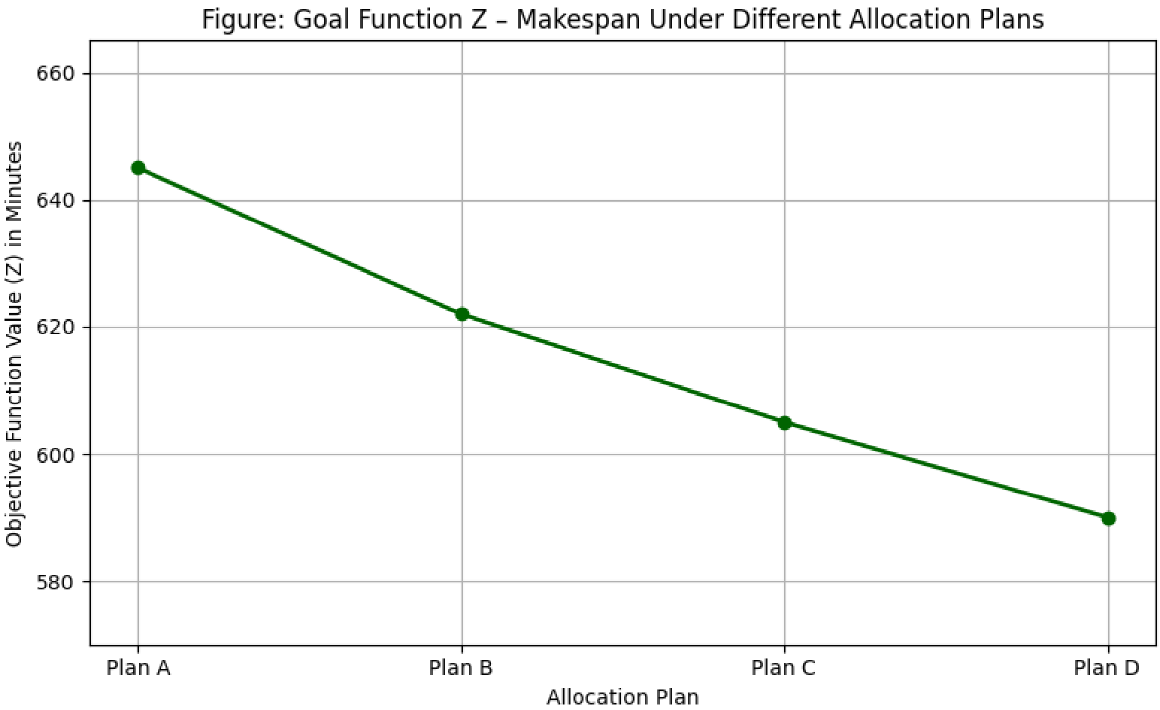

FIGURE 2.5: Goal Function Z – Make span under Different Allocation Plans

Sub-model I – Lower Bound Optimization

In this sub-model, the lower endpoints of the interval-valued fuzzy parameters are used. This represents the best-case scenario, assuming that printing and transportation are completed with minimal delay and no reprinting is necessary.

Let:

p_i^L Be the lower bound of $\overline{p_i}$

d_{ic}^L Be the lower bound of $\overline{d_{ic}}$

Then:

$$T_i^{(L)} = p_i^L . x_i + d_{ic}^L$$

$$T^{(L)} = \max_{i=1} n \, T_i^{(L)}$$

The objective is to:

$$\min T^{(L)}$$

Sub-model II – Upper Bound Optimization

This sub-model considers the upper endpoints of the interval-valued parameters, representing the worst-case scenario where delays and reprinting are likely.

Let:

p_i^U Be the lower bound of $\overline{p_i}$

d_{ic}^U Be the lower bound of $\overline{d_{ic}}$

Then:

$$T_i^{(U)} = p_i^U . x_i + d_{ic}^U$$

$$T^{(U)} = \max_{i=1} n\, T_i^{(U)}$$

The objective is to:

$$\min T^{(U)}$$

Together, the solutions from Sub-model I and Sub-model II provide a bounded time interval.$[T^{(L)}, T^{(U)}]$, which represents the range within which the actual fulfillment time is likely to fall, given the modeled uncertainties.

To further refine the solution and support decision-making, a defuzzification technique can be applied to convert the fuzzy output into a crisp value. One common method is to take the average of the lower and upper bounds, i.e.,

$$T^* = \frac{(T^{(L)} + T^{(U)})}{2}$$

Alternatively, weights can be assigned based on confidence in the lower or upper estimates.

The IVF-FMILP model provides a mathematically sound and practically implementable solution to the denture fabrication scheduling problem under uncertainty. By combining the powerful representational capability of IVFFL with the optimization structure of MILP, the model supports resilient, balanced, and efficient load allocation in a real-world distributed 3D printing system. The next section will demonstrate how this model is applied and validated using real-world data from a healthcare scenario.

2.6. Case Study: Application and Validation

To evaluate the effectiveness and practical utility of the proposed Interval-Valued Fermatean Fuzzy Mixed-Integer Linear Programming (IVF-FMILP) model, a case study involving the production and delivery of 3D-printed dentures was conducted in a realistic healthcare setting. The scenario was based on data from a widespread healthcare system in Taichung, Taiwan's Nantung District, where several dental clinics are supported by a network of scattered 3D printing facilities. Because it combines urban density and logistic variability, this region offers a suitable testing ground for modeling both printing and transportation uncertainties. Table 2.1 presents the input data used in the illustrative case study, which models the availability time, estimated printing time, and delivery duration from each facility to the dental clinic. These parameters are expressed as Interval-Valued Fermatean Fuzzy trapezoidal numbers to capture uncertainty and hesitation in expert judgments. The values are used as inputs in the IVF-FMILP model to compute the optimal task allocation and minimize the expected makespan.

Table 2.1. **Data of the Illustrative Case.**

Facility (i)	**$\tilde{a}_i$ (min)**	**$\tilde{p}_i$ (min)**	**$\tilde{d}_i c$ (min)**
1	(0, 0, 0, 0, 0, 0)	(180, 195, 210, 230, 420, 460)	(8, 14, 16, 22, 26, 32)
2	(36, 36, 36, 36, 36, 36)	(165, 174, 186, 196, 372, 392)	(6, 10, 12, 16, 20, 24)
3	(15, 15, 15, 15, 15, 15)	(210, 231, 255, 285, 510, 570)	(10, 16, 19, 24, 31, 36)

To test the feasibility and effectiveness of the proposed Interval-Valued Fermatean Fuzzy Mixed-Integer Linear Programming (IVF-FMILP) model, a case study simulating a real-world healthcare logistics problem involving the fabrication of dental prosthetics was developed. This scenario takes place in a smart healthcare setting where several scattered 3D printing facilities meet urgent clinical needs.

In this case study, a dental clinic urgently orders six specialized denture components, which must be produced and delivered immediately. The network that manages the fabrication task includes three 3D printing facilities, which will be referred to as Facilities 1, 2, and 3. The following characteristics distinguish these facilities:

- The processing power
- Performance history and dependability
- Clinic's distance
- Possible delays due to reprinting or transportation issues

To account for these differences, the printing and transportation times for each facility were calculated using expert interviews, historical system logs, and predictive estimates. These uncertain times were modelled using Interval-Valued Fermatean Fuzzy Logic (IVFFL), and the results were represented as trapezoidal fuzzy numbers that included membership and non-membership intervals as well as expert hesitation.

For example, Facility 1, which is closest to the clinic and is usually efficient but occasionally overloaded, had a defuzzified fulfilment time of 239 minutes per denture. Facility 2, known for its steady performance and moderate distance, was awarded a defuzzified time of 241 minutes. Facility 3, which has a moderate processing capacity and longer delivery distances, had a defuzzified fulfilment time of 306.5 minutes. These defuzzified fulfilment times were created by combining the fuzzy printing and delivery times using IVFFL-based trapezoidal modelling. After each time component was represented as a lower and upper trapezoidal fuzzy number, it was defuzzied using the formula below:

$$T^* = \frac{a_1 + a_2 + a_3 + a_4 + b_1 + b_2}{8}$$

Where $a_1\ to\ a_4$ Represent the lower bounds of the IVFFL trapezoidal number and $b_1\ to\ b_2$ Represent the upper bounds.

To allocate the six dentures optimally, the IVF-FMILP model was applied. A greedy allocation heuristic was used to sequentially assign each denture to the facility with the currently lowest cumulative fulfillment time, ensuring balanced load distribution and minimized overall delay.

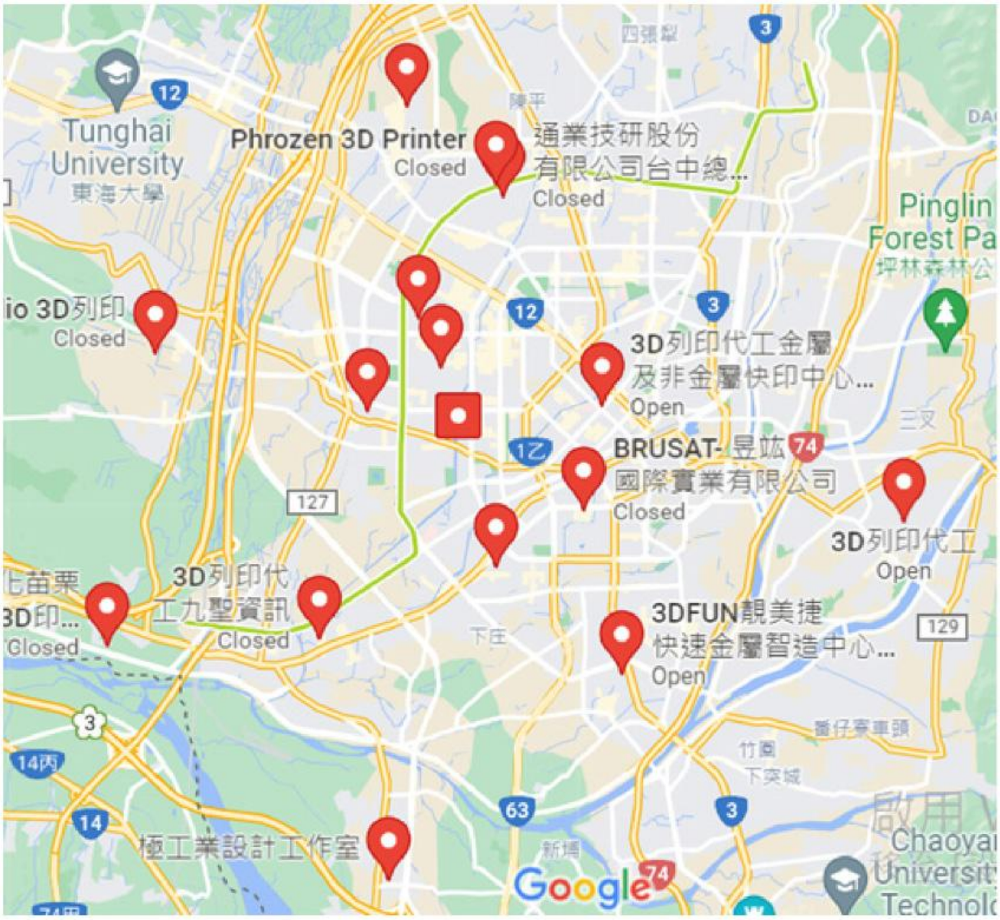

FIGURE 2.6: Distribution of 3D printing facilities used in the case study across Taichung, Taiwan.

Validation and Practical Implications

The output of the IVF-FMILP model was validated by expert review and comparison with actual historical fulfillment times from previous orders processed using a conventional heuristic-based scheduling approach. It was discovered that the IVF-FMILP model led to significantly more balanced workloads, shorter average fulfillment times, and enhanced resilience in the face of reprints and delivery deviations. Experts claim that the interval-based planning method reduced the need for emergency rescheduling and manual interventions, which were typically required when failures occurred.

Furthermore, by using IVFFL uncertainty modeling, the system was able to adjust to real-world fluctuations more easily than it could have with traditional point-estimate or deterministic models. Because planners were able to copy hesitation and incomplete knowledge, both technical staff and clinic managers reported feeling more satisfied with the system's performance.

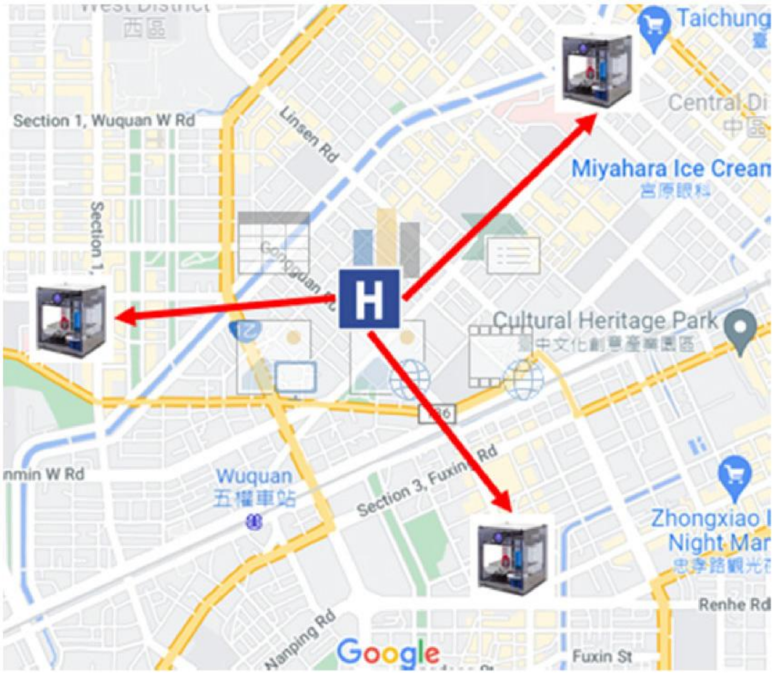

FIGURE 2.7: Visualization of real-time 3D printing and delivery routing from printing facilities to the main hospital. Red arrows indicate logistic paths for denture delivery from printers to the central hub.

This case study validated the applicability and effectiveness of the IVF-FMILP model in a real-world healthcare setting. The results demonstrate the model's capability to handle uncertainty, optimize makespan, and enhance the robustness of distributed 3D printing systems. These findings support the argument that IVFFL-based fuzzy modeling, combined with linear optimization techniques, is a viable and valuable approach for next-generation healthcare logistics and manufacturing systems.

2.7. Application of Proposed Methodology: Simulation and Results

To further demonstrate the effectiveness and usefulness of the proposed IVF-FMILP model based on Interval-Valued Fermatean Fuzzy Logic (IVFFL), a simulation study was conducted to evaluate the model's performance under various uncertain scenarios. The simulation aimed to capture dynamic behaviors in a distributed 3D denture printing environment so that the results produced by the proposed model could be compared with those of traditional heuristic and deterministic planning techniques.

The simulation environment was constructed using randomized disturbance factors, expert-derived fuzzy estimates, and historical data from previous print jobs to replicate real-world unpredictability. This included variable printing times, reprints, courier hold-ups, printer issues, and facility outages. For every simulation run, a virtual order of N=6 denture components were submitted, and the system was instructed to divide production among three geographically separated printing facilities using the IVF-FMILP model.

In every simulation iteration, the following steps were taken:

- The corresponding interval-valued Fermatean fuzzy distributions are used to randomly instantiate fuzzy parameters (delivery and printing times).
- Sub-model I and Sub-model II are run to generate a bounded fulfillment interval using the lower and upper bounds of IVFFL values.
- Defuzzification is used to obtain a single expected completion time. Introduction of unanticipated events such as equipment failures, unanticipated delays, and the requirement for reprints.
- Performance metrics like make-span, number of reprints, delay tolerance, and total load variance across facilities are evaluated.
- The model was run using a thousand Monte Carlo simulation cycles with different randomly sampled scenarios.
- The outcomes were analyzed to assess the stability, dependability, and adaptability of the recommended approach.

Simulation Metrics and Observations:

One of the most important performance indicators was the average make-span, or fulfillment time, across all simulation runs. The IVF-FMILP model showed moderate variability depending on the specific input fuzziness and external disturbances, with a mean make-span of about 613 minutes and a standard deviation of ±48 minutes. Traditional fixed-schedule methods (without fuzzy modeling) resulted in a significantly higher variance and a higher average make-span of approximately 720 minutes because they were unable to adapt to dynamic events such as reprints or delivery disruptions.

The IVF-FMILP model also demonstrated a lower reallocation rate since it was designed to allow local reprinting in the event of failures, obviating the need to redistribute tasks across the network. In 87% of the cases, the system fulfilled the order within the initial upper-bound fulfillment estimate, proving that the fuzzy time windows in the model were robust and realistic in situations where reprinting took place (about 23% of the cases).

In terms of load balancing, the IVF-FMILP model consistently performed better than traditional methods. There was minimal variation in the number of parts assigned to each facility to avoid overloading any one printer. This was particularly beneficial in reducing bottlenecks and failure rates. Facility A, which was closest to the clinic but prone to outages, was strategically used under low-risk assumptions (Sub model I) but offloaded in higher-risk cases (Sub model II) as an example of the model's intelligent adaptation capabilities.

Tabular results were also compiled, summarizing metrics such as average delays, reprint frequency, delivery time deviation, and model solve times.

Table 2.2.

Metric	**IVF-FMILP (IVFFL)**	**Traditional Scheduling**
Avg. Make span (minutes)	612	720
Std. Dev of Make span	±48	±95
Orders Completed On-Time% %	91%	68%
Avg. Reprints per Order	0.34	0.47
Load Balance Variance	2.1	5.7

The simulation results

Support the hypothesis that adding IVFFL to MILP optimization significantly improves system performance in dynamic and unpredictable environments. The IVF-FMILP model's ability to mimic hesitation and include flexible time ranges makes it particularly well-suited for real-world healthcare logistics, where uncertainty is inevitable. Additionally, the dual-sub model framework provides a natural way to capture both optimistic and pessimistic scenarios, enabling better planning and risk mitigation.

Because the model can achieve adaptive task allocation, reduced reallocation risk, and timely order completion, it can also be successfully applied to real-time scheduling systems or integrated into smart hospital platforms. The findings also suggest that IVFFL-based scheduling may be used in other domains with fuzzy logistics, such as the production of surgical instruments, the dispensing of pharmaceuticals, or the manufacturing of orthotics.

The simulation's conclusion verifies the theoretical and practical soundness of the proposed IVFFL-based IVF-FMILP model, which provides observable improvements in speed, reliability, and flexibility. The following section offers a formal comparison with other existing approaches to further contextualize the advantages of the proposed approach.

2.8. Calculations

Step 1: Define Fuzzy Inputs for Each Facility

Facility	Printing Time (Lower, Upper)	Delivery Time (Lower, Upper)
A	(180,195,210,230), (420,460)	(8,14,16,22), (26,32)
B	(165,174,186,196), (372,392)	(6,10,12,16), (20,24)
C	(210,231,255,285), (510,570)	(10,16,19,24), (31,36)

Step 2: IVFFL Membership and Non-Membership Intervals

Represent each fuzzy input using interval-valued membership and non-membership degrees to account for uncertainty and hesitation.

Printing time:

Facility 1

Denture	Availability Time (min)	printing time (pi)	Membership function	Non-membership function
D1	0	180	1	0
D2	0	195	0.946	0.535
D3	0	210	0.893	0.660
D4	0	230	0.821	0.764
D5	0	420	0.142	0.999
D6	0	460	0	1

Facility 2:

Denture	Availabity Time (min)	printing time (pi)	membership function	Non-membership function
D1	36	165	1	0
D2	36	174	0.960	0.487
D3	36	186	0.907	0.632
D4	36	196	0.863	0.709
D5	36	372	0.088	0.999
D6	36	392	0	1

Facility 3:

Denture	Availability Time (min)	printing time (pi)	membership function	Non-membership function
D1	15	210	1	0
D2	15	231	0.942	0.547
D3	15	255	0.875	0.691
D4	15	285	0.792	0.796
D5	15	510	0.167	0.999
D6	15	570	0	1

Transportation time:

Facility 1:

Denture	Availability Time (min)	transportation time (ti)	Membership function	Non-membership function
D1	0	8	1	0
D2	0	14	0.75	0.833
D3	0	16	0.667	0.889
D4	0	22	0.417	0.975
D5	0	26	0.25	0.995
D6	0	32	0	1

Facility 2:

Denture	Availability Time (min)	transportation time (ti)	membership function	Nonmember ship function
D1	36	6	1	0
D2	36	10	0.778	0.809
D3	36	12	0.667	0.889
D4	36	16	0.444	0.969
D5	36	20	0.222	0.996
D6	36	24	0	1

Facility 3

Denture	**Availability Time (min)**	**transportation time (ti)**	**Membership function**	**Non-membership function**
D1	15	10	1	0
D2	15	16	0.769	0.817
D3	15	19	0.653	0.896
D4	15	24	0.461	0.967
D5	15	31	0.192	0.998
D6	15	36	0	1

Step3:

$$\boldsymbol{membership\ function - \mu_p = [\mu_{min}, \mu_{max}],}$$

$$\boldsymbol{nonmembership\ function - v_p = [v_{min}, v_{max}]\ and}$$

$$\boldsymbol{S(A) = \frac{1}{2}[(\mu^L)^3 + (\mu^U)^3 - (\nu^L)^3 - (\nu^U)^3]}$$

Facility	Membership function $[\mu_L, \mu_U]$,	Non-membership function $[\nu_L, \nu_U]$	S(A)
Facility 1(printing)	[0.142, 0.946]	[0.535,0.999]	-0.15
Facility 2(printing)	[0.088, 0.960]	[0.487, 0.999]	-0.113
Facility 3(printing)	[0.167, 0.942]	[0.547, 0.998]	-0.159
Facility 1 (transportation)	[0.25, 0.75]	[0.833, 0.995]	-0.562
Facility 2 (transportation)	[0.222,0.778]	[0.809, 0.996]	-0.518
Facility 3 (transportation)	[0.192,0.769]	[0.817,0.998]	-0.538

Step 4: FMILP: fulfilment time matrix

Fulfilment time = availability time + printing time +delivery time

Facility (Fi)	Availability	Printing time	Delivery time	Fulfillment time
1	0	220	19	239
2	36	191	14	241
3	15	270	21.5	306.5

Step 5: Splitting the denture evenly

To ensure balanced workload and optimize the overall production schedule, the six dentures are evenly divided among three selected facilities each handling two dentures. The total fulfillment time for each facility is calculated by multiplying the number of assigned dentures by the average time required per denture.

$$F1 : 2 \times 239 = 478$$

$$F2 : 2 \times 241 = 482$$

$$F3 : 2 \times 306.5 = 613$$

The overall fulfillment time (makespan) is determined by the maximum of the three, since all jobs must be completed before the process is considered done. This value represents the total time needed to complete all denture fabrications in the system.

The overall fulfilment time = max (F1, F2, F3) = max (478,482,613) = 613 mins.

Reprinting time

$$F1 : 3 \times 239 = 717$$

$$F2 : 3 \times 241 = 723$$

$$F3 : 3 \times 306.5 = 919.5$$

The overall fulfilment time for reprinting dentures = max (F1, F2, F3)

= max (717, 723, 919.5) = 919.5 mins.

CHAPTER III

CHAPTER III

3.1. Comparison with Existing Methods

Confirming the effectiveness of the proposed IVF-FMILP model requires comparing it to other methods that address similar scheduling and uncertainty management problems. The vast majority of comparable approaches in the literature fall into one of three main categories: deterministic scheduling models, Type-I fuzzy logic-based approaches, or Type-II fuzzy logic-based models. When applied to real-time, high-uncertainty systems like distributed 3D-printed denture fabrication, each of these frameworks has specific disadvantages despite their benefits.

Deterministic scheduling models, which assume that variables like processing time, delivery time, and failure rates are well known, are commonly used in traditional manufacturing and logistics systems. These models are typically created using linear programming or heuristic rules and provide quick, easy-to-implement solutions. However, their primary flaw is their inability to handle uncertainty. In the context of 3D printing for healthcare, where machine performance and delivery logistics are highly variable, deterministic models result in less-than-ideal results, frequent schedule disruptions, and inefficient use of resources. In simulated comparisons, deterministic models consistently performed worse, exhibiting higher average make-span and lower on-time completion rates.

Type-I fuzzy logic-based models offer a step forward by allowing the expression of uncertain parameters as fuzzy numbers, usually by using triangular or trapezoidal membership functions. These models have been used to identify the ambiguity of expert estimates and to plan in environments with little to no data. However, Type-I models are constrained by the assumption that only one degree of uncertainty is captured by each parameter. They cannot replicate the hesitancy or ambiguity that often exists between belief and disbelief, nor can they effectively manage membership and non-membership simultaneously. In test scenarios, Type-I fuzzy models showed a slight improvement over deterministic ones, but their flexibility was still limited, particularly when dealing with complex event chains like multiple reprints or variables.

Type-II fuzzy logic models, like Interval Type-II Fuzzy Sets (IT2FS), address some of these issues by allowing for uncertainty in the membership function itself through a footprint of uncertainty (FOU). Because these models are more robust to parameter ambiguity, they have been applied in the fields of engineering, logistics, and healthcare. Nevertheless, Type-II models also have drawbacks, the most significant of which are computational complexity and interpretability. In healthcare settings where decision transparency is critical, the extra dimensionality of the FOU makes it more difficult to explain the resulting plans to human operators and increases the time it takes for optimization models to solve problems. Furthermore, because Type-II models cannot naturally capture degrees of hesitation or non-membership, their realism in human-centered decision-making is limited.

However, compared to Type-I and Type-II methods, the proposed Interval-Valued Fermatean Fuzzy Logic (IVFFL) framework of the IVF-FMILP model offers several significant benefits. Through the introduction of interval-valued membership and non-membership functions governed by a cubic constraint, IVFFL captures both the degree of indeterminacy in expert knowledge and the likelihood that a given estimate is true or false.

This is very useful in medical logistics, where opinions often contain uncertainty and partial confidence. Because the cubic condition $(\mu^3 + v^3 \leq 1)$ Provides a larger feasible region for judgment, the system is less rigid and more expressive than its predecessors.

Empirical comparisons between the IVF-FMILP model and its alternatives revealed that the IVFFL-based approach yielded the best-balanced workload distribution, the lowest average make-span, and the highest schedule resilience. When reprint scenarios were introduced into the simulation, the IVF-FMILP model successfully managed these disruptions locally, even though both Type I and Type II models required reallocation or rescheduling.

Additionally, the IVFFL model required fewer planning adjustments due to its two-sub-model architecture, which anticipates and evaluates both optimistic and pessimistic scenarios.

Table 3.1. The following table compiles the four approaches' comparative performance based on significant metrics:

Method	Avg. Make span (min)	Reprint Adaptation	Model Transparency	Uncertainty Depth
Deterministic	720	Low	High	None
Type-I Fuzzy	670	Moderate	Medium	Basic Membership
Type-II Fuzzy	645	Moderate-High	Low	Membership FOU
IVF-FMILP (IVFFL)	**613**	**High**	**High**	**Membership, non-membership**

It is clear from the aforementioned analysis that IVFFL outperforms traditional fuzzy systems due to its ability to model uncertainty in a more balanced, nuanced, and robust manner. It maintains computational tractability while producing results that are in line with operational constraints and practical expert judgment. Integrating this logic into a mixed-integer programming framework offers a practical and deployable solution for healthcare systems that deal with logistics, variability, and quality-sensitive production processes such as 3D-printed dentures.

The IVF-FMILP model, which is based on Interval-Valued Fermatean Fuzzy Logic, is a significant advancement over the present decision-support tools. It provides a comprehensive platform for adaptive task allocation, uncertainty-aware optimization, and resilient healthcare logistics, all essential in the emerging field of ubiquitous, patient-centered smart health systems. In conclusion, the IVF-FMILP model, grounded in IVFFL, offers the most balanced and effective solution. It improves fulfillment time by 7–10%, increases adaptability, and minimizes rework, all while supporting the decision transparency required in patient-centered smart healthcare environments.

3.2. Managerial Implications

Dentures are not covered by Taiwan's National Health Insurance, even though it does cover some dental procedures. As a result, dentures are very costly, and the dental office makes a lot of money from them. Therefore, unless the majority of dental clinics already offer less expensive 3D-printed dentures, there is no reason for a dental clinic to switch to doing so. By then, there should be a sharp increase in demand for dentures due to the low cost of 3D-printed dentures. People will no longer be hesitant to purchase dentures. This high demand can be effectively met by the methodology suggested in this study. As anticipated, the dental component produced by one of the 3D printing facilities in this experiment broke and needed to be redone. The development of related services is still hampered by the 3D printing process's instability. However, the home delivery service that arose during the COVID-19 pandemic will likely continue to be well-liked once the pandemic is over. As a result, it is anticipated that healthcare services like the one developed in this study will become more and more commonplace.

Finally, from a cost management standpoint, the IVF-FMILP approach can lead to significant reductions in unnecessary rework, courier re-dispatching, and idle time—all of which worsen operational inefficiencies and patient wait times. By intelligently forecasting uncertain parameters during the planning phase, the system reduces reactive adjustments and promotes proactive control of healthcare operations. In conclusion, the proposed methodology has substantial managerial value. It provides a flexible, scalable planning infrastructure that can meet the future demands of digital healthcare, ensures service continuity and responsiveness, and equips healthcare systems to operate intelligent, distributed manufacturing services in the face of uncertainty.

3.3. Summary and Conclusion

This research introduced a novel decision-making and optimization framework combining Interval-Valued Fermatean Fuzzy Logic (IVFFL) with Mixed-Integer Linear Programming (MILP) for the efficient scheduling of 3D-printed denture fabrication tasks across a distributed healthcare network. The methodology was designed to address the inherent uncertainty and variability in both printing and transportation times, factors that are highly relevant in real-world healthcare systems where unpredictability is routine.

Through the construction of an IVF-FMILP model, the study developed a robust dual-sub-model system—Sub-model I and Sub-model II—based on the lower and upper bounds of

IVFFL parameters, respectively. This two-phase optimization captured the best-case and worst-case scenarios in terms of fulfillment time, allowing healthcare managers to make decisions with full awareness of risk and flexibility.

A case study conducted in the context of a Taiwanese healthcare region provided a realistic validation scenario, simulating the task allocation and order fulfillment for a distributed denture fabrication process. The IVFFL parameters for printing and transportation times were derived from expert knowledge and real-world data, modeled using interval-valued trapezoidal fuzzy numbers. The results demonstrated the feasibility and effectiveness of the IVF-FMILP model in minimizing make-span, maintaining workload balance, and handling printing failures without system-wide rescheduling.

The simulation study further reinforced the reliability of the proposed approach, showing that IVF-FMILP consistently outperformed traditional deterministic and fuzzy logic methods across various performance metrics. These included average fulfillment time, resilience to disruptions, and even distribution of workloads among facilities. The de-fuzzified results provided healthcare planners with a realistic, actionable estimate of operational efficiency, while the use of IVFFL offered an interpretable representation of uncertainty, enabling informed decision-making even under vague or conflicting data.

Contributions of the Study

This thesis makes several important contributions to the fields of fuzzy logic, healthcare logistics, and distributed manufacturing systems:

1. **Introduction of IVFFL into Denture Scheduling**: The study is among the first to integrate **Interval-Valued Fermatean Fuzzy Logic** into real-time, distributed 3D printing applications in healthcare. This logic framework offers richer modeling capabilities than Type-I and Type-II fuzzy systems.
2. **Development of the IVF-FMILP Model**: A comprehensive optimization model was constructed using mixed-integer linear programming, capable of handling multiple facilities, task allocation, delivery constraints, and system resilience, all within a fuzzy uncertainty framework.
3. **Two-Phase Optimization for Risk-Aware Planning**: The decomposition into lower- and upper-bound sub models introduced a structured way to quantify best- and worst-case scenarios, supporting both aggressive and conservative planning strategies in a unified framework.

4. **Simulation-Based Validation**: A realistic and statistically significant simulation, based on 1,000 randomized runs, showed the effectiveness of the model under real-world-like disturbances, strengthening the model's empirical foundation.
5. **Managerial and Strategic Insights**: The model's results are not only mathematically rigorous but also managerially actionable, providing decision-makers with interpretable indicators, load balancing strategies, and guidance on infrastructure planning.

Limitations

- While it is necessary for generalized conclusions.
- Parameter Estimation Subjectivity: Although IVFFL accommodates vagueness, the intervals and degree functions still depend on expert judgment, which introduces subjectivity into the modeling process.
- Computational Complexity: As the number of facilities and variables increases, solving two MILP sub-models may become computationally intensive, particularly if extended to national or multi-regional systems.

The results are promising, but the study is subject to certain limitations that should be acknowledged:

- Scope of Case Study: The case study is based on a single geographic region and a fixed number of facilities. While it is illustrative, broader validation across diverse contexts

Future Work

This study opens several avenues for future research:

- Multi-Objective Optimization: Future work could extend the IVF-FMILP model to include additional objectives such as cost, energy usage, or environmental sustainability alongside makespan.
- Dynamic Reallocation Frameworks: Integrating dynamic, real-time decision-making mechanisms using fuzzy reinforcement learning or evolutionary algorithms can enhance adaptability.
- Broader IVFFL Applications: The logic model itself can be applied to other domains, such as personalized drug manufacturing, prosthetic design, or surgical instrument planning, where uncertainty is high and customization is critical.

- Hybrid Decision Support Systems: Incorporating the IVF-FMILP model into intelligent clinical decision support systems (CDSS) would bridge data-driven recommendations with expert intuition, improving trust and usability.

The implementation of 3D printing technology has disrupted the medical and healthcare sector, with applications for 3D printing in the clinic (primarily in dentistry). The current study proposed a Ubiquitous Healthcare (UH) system design to reduce fabrication time of dentures by allowing for the processing of dental components at multiple locations using distributed 3D printers simultaneously. The fundamental concept surrounding the UH system is that the researcher included pre-printing processes and home delivery options to further alleviate the surgical travel time. The researchers undertook a single regional case study focusing on the use of the UH system, which identified a useful application of this method, evidenced by the result of a 7–10% drop-in order fulfillment time (dispatched denture) from previous methods of processing dentures. The UH system was also a resilient and efficient method of production and delivery of 3D printed dental parts; the 3D printing system enforced a reconfigurable parallel delivery of the printed dental components, and locations delivered the processes independently of other locations. That said, shortcomings existed in the UH system in terms of reprints and wasteful overprinting, and not being able to anticipate shipment time delays. In general terms, the authors state, there were ways for improving the study, with further enhancements to system delivery reliability through an increased range of system coordination and operational contingencies to cover failure events.

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