Chapter 4

J-Separation Axioms in Topological Spaces

§ 4.1. Introduction

Topological spaces are classified according to which such conditions, called separation axioms. Separation axioms are one among the most common, important and interesting concepts in Topology. They can be used to define more restricted classes of topological spaces. The separation axioms of topological spaces are usually denoted with the letter "T" after the German "Trennung" which means separation.In this Chapter, we have introduced some exciting new spaces like JTδ-space, JTC-space, JTδg-space and JTg-space, JTδg*-space, gδTJ-space and the dependence as well as independence of new spaces with other existing separation axioms are analysed. Also interrelations of these new six spaces are investigated.

§ 4.2. J-Separation Axioms

As an outcome of J-closed sets, six new spaces are introduced and interrelations of these new six spaces are investigated.

Definition 4.2.1. A topological space (Y,ζ) is known as **JTδ-space** when each J-closed set is a δ-closed set in (Y,ζ).

Example 4.2.2. Consider Y={p,q,r,s},ζ={ϕ,Y,{p},{q},{r},{p,q},{q,r},{p,r},{p,q,r},{p,q,s}}.Here J-closed sets coincide with δ-closed sets.

Definition 4.2.3. A topological space (Y,ζ) is known as **JTg- space** when each J-closed set is a g-closed set in (Y,ζ).

Example 4.2.4. Consider Y={p,q,r,s},ζ={ϕ,Y,{p}}.Here J-closed sets coincide with g-closed sets.

Definition 4.2.5 A topological space (Y,ζ) is known as **JTC-space** when each J-closed set is a closed set in (Y,ζ).

Example 4.2.6. Consider Y={p,q,r},ζ={ϕ,Y,{p},{p,q},{q}}.Here J-closed sets coincide with closed sets.

Definition 4.2.7. A topological space (Y,ζ) is known as **JTδg*-space** when each J-closed set is a δg*-closed set in (Y,ζ).

Example 4.2.8. Consider Y={p,q,r,s},ζ={ϕ,Y,{p,q},{r},{p,q,r}}.Here J-closed sets coincide with δg*-closed sets.

Definition 4.2.9. A topological space (Y,ζ) is known as **JTδg-space** when each J-closed set is a δg-closed set in (Y,ζ).

Example 4.2.10. Consider Y={p,q,r},ζ={ϕ,Y,{p},{q,r}}.Here J-closed sets coincide with δg-closed sets.

Definition 4.2.11. A topological space (Y,ζ) is known as **gδTJ- space** when each gδ-closed set is a J-closed set in (Y,ζ).

Example 4.2.12. Consider Y={p,q,r,s},ζ={ϕ,Y,{p,q,r}}.Here J-closed sets coincide with gδ-closed sets.

§ 4.3. JTδ-space

A JTδ-space is a very strong tool to characterise the following Propositions.

Proposition 4.3.1. If (Y,ζ) is a JTδ-space, then it becomes a JTC-space, a JTδg*-space, a JTδg -space and a JTg-space.

Proof By **Note 2.2.21.** and **Remark 2.3.20.** closed

δ-closed → δg*-closed → δg-closed → g-closed → J-closed and from the **Definition 4.2.1.,4.2.5.,4.2.7,4.2.9. and 4.2.3.**,the **Proposition 4.3.1.** follows immediately.

Result 4.3.2. The converse of the above Proposition 4.3.1. does not hold good. It can be shown by the following Counter Examples.

Counter Example 4.3.3. Consider Y={u,v,w,x},ζ ={ϕ, Y, {u,v},{w},{u,v,w}}.Here (Y,ζ) represents JTδg*-space but it is not JTδ -space. Because {u,x} is J-closed but it is not δ-closed.

Counter Example 4.3.4. Consider Y={u,v,w},ζ ={ϕ, Y, {u},{v,w}}.Here (Y,ζ) represents JTδg-space and JTg-space but it is not JTδ-space. Because {w} is J-closed but it is not δ-closed.

Note 4.3.5. A JTδ-space is also a JTC-space, a JTδg - space, a JTg -space and a JTδg* - space.

Remark 4.3.6.(i) Since δ-closed ⟶ closed ⟶ J-closed(by Note 2.2.21. and Remark 2.3.20.). In a JTδ-space, the collection of all δ-closed sets, all closed sets and all J-closed sets coincide.

(ii) Since δ-closed ⟶ closed ⟶ g-closed ⟶ J-closed (by Note 2.2.21. and Remark 2.3.20.). In a JTδ-space, the collection of all δ-closed sets, all closed sets, all g-closed sets and all J-closed sets coincide.

(iii) Since δ-closed ⟶ δg*- closed ⟶ δg-closed ⟶ g -closed ⟶ J -closed (by Note 2.2.21. and Remark 2.3.20.). In a JTδ-space, the collection of all δ-closed sets, all δg*-closed sets, all δg-closed sets and all g-closed sets, all J-closed sets coincide.

Proposition 4.3.7. If (Y,ζ) is a JTδ-space, then it becomes a gδTJ-space.

Proof In a JTδ-space, every J-open set is δ-open. By **Remark 4.3.6.(i)**, in a JTδ-space, δO(Y,ζ) = ζ = JO(Y,ζ) -----(1).Consider a gδ-closed set D in (Y,ζ).By definition of gδ-closed set,Cl(D)⊆ M whenever D⊆M where M is δ-open which is open by (1). Hence D is a g-closed set. By **Proposition 2.3.10**.,D is J-closed. Therefore (Y,ζ) is a gδTJ- space.

The converse of the above Proposition 4.3.7. is not true. It can be shown by the following Counter Example.

Counter Example 4.3.8. Consider Y={u,v,w,x},ζ ={ϕ, Y, {u,v},{w},{u,v,w}}.Here (Y,ζ) represents gδTJ-space but it is not a JTδ -space. Because {v,x} is J-closed but it is not δ-closed.

Now to analyse a JTδ-space with existing spaces.

Proposition 4.3.9. If (Y,ζ) is a JTδ -space, then it becomes $T_{3/4}$ -space.

Proof Let (Y,ζ) be a JTδ-space. Consider a δg-closed subset H of Y. By **Proposition 2.3.8.**, we get H is a J-closed set. In a JTδ-space, H is δ-closed. Therefore H is a $T_{3/4}$-space.

Remark 4.3.10. Every $T_{3/4}$-space is a digital line ((Khalimsky,1990),(Kong,1991)).

Corollary 4.3.11. JTδ-space is a digital line.

Note 4.3.12. The converse of above Proposition 4.3.9. does not hold good. The following Counter Example explains it.

Counter Example 4.3.13. Consider Y={u,v,w,x},ζ ={ϕ, Y,{u},{v},{u,v}, {u,v,w}, {u,v,x}}.Here (Y,ζ) represents $T_{3/4}$-space (since δg-closed sets coincide with δ-closed sets in this topology) but it is not a JTδ -space. Because {v,w} is J-closed not δ-closed.

Proposition 4.3.14. If (Y,ζ) is a $T_{3/4}$ and a JTδg-space, then it becomes a JTδ-space.

Proof Consider a J-closed subset H of Y. In JTδg-space, H is δg-closed. In a $T_{3/4}$ -space, H is δ-closed. Therefore (Y,ζ) is a JTδ-space.

Proposition 4.3.15. If (Y,ζ) is a JTδ-space, then it becomes $T_{1/2}$ - space.

Proof Given (Y,ζ) is a JTδ-space. Consider a g-closed subset H of Y. By **Proposition 2.3.10.**,H is J-closed. In JTδ-space, H is δ-closed. Always δ-closed is closed. From the above discussion, we get g-closed sets coincide with closed sets. Therefore H is a $T_{1/2}$-space.

Corollary 4.3.16. Each JTδ-space is a Kolmogorov-space or T_0-space by Theorem 5.2(Levine,1970).

Note 4.3.17. The converse of above Proposition 4.3.15. does not hold. From the following Counter Example, we get the clear idea.

Counter Example 4.3.18. Consider Y={u,v,w},ζ ={ϕ, Y,{u},{v},{u,v},{u,w}}.Here (Y,ζ) represents $T_{1/2}$-space(since g-closed sets coincide with closed sets in this topology) but it is not a JTδ-space. Because {w} is J-closed not δ-closed.

Result 4.3.19. Each JTδ -space is a semi-T_1-space.

Proof By **Proposition 4.3.9.**,every JTδ-space is a $T_{3/4}$-space.Now by Theorem 4.9 of (Dontchev,1996),every $T_{3/4}$-space is a semi-T_1-space.Hence JTδ-space is a semi-T_1-space.

Result 4.3.20. Each JTδ -space is a semi-pre-$T_{1/2}$-space.

Proof By **Proposition 4.3.9.**,every JTδ-space is a $T_{3/4}$-space.Now by Theorem 4.15 of (Dontchev,1996),every $T_{3/4}$ -space is a semi-pre-$T_{1/2}$-space. Hence JTδ-space is a semi-pre-$T_{1/2}$-space.

Note 4.3.21. The converse of above Results 4.3.19. and 4.3.20. do not hold. From the following Counter Examples, we get clear idea.

Counter Example 4.3.22. Consider Y={u,v,w},ζ ={ϕ, Y, {u}}.Here (Y,ζ) represents semi-T_1-space (since semi-generalized closed sets coincide with semi-closed sets in this topology)but it is not a JTδ-space. Because {u,v} is J-closed but it is not δ-closed.

Counter Example 4.3.23. Consider Y={u,v,w},ζ ={ϕ, Y, {u},{v,w}}.Here (Y,ζ) represents semi-pre-$T_{1/2}$-space (since gsp- closed sets coincide with semi-pre closed sets in this topology)but it is not a JTδ-space. Because {w} is J-closed but it is not δ-closed.

Proposition 4.3.24. If (Y,ζ) is a T_δ-space, then it becomes JTδ-space.

Proof Given (Y,ζ) is a T_δ-space. Consider a J-closed subset H of Y. By **Proposition 2.3.12.**,H is gδ-closed. In T_δ -space, we get H is a δ-closed set. Therefore H is a JTδ-space.

Proposition 4.3.25. If (Y,ζ) is a JTδ-space, then it becomes a *$T_{1/2}$-space.

Proof Consider a g-closed subset H of Y. By **Proposition 2.3.10.**,H is J-closed. In a JTδ -space, we get H is a δ-closed set. By Proposition 2.2.9 (Sudha,2014),H is g*-closed Therefore H is a *$T_{1/2}$-space.

Note 4.3.26. The converse of above Proposition 4.3.25. does not hold good. The following Counter Example explains it.

Counter Example 4.3.27. Consider Y={p,q,r},ζ={ϕ,Y,{p},{p,q},{p,r},{q}}.Here (Y,ζ) represents a *$T_{1/2}$-space(since g-closed sets coincide with g*-closed sets) ,but it is not a JTδ-space. Because {p,q} is J-closed not δ-closed.

Proposition 4.3.28. If (Y,ζ) is a JTδ-space, then it becomes a $T_{1/2}$*-space.

Proof Consider a g*-closed subset H of Y. By **Proposition 2.3.14.**,H is J-closed. In a JTδ -space, we get H is a δ-closed set. By **Note 2.2.21.**,H is closed. Therefore H is a $T_{1/2}$*-space.

Note 4.3.29. The converse of above Proposition 4.3.28. does not hold good. The following Counter Example explains it.

Counter Example 4.3.30. Consider Y={p,q,r},ζ={ϕ,Y,{p},{p,q},{p,r},{q}}.Here (Y,ζ) represents a $T_{1/2}$*-space (since g*-closed sets coincide with closed sets),but it is not a JTδ-space. Because {p,q} is J-closed not δ-closed.

Proposition 4.3.31. If (Y,ζ) is a JTδ-space, then it becomes a $_{\delta g*}T_{\delta}$-space.

Proof Consider a δg*-closed subset H of Y. By **Proposition 2.3.6.**, H is J-closed. In a JTδ -space, we get H is a δ-closed set. Therefore H is a $_{\delta g*}T_{\delta}$-space.

Note 4.3.32. The converse of above Proposition 4.3.31. does not hold good.The following Counter Example explains it.

Counter Example 4.3.33. Consider Y={p,q,r},ζ={ϕ,Y,{p},{q,r}}.Here (Y,ζ) represents a $_{\delta g*}T_{\delta}$-space (since δg*-closed sets coincide with δ-closed sets),but it is not a JTδ-space. Because {p,r} is J-closed not δ-closed.

Proposition 4.3.34. If (Y,ζ) is a JTδ-space, then it becomes a $_{\delta g}T_{\delta g*}$-space.

Proof Consider a δg-closed subset H of Y. By **Proposition 2.3.8.**, H is J-closed. In a JTδ -space, we get H is a δ-closed set,by Proposition 2.2.2. (Sudha,2014).Therefore H is a $_{\delta g}T_{\delta g*}$-space.

Note 4.3.35. The converse of above Proposition 4.3.34. does not hold good. The following Counter Example explains it.

Counter Example 4.3.36. Consider Y={p,q,r},ζ={ϕ,Y,{p},{p,q}}.Here (Y,ζ) represents a $_{\delta g}T_{\delta g*}$-space (since δg-closed sets coincide with δg*-closed sets),but it is not a JTδ-space. Because {q,r} is J-closed not δ-closed.

Proposition 4.3.37. If (Y,ζ) is a JTδ-space, then it becomes a $_{g}T_{\delta g*}$-space.

Proof Consider a g-closed subset H of Y. By **Proposition 2.3.10.**, H is J-closed. In a JTδ -space, we get H is a δ-closed set, by Proposition 2.2.2. (Sudha,2014). Therefore H is a ${}_{g}T_{\delta g*}$-space.

Note 4.3.38. The converse of above Proposition 4.3.37. does not hold good.The following Counter Example explains it.

Counter Example 4.3.39. Consider Y={p,q,r},ζ={ϕ,Y,{p,q}}.Here (Y,ζ) represents a ${}_{g}T_{\delta g*}$-space (since g-closed sets coincide with δg*-closed sets),but it is not a JTδ-space. Because {p,q} is J-closed not δ-closed.

Proposition 4.3.40. If (Y,ζ) is a JTδ-space, then it becomes a ${}_{g*}T_{\delta g*}$-space.

Proof Consider a g*-closed subset H of Y. By **Proposition 2.3.14.**, H is J-closed. In a JTδ -space, we get H is a δ-closed set by Proposition 2.2.2. (Sudha,2014). Therefore H is a ${}_{g*}T_{\delta g*}$-space.

Note 4.3.41. The converse of above Proposition 4.3.40. does not hold good.The following Counter Example explains it.

Counter Example 4.3.42. Consider Y={p,q,r},ζ={ϕ,Y,{p,q}}.Here (Y,ζ) represents a ${}_{g*}T_{\delta g*}$-space,but it is not a JTδ-space. Because {p,q} is J-closed not δ-closed.

Remark 4.3.43. T_b and T_d -spaces are independent with JTδ-space. This can be seen from the following Counter Examples.

Counter Example 4.3.44. Consider Y={p,q,r},ζ={ϕ,Y,{p},{p,q},{q}}.Here (Y,ζ) represents JTδ-space,but not T_b-space(since gs-closed sets coincide with closed sets) and T_d-space(since gs-closed sets coincide with g-closed sets). Because {p} is gs-closed not closed and {q} is gs-closed but not g-closed respectively.

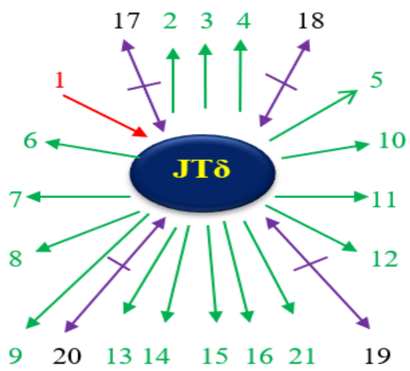
Counter Example 4.3.45. Consider Y={u,v,w},ζ ={ϕ, Y,{u},{v},{u,v},{u,w}}.Here (Y,ζ) represents both T_b-space (since gs-closed sets coincide with closed sets) and T_d-space (since gs-closed sets coincide with g-closed sets) but it is not JTδ-space. Because {w} is J-closed not δ-closed.

Note 4.3.46. T_c and ${}_{gs}T_{\delta g*}$ spaces are independent with JTδ-space.It can be seen from the following Counter Examples.

Counter Example 4.3.47. Consider Y={p,q,r},ζ={ϕ,Y,{p},{p,q},{q}}.Here (Y,ζ) represents JTδ-space,but not T_c-space,because {p} is gs-closed not g*-closed and $_{gs}T_{\delta g^*}$-space as {q} is gs-closed but not δg*-closed respectively.

Counter Example 4.3.48. Consider Y={u,v,w},ζ ={ϕ, Y,{u,v}}.Here (Y,ζ) represents T_c and $_{gs}T_{\delta g^*}$-space but it is not JTδ-space. Because {u} is J-closed not δ-closed.

Note 4.3.49. The following picture gives the relations of JTδ-space with other spaces.



1.T_δ-space	2.JTC-space	3. JTδg-space
4. JTg-space	5. JTδg*-space	6. gδTJ-space
7.$T_{1/2}$-space	8.T_0-space	9.$T_{3/4}$-space
10.*$T_{1/2}$-space	11. semi-T_1-space	12.semi-pre-$T_{1/2}$-space
13.$T_{1/2}$*-space	14. $_{\delta g}T_{\delta g^*}$-space	15. $_{\delta g^*}T_\delta$-space
16.$_gT_{\delta g^*}$-space	17.T_c-space	18.$_{gs}T_{\delta g^*}$-space
19. T_b-space	20.T_d-space	21. $_{g^*}T_{\delta g^*}$-space

§ 4.4. JTg-space

This chapter is devoted to analyse the comparison to analyse of a JTg-space with newly defined spaces and existing spaces.

Proposition 4.4.1. If (Y,ζ) is a $T_{3/4}$ and a JTδg-space, then it becomes a JTg-space.

Proof By **Proposition 4.3.14.**, (Y,ζ) is a JTδ-space which inturn becomes JTg-space by **Note 4.3.5.**

Proposition 4.4.2. If (Y,ζ) is a T_δ-space, then it becomes JTg-space.

Proof By **Proposition 4.3.24.**, a T_δ-space is a JTδ-space.Then by **Note 4.3.5.,**it is a JTg-space also.

Note 4.4.3. The converse of above Proposition 4.4.2. does not hold good.The following Counter Example explains the concept.

Counter Example 4.4.4. Consider Y={u,v,w},ζ={ϕ,Y,{u}}.Here (Y,ζ) represents a JTg-space but it is not T_δ -space. Because {u} is a gδ -closed set not a δ-closed set.

Proposition 4.4.5. If (Y,ζ) is both a JTg-space and a $T_{1/2}$-space, then it becomes a JTC-space.

Proof Consider a J-closed subset H of Y. In a JTg-space,H becomes a g-closed set.In a $T_{1/2}$-space,g-closed sets coincide with closed sets which in turn gives that H is closed. Therefore (Y,ζ) is a JTC-space.

Proposition 4.4.6. If (Y,ζ) is both a T_d-space and a JTg-space, then the following closed sets are equivalent:

(i) A gs-closed set
(ii) A J-closed set.

Proof (i) ⇒ (ii) In a T_d-space, gs-closed sets coincide with g-closed sets.By **Proposition 2.3.10.**,g-closed sets is a J-closed set.

(ii) ⇒ (i) In a JTg-space,J-closed sets coincide with g-closed sets.In T_d-space, gs-closed sets coincide with J-closed sets.

Proposition 4.4.7. If (Y,ζ) is both a T_b-space and a JTg-space, then the following closed sets are equivalent:

(i) A gs-closed set
(ii) A J-closed set.

Proof The proof follows **Proposition 4.4.6.**,since every T_b-space is T_d-space.

Proposition 4.4.8. A topological space (Y,ζ) is a semi-regular space, then (Y,ζ) becomes a JTg -space.

Proof Given (Y,ζ) is a semi-regular space. Consider a J-closed subset H of Y. By **Proposition 2.3.12.,**H is gδ-closed. In a semi-regular space, H is g-closed by Theorem 5.8 (Dontchev,1996).Hence (Y,ζ) becomes a JTg-space.

The converse of above Proposition 4.4.8. does not hold good.

Counter Example 4.4.9. Consider Y={u,v,w},ζ={ϕ,Y,{u}}.Here (Y,ζ) represents a JTg-space but it is not a semi-regular space because {v,w} is closed not δ-closed.

Proposition 4.4.10. If (Y,ζ) is both a JTg-space and a $T_{1/2}$-space, then it becomes a $T_{1/2}$*-space.

Proof Consider a g*-closed subset H of Y. By **Proposition 2.3.14.**,H is a J-closed set. In a JTg-space,H becomes a g-closed set.In a $T_{1/2}$-space,g-closed sets coincide with closed sets which in turn gives that H is closed.Therefore (Y,ζ) is a $T_{1/2}$*-space.

The converse of the above Proposition 4.4.10. is not true. It can be seen from the following Counter Example.

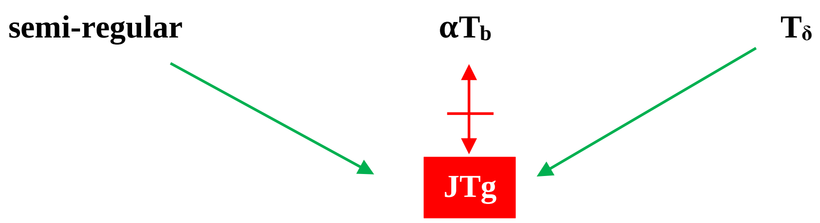
Counter Example 4.4.11. Consider Y={u,v,w},ζ={ϕ,Y,{u}}.Here (Y,ζ) represents a $T_{1/2}$*-space(since g*-closed sets coincide with closed sets), a JTg-space but it is not a $T_{1/2}$ -space.Since {q} is a g-closed set but not a closed set.

Remark 4.4.12. A JTg -space is independent with a αT_b-space. This can be seen from the following Counter Example.

Counter Example 4.4.13. Consider Y={u,v,w},ζ={ϕ,Y,{u}}.Here (Y,ζ) represents a JTg-space but it is not a αT_b -space(since αg-closed sets coincide with closed sets) because {v} is αg-closed not closed.

Counter Example 4.4.14. Consider Y={u,v,w},ζ={ϕ,Y,{u},{v},{u,v},{u,w}}.Here (Y,ζ) represents a αT_b -space(since αg-closed sets coincide with closed sets), but it is not a JTg -space. Because {u} is J-closed not g-closed.

Note 4.4.15. From the above discussions JTg-space is dependent with different existing separation axioms in the following manner. The following figure depicts the isolation of JTg-space with αT_b-space.



§ 4.5. JTC-space

A JTC-space is an interesting application of J-closed sets. In this section we analyse this space with some spaces in literature.

Proposition 4.5.1. If (Y,ζ) is a JTC-space, then it becomes JTg-space.

Proof Given (Y,ζ) is a JTC-space. Consider a J-closed subset H of Y. In a JTC-space, H is closed. In general every closed set is g-closed. Hence the result follows.

Result 4.5.2. The converse of the above Proposition 4.5.1. is not true. It can be shown by the following Counter Example.

Counter Example 4.5.3. Consider Y={u,v,w},ζ ={ϕ, Y, {u},{v,w}}.Here (Y,ζ) represents JTg-space but it is not a JTC-space. Because {v} is J-closed but it is not closed.

Proposition 4.5.4. If (Y,ζ) is a JTC-space, then it becomes $T_{1/2}$ -space.

Proof Given (Y,ζ) is a JTC-space. Consider a g-closed subset Hof Y. By **Proposition 2.3.10.**,Then H is J-closed. In JTC-space, H is closed. Therefore H is a $T_{1/2}$-space.

Corollary 4.5.5. Each JTC-space is a T_0 -space.

Note 4.5.6. The converse of above Proposition 4.5.4. does not hold. From the Counter Example, we get clear idea.

Counter Example 4.5.7. Consider Y={u,v,w},ζ ={ϕ, Y,{u},{v},{u,v},{u,w}}.Here (Y,ζ) represents a $T_{1/2}$-space(since g-closed sets coincide with closed sets in this topology) but it is not a JTC-space. Because {u,v} is J-closed but not closed.

Proposition 4.5.8. If a given space (Y,ζ) is a T_b-space and a JTC-space, then gs-closed sets coincide with J-closed sets.

Proof The proof follows from the definitions of T_b -space and JTC-space.

Proposition 4.5.9. If (Y,ζ) is a JTC-space, then it becomes *$T_{1/2}$ -space.

Proof Given (Y,ζ) is a JTC-space. Consider a g-closed subset H of Y. By **Proposition 2.3.10.**,H is J-closed.In JTC-space, H is closed. Since closed set is a g*-closed set (Veerakumar, 2000).Therefore H is a *****$T_{1/2}$-space.

Note 4.5.10. The converse of above Proposition 4.5.9. does not hold. From the following Counter Example, we get clear idea.

Counter Example 4.5.11. Consider Y={u,v,w},ζ ={ϕ, Y,{u},{u,v}}.Here (Y,ζ) represents *$T_{1/2}$-space(since g-closed sets coincide with g*-closed sets in ζ) but it is not a JTC-space because {u,v} is J-closed not closed.

Proposition 4.5.12. If (Y,ζ) is a JTC - space, then it becomes $T_{1/2}$*-space.

Proof Given (Y,ζ) is a JTC-space. Consider a g*-closed subset H of Y. By **Proposition 2.3.14.**, H is J-closed. In JTC-space, H is closed. Therefore H is a $T_{1/2}$*-space.

Note 4.5.13. The converse of above Proposition 4.5.12. does not hold. From the following Counter Example, we get clear idea.

Counter Example 4.5.14. Consider Y={u,v,w},ζ ={ϕ, Y,{u}}.Here (Y,ζ) represents $T_{1/2}$*-space(since g*-closed sets coincide with closed sets in ζ) but it is not a JTC-space. Because {v} is J-closed not closed.

Proposition 4.5.15. If (Y,ζ) is a T_δ-space, then it becomes JTC-space.

Proof Consider a J-closed subset H of Y. By **Proposition 2.3.12.**,H is gδ-closed. In a T_δ-space, H is δ-closed. Since each δ-closed set is a closed set. Therefore H is a JTC-space.

Proposition 4.5.16. If (Y,ζ) is an almost weakly Hausdorff space, then it becomes JTC - space.

Proof Consider a J-closed subset H of Y. By **Proposition 2.3.12.**,H is gδ-closed. In an almost weakly Hausdorff space, each gδ-closed set is a closed set (Theorem 5.1(Dontchev,2000)) which gives H is closed. Therefore H is a JTC-space.

Corollary 4.5.17. In an almost weakly Hausdorff space (Y,ζ), the following statements are equivalent. (i) (Y,ζ) is JTC-space. (ii) (Y,ζ) is gδTJ-space.

Proposition 4.5.18. In a $_{gs}T_{\delta g*}$-space, every gs-closed set is a J-closed set.

Proof In a $_{gs}T_{\delta g*}$-space, gs-closed sets coincide with δg*-closed sets (Sudha,2014).By **Proposition 2.3.6.**,every δg*-closed set is a J-closed set. Hence every gs-closed set is a J-closed set.

Counter Example 4.5.19. Consider Y={u,v,w},ζ ={ϕ, Y,{u,v}}.Here (Y,ζ) is a $_{gs}T_{\delta g*}$-space. But JC(Y,ζ) = P(Y) and gsC(Y,ζ) = ={ϕ, Y,{u,w},{w},{v,w}}.

Remark 4.5.20. T_d-space is independent with JTC-space. This can be seen from the following Counter Examples.

Counter Example 4.5.21. Consider Y={u,v,w},ζ ={ϕ, Y,{u},{v},{u,v}}.Here (Y,ζ) represents a JTC-space, but not a T_d -space (since gs-closed sets coincide with g-closed sets in this topology). But {u} is a gs-closed set but not a g-closed set.

Counter Example 4.5.22. Consider Y={u,v,w},ζ ={ϕ, Y,{u},{v},{u,v},{u,w}}.Here (Y,ζ) represents a T_d-space(since gs-closed sets coincide with g-closed sets in this topology) but it is not a JTC-space. Because {u} is J-closed not closed.

Remark 4.5.23. T_b-space is independent with JTC-space. This can be seen from the following Counter Examples.

Counter Example 4.5.24. Consider Y={u,v,w},ζ ={ϕ, Y,{u},{u,v},{u,w}}.Here (Y,ζ) represents a T_b -space (since gs-closed sets coincide with closed sets in this topology), but not a JTC-space. But {u} is a J-closed set but not a closed set.

Counter Example 4.5.25. Consider Y={u,v,w},ζ ={ϕ, Y,{u},{v},{u,v}}.Here (Y,ζ) represents a JTC-space but it is not a T_b -space(since gs-closed sets coincide with closed sets in this topology). Because {v} is gs-closed not closed.

Theorem 4.5.26. Let (Y,ζ) be a topological space. Then the following conditions are equivalent.

(a) (Y,ζ) is a JTC-space.

(b) Every singleton set is either open or η*-closed in (Y,ζ).

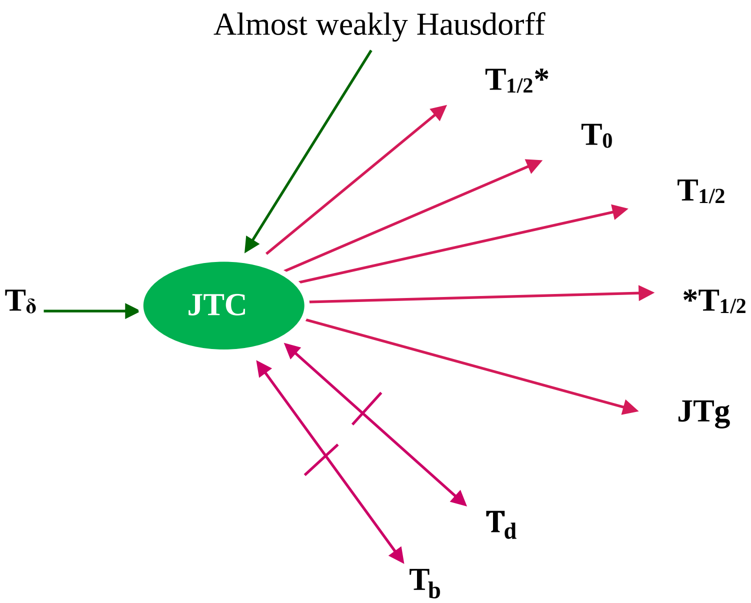
Proof (a) ⇒ (b) By **Theorem 2.3.90.**,every singleton set is either η*-closed or J-open in (Y,ζ).If {x} is η*-closed, then **(b)** holds. If not, {x} is J-open. In a JTC-space is open i.e. {x} is open.

(b) ⇒ (a) Let D ⊆Y be J-closed. Let y∈Cl(D). We consider the following two cases:

Case (i) Let {y} be an open set. Since y∈Cl(D), then {y}∩D ≠ ϕ, since {y} is a neighbourhood of y as {y} is open. This gives that y∈D, Cl(D)⊆D, D = Cl(D). Hence D is closed.

Case (ii) Let {y} be η*-closed. Suppose y∉D, y∈ Cl(D) − D. By **Theorem 2.3.55.**, Cl(D) − D does not contain a non-empty η*-closed set. Therefore it is a contradiction. Hence y∈D. Hence D = Cl(D) which implies every J-closed set is closed. Hence (Y,ζ) is JTC-space.

Note 4.5.27. The following picture gives the relations of JTC-space with other spaces.



§ 4.6. JTδg*-space

This is an important tool to examine the properties, dependence and independence of JTδg*-space with other new defined spaces and existing spaces.

Proposition 4.6.1. If (Y,ζ) is a JTδg*-space, then it becomes a JTδg-space and a JTg-space respectively.

Proof Consider a J-closed subset H of Y. In a JTδg*-space, H is δg*-closed.By Proposition 2.2.4. and Proposition 2.2.6 (Sudha,2014),H is δg-closed and g-closed respectively.We get the proof.

The converse of the above Proposition 4.6.1. does not hold good.

Counter Example 4.6.2. Consider Y={u,v,w},ζ ={ϕ,Y,{u}}.Here (Y,ζ) represents a JTδg-space and a JTg-space but not a JTδg*-space.Since {u,v} is a J-closed set but not a δg*-closed set.

Proposition 4.6.3. When a space is JTδg* and $_{\delta g^*}T_{\delta}$,then it is a $T_{3/4}$ -space.

Proof In a JTδg*-space,J-closed sets coincide with δg*-closed sets. In a $_{\delta g^*}T_{\delta}$-space, δg*-closed sets coincide with δ-closed sets.Hence (Y,ζ) is JTδ.By **Proposition 4.3.9.**, (Y,ζ) is a $T_{3/4}$ -space.

Counter Example 4.6.4. Consider Y={u,v,w,x},ζ ={ϕ,Y,{u},{v}, {u,v},{u,v,w}, {u,v,x}}.Here (Y,ζ) represents a $T_{3/4}$ -space(since δg-closed sets coincide with δ-closed sets) but not JTδg* and $_{\delta g^*}T_{\delta}$ (since δg*-closed sets coincide with δ-closed sets).

Proposition 4.6.5. If (Y,ζ) is a JTδg*-space, then it becomes a *$T_{1/2}$-space.

Proof Consider a g-closed subset H of Y. By **Proposition 2.3.10.,**H is J-closed.In a JTδg*-space, H is δg*-closed. By Proposition 2.2.9. (Sudha,2014),H is g*-closed.Hence we get the proof.

Counter Example 4.6.6. Consider Y={u,v,w},ζ={ϕ,Y,{u},{u,v}}.Here (Y,ζ) represents a *$T_{1/2}$-space(since g-closed sets coincide with g*-closed sets in this topology), but not a JTδg*-space. Since {v} is a J-closed set but not a δg*-closed set.

Proposition 4.6.7. If (Y,ζ) is both a T_d-space and a JTδg*-space, then the following closed sets are equivalent:

- **(i) A gs-closed set**
- **(ii) A J-closed set**
- **(iii) A δg*-closed set.**

Proof (i) ⇒ (ii) In a T_d-space,gs-closed sets coincide with g-closed sets.By **Proposition 2.3.10.**,g-closed sets is a J-closed set. **(ii) ⇒ (iii)** In a JTδg*-space,J-closed set is a δg*-closed set.Hence **(ii) ⇒ (iii). (iii) ⇒ (i)** By Proposition 2.2.11 (Sudha,2014).

Proposition 4.6.8. If (Y,ζ) is both a T_b-space and a JTδg*-space, then the following closed sets are equivalent:

(i) A gs-closed set

(ii) A J-closed set

(iii) A δg*-closed set.

Proof Same as the above Proof from the fact that every closed is J-closed (by **Proposition 2.3.2.**) and by Proposition 2.2.11(Sudha,2014).

Proposition 4.6.9. If (Y,ζ) is both a T_c-space and a JTδg*-space, then the following closed sets are equivalent:

(i) A gs-closed set

(ii) A J-closed set

(iii) A δg*-closed set.

Proof Same as the proof of **Proposition 4.6.7.** from the fact that every g*-closed is J-closed (by **Proposition 2.3.14.**) and from Proposition 2.2.11(Sudha,2014).

Another way of characterization of a JTδg*-space is given below:

Remark 4.6.10. From the above Propositions 4.6.7. to 4.6.9.,

(i) If a space (Y,ζ) is both a T_d-space and a JTδg*-space,then (Y,ζ) is a ${}_{gs}T_{\delta g*}$ -space.

(ii) If a space (Y,ζ) is both a T_b-space and a JTδg*-space,then (Y,ζ) is a ${}_{gs}T_{\delta g*}$ -space.

(iii) If a space (Y,ζ) is both a T_c-space and a JTδg*-space,then (Y,ζ) is a ${}_{gs}T_{\delta g*}$ -space.

Proposition 4.6.11. If (Y,ζ) is a T_δ-space, then it becomes a JTδg*-space.

Proof Consider a J-closed subset Hof Y. By **Proposition 2.3.12.**,H is a gδ-closed set. In a T_δ-space,H is δ-closed.By Proposition 2.2.2 (Sudha,2014), H is a δg*-closed set.

The converse of the above Proposition 4.6.11. is not hold good.

Counter Example 4.6.12. Consider Y={u,v,w,x},ζ ={ϕ,Y,{u,v},{w},{u,v,w}}.Here (Y,ζ) represents a JTδg*-space, but not a T_δ -space (since gδ-closed sets coincide with δ-closed sets in this topology). But {u,x} is a gδ-closed set but not a δ-closed set.

Remark 4.6.13. A JTδg*-space is independent with a $T_{1/2}$-space. This can be seen from the following Counter Examples.

Counter Example 4.6.14. Consider Y={u,v,w},ζ ={ϕ, Y,{u},{u,v},{u,w}}.Here (Y,ζ) represents a $T_{1/2}$-space(since g-closed sets coincide with closed sets), but not a JTδg*-space,because {u,v} is a J-closed set but not a δg*-closed set.

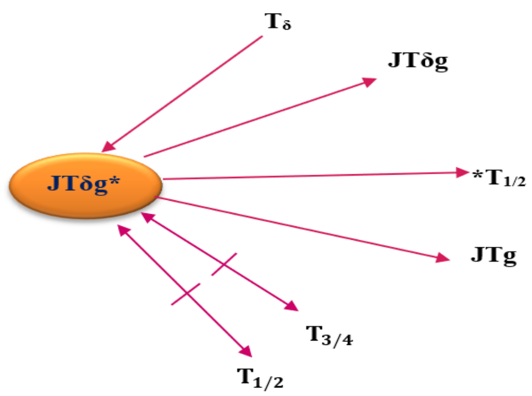
Counter Example 4.6.15. Consider Y={u,v,w,x},ζ ={ϕ, Y,{w},{u,v},{u,v,w}}.Here (Y,ζ) represents a JTδg*-space but it is not a $T_{1/2}$-space(since g-closed sets coincide with closed sets),because {u,x} is g-closed not closed.

Remark 4.6.16. A JTδg*-space is independent with a $T_{3/4}$-space. This can be seen from the following Counter Examples.

Counter Example 4.6.17. Consider Y={u,v,w,x},ζ ={ϕ, Y,{u},{v},{u,v},{u,v,w},{u,v,x}}.Here (Y,ζ) represents a $T_{3/4}$-space(since δg-closed sets coincide with δ-closed sets), but not a JTδg*-space, because {u,v,x} is a J-closed set but not a δg*-closed set.

Counter Example 4.6.18. Consider Y={u,v,w,x},ζ ={ϕ, Y,{w},{u,v},{u,v,w}}.Here (Y,ζ) represents a JTδg*-space but it is not a $T_{3/4}$-space(since δg-closed sets coincide with δ-closed sets),because {v,w,x} is δg-closed not δ-closed.

Note 4.6.19. The following picture gives the relations of JTδg*-space with other spaces.



§ 4.7. JTδg-space

Proposition 4.7.1. If (Y,ζ) is a JTδg-space, then it becomes JTg-space.

Proof Let (Y,ζ) be a JTδg-space. Let D be a J-closed set. In a JTδg-space, every J-closed set is a δg-closed set. Then D is a δg-closed set and hence by Theorem 3.1(iii) (Dontchev,1996),Dis a g-closed set. Thus (Y,ζ) is a JTg-space.

Proposition 4.7.2. If (Y,ζ) is a T_δ-space, then it becomes JTδg-space.

Proof Let (Y,ζ) be a T_δ-space. Let D be a J-closed set. In a T_δ-space, every gδ-closed set is a δ-closed set. By **Proposition 2.3.12.,**D is a gδ-closed set.Hence δ-closed.By Theorem 3.1(i) (Dontchev,1996),D is δg-closed.Thus (Y,ζ) is a JTδg-space.

Note 4.7.3. The converse of above Proposition 4.7.2. is not true. It can be seen from the following.

Counter Example 4.7.4. Consider Y={u,v,w},ζ ={ϕ, Y,{u}}.Here (Y,ζ) is a JTδg-space but not a T_δ-space(since gδ-closed sets coincide with δ-closed sets).Since gδC(Y,ζ) = P(Y) ≠ δC(Y,ζ) is a trivial topology.

Proposition 4.7.5. If (Y,ζ) is a JTδg and a T_b-space, then it becomes a JTC-space.

Proof Consider a J-closed subset H in (Y,ζ).In a JTδg-space,J-closed sets coincide with δg-closed sets which implies H is δg-closed.By Theorem 3.1(iii) (Dontchev,1996), a δg-closed set is a gs-closed set. Therefore H is gs-closed. In a T_b-space, every gs-closed set is closed which gives H is closed. Hence (Y,ζ) is a JTC-space.

The converse of the above Proposition 4.7.5. is not true.

Counter Example 4.7.6. Consider Y={u,v,w},ζ ={ϕ, Y,{u},{v},{u,v}}.Here (Y,ζ) represents a JTC-space and a JTδg-space but it is not a T_b-space(Since every gs-closed set is closed),because {p} is gs-closed not closed in (Y,ζ).

Another way of characterization of JTC-space is given below.

Proposition 4.7.7. If (Y,ζ) is a JTδg and a $T_{1/2}$-space, then it becomes a JTC-space.

Proof Consider a J-closed subset H in (Y,ζ).In a JTδg-space,J-closed sets coincide with δg-closed sets which implies H is δg-closed.By Theorem 3.1(ii) (Dontchev,1996), a δg-closed set is a g-closed set.Therefore H is g-closed. In a $T_{1/2}$-space, every g-closed set is closed which gives H is closed. Hence (Y,ζ) is a JTC-space.

Proposition 4.7.8. If (Y,ζ) is JTδg and T_d-space, then it becomes JTg-space.

Proof Consider a J-closed set in (Y,ζ).In a JTδg-space,J-closed sets coincide with δg-closed sets. In general a δg-closed set is a gs-closed set. In a T_d-space, every gs-closed set is g-closed. Hence (Y,ζ) is a JTg-space.

Proposition 4.7.9. If (Y,ζ) is a JTδg-space and a T_δ-space, then it becomes a gδTJ-space.

Proof Consider a gδ-closed set H in (Y,ζ).In a T_δ-space, gδ-closed set H is a δ-closed set.By **Proposition 2.3.4.**, δ-closed set H is a J-closed set.In a JTδg-space,J-closed sets H coincide with δg-closed sets.By **Proposition 2.3.8.,**H is J-closed.Hence (Y,ζ) is a gδTJ-space.

The converse of above Proposition 4.7.9. is not true.

Counter Example 4.7.10. Consider Y={u,v,w,x},ζ={ϕ,Y,{u},{v},{u,v}, {u,v,w}, {u,v,x}}.Here (Y,ζ) represents a gδTJ-space but it is not a JTδg-space and a T_δ-space (since gδ-closed sets coincide with δ-closed sets) because {u,v,w} is J-closed not δg-closed and gδ-closed but not δ-closed.

Proposition 4.7.11. If (Y,ζ) is both a T_d-space and a JTδg-space, then the following closed sets are equivalent:

(i) **A gs-closed set**
(ii) **A J-closed set**
(iii) **A δg-closed set.**

Proof (i) ⇒ (ii) In a T_d-space,gs-closed sets coincide with g-closed sets.By **Proposition 2.3.10.**,g-closed sets is a J-closed set. **(ii) ⇒ (iii)** In a JTδg-space,J-closed set is a δg-closed set. **(iii) ⇒ (i)** By Theorem 3.1(iii) (Sudha,2014).

Proposition 4.7.12. If (Y,ζ) is both a T_b-space and a JTδg-space, then the following closed sets are equivalent:

(i) **A gs-closed set**
(ii) **A J-closed set**
(iii) **A δg-closed set.**

Proof Same as the above Proof from the fact that every closed is J-closed (by **Proposition 2.3.2.**).

Proposition 4.7.13. If (Y,ζ) is a T_c and JTδg-space, then the following implications are equivalent.

- **(i) A gs-closed set**
- **(ii) A J-closed set**
- **(iii) A δg-closed set.**

Proof (i) ⇒(ii) In a general a T_c-space,every gs-closed set is a g*-closed set.By **Proposition 2.3.14.**,every g*-closed sets is a J-closed set. **(ii) ⇒(iii)** Now in a JTδg-space, J-closed set is a δg-closed set**.**Hence the result. **(iii) ⇒ (i)** By Theorem 3.1(iii) (Sudha,2014).

Result 4.7.14. JTδg-space is independent with $T_{1/2}$ and $T_{3/4}$-space.Next Counter Example shows it.

Counter Example 4.7.15. Consider Y={u,v,w},ζ ={ϕ, Y,{u}}.Here (Y,ζ) represents JTδg-space but it is not a $T_{1/2}$-space(since g-closed sets coincide with closed sets) and $T_{3/4}$-space(since δg-closed sets coincide with δ-closed sets). Because {v} is g-closed not closed and δg-closed but not δ-closed also.

Counter Example 4.7.16. Consider Y={u,v,w},ζ ={ϕ, Y,{u},{u,v},{u,w}}.Here (Y,ζ) represents a $T_{1/2}$- space(since g-closed sets coincide with closed sets) but it is not a JTδg -space. Because {v} is J-closed not δg-closed.

Counter Example 4.7.17. Consider Y={u,v,w,x},ζ ={ϕ, Y,{u},{v},{u,v},{u,v,w}, {u,v,x}}.Here (Y,ζ) represents a $T_{3/4}$-space(since δg-closed sets coincide with δ-closed sets) but it is not a JTδg- space. Because {u,v,w} is J-closed but not δg-closed.

Remark 4.7.18. JTδg-space is independent with *$T_{1/2}$-space from the following Counter Examples.

Counter Example 4.7.19. Consider Y={u,v,w},ζ ={ϕ, Y,{u},{u,v}}.Here (Y,ζ) represents *$T_{1/2}$-space in which g-closed sets coincide with g*-closed sets but it is not JTδg. Because {v} is J-closed not δg-closed.

Counter Example 4.7.20. Consider Y={u,v,w},ζ ={ϕ, Y,{u},{v,w}}.Here (Y,ζ) represents JTδg-space but it is not $*T_{1/2}$- space (since g-closed sets coincide with g*-closed sets). Because {w} is g-closed not g*-closed.

Remark 4.7.21. JTδg-space is independent with $T_{1/2}$*-space from the following Counter Examples.

Counter Example 4.7.22. Consider Y={u,v,w},ζ ={ϕ, Y,{u},{v},{u,v},{u,w}}.Here (Y,ζ) represents $T_{1/2}$*-space (since g*-closed sets coincide with closed sets) but it is not a JTδg -space. Because {u} is J-closed not δg-closed.

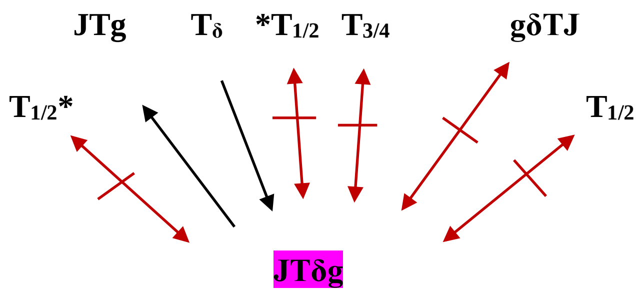
Counter Example 4.7.23. Consider Y={u,v,w,x},ζ ={ϕ, Y,{u,v}}.Here (Y,ζ) represents a JTδg-space but it is not a $T_{1/2}$*-space(since g*-closed sets coincide with closed sets). Because {u,w,x} is g*-closed not closed.

Remark 4.7.24. JTδg-space is independent with gδTJ-space from the following Counter Examples.

Counter Example 4.7.25. Consider Y={u,v,w},ζ ={ϕ, Y,{u}}.Here (Y,ζ) represents a JTδg-space but it is not a gδTJ-space. Because {u} is gδ-closed not J-closed.

Counter Example 4.7.26. Consider Y={u,v,w},ζ ={ϕ, Y,{u,v}}.Here (Y,ζ) represents a gδTJ-space but it is not a JTδg-space. Because {u,w,x} is J-closed not δg-closed.

Note 4.7.27. The following picture gives the relations of JTδg-space with other spaces.



4.8. gδTJ-space

Proposition 4.8.1. A topological space (Y,ζ) is a gδTJ-space and a JTC-space, then (Y,ζ) becomes a $T_{1/2}$ - space.

Proof Given (Y,ζ) is gδTJ and JTC-space. Consider a g-closed subset H of Y. By Theorem 3.2 (Dontchev,2000),H is gδ-closed. Hence in a gδTJ-space, H is J-closed. Also it is JTC-space and therefore H is closed. Hence (Y,ζ) becomes a $T_{1/2}$-space.

The converse of above Proposition 4.8.1. does not hold good.

Counter Example 4.8.2. Consider Y={u,v,w},ζ={ϕ,Y,{u},{v},{u,v},{u,w}}.Here (Y,ζ) represents a $T_{1/2}$-space and a gδTJ-space but it is not a JTC-space because {u,v} is J-closed not closed.

Proposition 4.8.3. A topological space (Y,ζ) is a JTδ-space and a gδTJ-space, then (Y,ζ) becomes a $T_{3/4}$ - space.

Proof Given (Y,ζ) is gδTJ and JTδ-space. Consider a δg-closed subset H of Y. By Corollary 3.3 of (Dontchev,2000),H is gδ-closed. Hence in a gδTJ-space, H is J-closed. Also it is JTδ-space and therefore H is δ-closed. Hence (Y,ζ) becomes a $T_{1/2}$ - space.

The converse of above Proposition 4.8.3. does not hold good.

Counter Example 4.8.4. Consider Y={u,v,w,x},ζ={ϕ,Y,{u},{v},{u,v}, {u,v,w}, {u,v,x}}.Here (Y,ζ) represents a $T_{3/4}$-space(since δg-closed sets coincide with δ-closed sets) and a gδTJ-space but it is not a JTδ-space because {u,v,w} is J-closed not δ-closed.

Proposition 4.8.5. If (Y,ζ) is a semi-regular space, then it becomes a gδTJ-space.

Proof Consider a gδ-closed subset H in (Y,ζ).In a semi-regular space,each gδ-closed set is g-closed by Theorem 5.8 of (Dontchev,2000).By **Proposition 2.3.10.**,H is J-closed. Hence (Y,ζ) is a gδTJ-space.

The converse of above Proposition 4.8.5. does not hold good.

Counter Example 4.8.6. Consider Y={u,v,w},ζ={ϕ,Y,{u,v}}.Here (Y,ζ) represents a gδTJ-space but it is not a semi-regular space (since closed sets coincide with δ-closed sets)because {w} is closed but not δ-closed.

Proposition 4.8.7. If (Y,ζ) is a T_δ-space, then it becomes a gδTJ-space.

Proof Consider a gδ-closed subset H in (Y,ζ).In a T_δ-space, gδ-closed sets coincide with δ-closed sets which implies H is δ-closed. By **Proposition 2.3.4.,**H is J-closed. Hence (Y,ζ) is a gδTJ-space.

The converse of above Proposition 4.8.7. is not true.

Counter Example 4.8.8. Consider Y={u,v,w},ζ={ϕ,Y,{u,v}}.Here (Y,ζ) represents a gδTJ-space but it is not a T_δ-space(since gδ-closed sets coincide with δ-closed sets) because {u,w} is gδ-closed not δ-closed.

Proposition 4.8.9. If (Y,ζ) is a $_{g\delta}T_{\delta g*}$- space, then it becomes a gδTJ-space.

Proof Consider a gδ-closed subset H of Y. In a $_{g\delta}T_{\delta g*}$-space,H is a δg*-closed set.By **Proposition 2.3.6.**,H is J-closed. Hence (Y,ζ) is a gδTJ-space.

Note 4.8.10. The converse of above Proposition 4.8.9. is not true. From the counter example, we get clear idea.

Counter Example 4.8.11. Consider Y={u,v,w},ζ ={ϕ, Y,{u,v}}.It is a gδTJ-space but not a $_{g\delta}T_{\delta g*}$- space(since gδ-closed sets coincide with δg*-closed sets),because {v} is gδ-closed not δg*-closed.

Remark 4.8.12. gδTJ-space is independent with semi-$T_{1/2}$-space from the following Counter Example.

Counter Example 4.8.13. Consider Y={u,v,w},ζ ={ϕ, Y,{u,v}}.Here sgC(Y,ζ)={ϕ, Y,{w},{u,w},{v,w}}. and sC(Y,ζ) = {ϕ,Y,{w}}. It is a gδTJ-space, but it is not a semi-$T_{1/2}$-space (Since every sg-closed sets coincide with semi-closed sets).Since {u,w} is sg-closed but not semi-closed.

Counter Example 4.8.14. Consider Y={u,v,w},ζ ={ϕ, Y,{u},{u,v}}. Here (Y,ζ) represents semi-$T_{1/2}$-space (Since every sg-closed sets coincide with semi-closed sets) but it is not a gδTJ-space. Because {u} is gδ-closed not J-closed.

Remark 4.8.15. gδTJ-space is independent with T_d-space from the following Counter Example.

Counter Example 4.8.16. Consider Y={u,v,w},ζ ={ϕ, Y,{u}}.Here gδC(Y,ζ)=P(Y) and JC(Y,ζ) = gsC(Y,ζ) = gC(Y,ζ) = P(Y)−{u}. It is a T_d-space(since gs-closed sets coincide with g-closed sets), but it is not a gδTJ-space. Since {u} is gδ-closed but not J-closed.

Counter Example 4.8.17. Consider Y={u,v,w},ζ ={ϕ,Y,{u},{v},{u,v}}.Here gδC(Y,ζ)= JC(Y,ζ) ={ϕ,Y,{w},{u,w},{v,w}} = gC(Y,ζ) and gsC(Y,ζ) = P(Y)−{u,v}.It is a gδTJ-space, but it is not a T_d -space(since gs-closed sets coincide with g-closed sets). Since {v} is gs-closed but not g-closed.

Note 4.8.18. The following picture gives the relations of gδTJ-space with other spaces.