Avinashilingam Institute for Home Science and Higher Education for Women

(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)
Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B
Coimbatore - 641 043, Tamil Nadu, India

Bachelor's Degree Examination – January 2021
V Semester

Class	**: III UG**	**Time**	**: 3 Hours**
Major	**: Mathematics/ Spl. Education and Mathematics**	**Max. Marks**	**: 100**

18BMAC13/18BSMC10 Abstract Algebra-I

PART A **10 x 1 = 10**
Choose the Correct Answer

1. Let H be a group of all real 2x2 matrices with $\det(H) \neq 0$ under matrix multiplication and let $K = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}$. Then K is
   a. group of H b.subgroup of H c. normal group of H d. none

2. If G is a finite group and $a \in G$, then
   a. $a^{o(G)} \neq e$ b. $a^{o(G)} = e$ c. $o(G)^a \neq e$ d. $o(G)^a = e$

3. If G is a group and H is a sub group of index 2, then H is a _________ of G.
   a. quotient group b. normal group c. normal subgroup d. quotient subgroup

4. A homomorphism $\varphi: G \dashrightarrow \bar{G}$ with kernel K_φ is an isomorphism of G into $\bar{G}$ if and only if
   a. $K_\varphi = (e)$ b. $K_\varphi = (0)$ c. $K_\varphi \neq (e)$ d.$K_\varphi \neq (0)$

5. An isomorphism of a group G onto itself is called as
   a. monomorphism b. automorphism c. homomorphism d. epimorphism

6. Every finite group having more than two elements has a _________ automorphism
   a. one b. trivial c. nontrivial d. two

7. A commutative ring is an integral domain if it has
   a. zero divisor b. no zero divisor c. field d. division ring

8. A homomorphism of a ring R into a ring R' is said to be isomorphism, if it is ___________
   a. onto b. one- to- one c. normal d. identity

9. Every maximal ideals correspond to the points on the ________interval.
   a. closed, unit b. open, unit c. intermediate d. boundary

10. Every integral domain can be imbedded in a _______
    a. ring b. field c. ideal d. kernel

Part B **5 x 6 = 30**

Answer ALL questions
Each answer should not exceed 400 words or two pages

11.a. Prove that if *H* is a nonempty finite subset of a group *G* and *H* is closed under multiplication then *H* is a subgroup *G*.

(or)

11.b. Show that if *G* is a finite group whose order is a prime number p, then *G* is a cyclic group.

12.a. Show that a subgroup N of G is a normal subgroup of G if and only if product of two right cosets of N in G is again a right cosets of N in G.

(or)

12.b. Let G be a group of all real 2x2 matrices with nonzero determinant, under matrix multiplication. Let $\bar{G}$ be a group of all non-zero real numbers under multiplication. Define $\varphi: G \dashrightarrow \bar{G}$ by $\varphi\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$. Check whether φ is homomorphism of G onto $\bar{G}$?

13.a. Are the following mappings automorphisms of their respective groups?

i) G group of integers under addition $T: x \dashrightarrow -x$.

ii) G group of positive reals under multiplication $T: x \dashrightarrow x^2$.

(or)

13.b. Prove $\mathcal{J}(G) \approx$ G/Z, where $\mathcal{J}(G)$ is the group of inner homomorphism of G and Z is center of G.

14.a. Give an example of an integral domain which has an infinite number of elements, it is of finite characteristic.

(or)

14.b. Prove that any homomorphism of a field is either an isomorphism or takes each element into 0.

15.a. Let R be a commutative ring with unit element whose only ideals are $\{0\}$ and R itself. Then prove that R is a field.

(or)

15.b. Prove that the intersection of two left ideals of ring R is a left ideal of R.

Part C **5 x 12 = 60**

Answer ALL questions

Each answer should not exceed 800 words or fourpages

16.a. State and prove the Lagrange's theorem.

(or)

16.b. If H and K are finite subgroups of G of orders $o(H)$ and $o(K)$, respectively, then show that

$$o(HK) = \frac{O(H)O(K)}{O(H \cap K)}.$$

17.a. Show that if G is a finite group and N is a normal subgroup of G, then $o(G|N) = o(G)|o(N)$.

(or)

17.b. Let φ be a homomorphism of G onto $\bar{G}$ with kernel K. Then prove that $G|K \approx \bar{G}$.

18.a. If G is a group, then show that $\mathcal{A}(G)$, the set of all automorphisms of G, is also a group.

(or)

18.b. State and prove the Cayle's theorem.

19.a. If every $x \in R$ satisfies $x^2 = x$, prove that R must be commutative. Also, prove that if p is a prime number then Jp , the ring of integers mod p, is a field.

(or)

19.b. Define ring homomorphism. Also, if φ is a homomorphism of R into R' with kernel $I(\varphi)$ then prove

i) $I(\varphi)$ is a subgroup of R under addition.

ii) If $a \in I(\varphi)$ and $r \in R$ then ar and ra are in $I(\varphi)$.

20.a. Define ideal of the ring R. Also, prove that if U is an ideal of the ring R, then R/U is also a ring and is a homomorphic image of R.

(or)

20.b. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of that R if and only if that R/M is a field.

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