REVIEW OF LITERATURE

Topology is a branch of mathematics concerned with the study of surfaces with spatial properties preserved under bi-continuous deformation. It emerged through the development of concepts from geometry and set theory. It is the study of continuity and connectivity. The topological structures on the collection of data are suitable mathematical models for not only the quantitative data but also for the qualitative data.

Primarily, the topological spaces were characterized by open sets. Later Stone (1937) introduced regular openness which is stronger than openness. In 1963, Norman Levine introduced the notion of semi-openness which is weaker than the notion of openness in topological spaces. Since then several interesting generalized open sets have been introduced by many topologists.

Njastad (1965) established α-closed sets. Norman Levine (1970) introduced the notion of generalized closed (briefly g-closed) sets in topological spaces and showed that compactness, countably compactness, para compactness and normality were all g-closed hereditary. Generalized closed sets are a strong tool in the characterization of topological spaces satisfying weak separation axioms.

In 1968, Velicko introduced δ-open sets, which are stronger than open sets, in order to investigate the characterization of H-closed spaces in terms of arbitrary filterbases and showed that the collection of all δ-open sets is a topology on X such that $\tau_\delta \subseteq \tau$. But in a semi-regular space, $\tau = \tau_s = \tau_\delta$. Since then δ-open sets have been widely used in order to introduce new spaces and functions.

Since the number of research papers published on various closed sets in topological spaces is numerous, a brief review of literature on some of the important articles published on this topic.

- ♣ **Title : ** Generalised Closed Sets in Topology.
- ♣ **Author :** Norman Levine (1970)
- ♣ **Inference observed :** The author introduced generalized closed sets, briefly denoted by g-closed sets and studied their characterizations as well as their behaviour relative to unions, intersections and subspaces. Further it was proved that the hereditary properties of closed sets in Normal space, Complete Uniform space, Locally Compact space and Hausdorff space were preserved by g-closed sets. Further, a $T_{1/2}$-space in which closed sets and g-closed sets coincide was introduced and proved that it lies strictly between T_0 and T_1 -spaces. Furthermore a new space namely, Symmetric space was introduced and it was shown that singleton sets are g-closed sets in this space. The images and inverse images of g-closed sets and g-open sets under continuous and closed transformations were also explored in this paper.

♣ **Title :** On δ-Generalized Closed Sets and $T_{3/4}$-Spaces .

♣ **Authors :** Julian Dontchev and Maximilian Ganster (1996)

♣ **Inference observed :** Through the semi regularization of a given topology and the associated δ-closure operator, a stronger form of g-closedness, properly placed between δ-closedness and g-closedness called δ-generalised closed sets, briefly denoted by δg-closed sets was introduced in this article. Also a new separation axiom, namely, $T_{3/4}$, which is properly placed between $T_{1/2}$and T_1-spaces was defined and proved that every δg-closed set coincides with δ-closed set in this space. Further, the concept of δg-continuous and δg-irresolute functions were introduced and investigated in this paper.

♣ **Title** : Generalized Locally Closed Sets and GLC-Continuous Functions.

♣ **Authors :** Balachandran, K, Sundaram, P. and Maki, H. (1996)

♣ **Inference observed :** In this paper, generalised locally closed (briefly, glc) sets and different notions of generalization of continuous maps in topological spaces were introduced and dicussed their properties by continuing the concept of locally closed sets, Ganster et.al.,(1989).

♣ **Title** : On Generalized δ-Closed Sets and Almost Weakly Hausdorff Spaces.

♣ **Authors :** Julian Dontchev, Arokiarani, I. and Balachandran, K. (2000)

♣ **Inference observed :** In this article, two new classes of generalized closed sets namely, gδ-closed sets and $\delta g^{\ddagger}$-closed sets were introduced and new characterizations of almost weakly Hausdorff spaces and thus of the digital line were explored. Both concepts were based on the δ-closure operator which was initiated by Veliko (1968). Further, a stronger form of semi-regularity, called T_δ -spaces was introduced and it was proved that it is equal to semi-regulrity plus almost weakly Hausdorffness. Also gδ-continuous and gδ-irresolute functions were introduced and studied in this paper.

In the year 1977, Dunhan showed that $T_{1/2}$-spaces are precisely the spaces in which singletons are open or closed. Mashhour et al (1982) defined the notion of pre open sets in topological spaces and obtained various properties. Using pre open sets, they introduced and investigated modified continuous functions called pre continuous functions and weak pre continuous functions.

In 1983, semi pre open sets were introduced by Abd El-Monsef et.al. By continuing the work of Veliko (1968), several new and important results related to δ-open sets were obtained by Noiri (1980).

Intensive research on the field of generalized closed sets was done as the theory was developed by

- Arya, et. Al.,(1990) – Generalized semi-closed sets.
- Palaniappan, et. al.,(1993) – Regular generalized closed sets.
- Maki, et.al.,(1994)-αg-closed sets.
- Julian Dontchev, et. al.,(1996)-δg-closed sets.
- Gnanambal (1997)-Generalised pre closed sets.
- Veerakumar (2000) – g*-closed sets.
- Lellis Thivagar (2010)- $\delta\hat{g}$ -closed sets.
- Pushpalatha, et. al.,(2011) – g^*s-closed sets.
- R.Sudha (2012)-δg^*-closed sets.

Separation axioms is one of the most important and interesting concept in topological spaces. One of the most well known low separation axioms is the T_1 separation axiom in which singleton sets are closed. Several topological spaces that fail to be T_1 are very often of significant importance in the study of the geometric and topological properties of digital images.

Several new separation axioms were defined in the course of the investigation of generalized closedness. Maki,et.al.,(1986) introduced T_b and T_d-spaces. Julian Dontchev (1996) considered a new separation axiom $T_{3/4}$, as the class of topological spaces where every δg-closed set is δ-closed and hence closed in the semi-regularization of the given topology. The class of $T_{3/4}$-spaces is properly placed between the classes of $T_{1/2}$- and

T_1-spaces. The Khalimsky line, a space which is widely used in the application of point set topology in computer graphics, is a $T_{3/4}$-space but not T_1.

Lellis Thivagar et.al., (2011) defined a new class of functions called $\delta\hat{g}$-closed maps, $\delta\hat{g}$-continuous functions, and $\delta\hat{g}$ ($\delta\hat{g}c$) homeomorphisms and studied their properties, applications and their group structure. In his later paper (2012), contra $\delta\hat{g}$-continuous functions were introduced and several characterizations were obtained.

According to Bourbaki (1966) a subset of a topological space is called a locally closed if it is the intersection of an open set and a closed set. Stone (1980) has used the term FG for a locally closed subset. Using the concept of locally closedness, Ganster and Reilly (1989) obtained different notions of generalized continuity. Balachandran et. al., (1996) introduced the concept of generalized locally closed sets and obtained different notions of generalized continuity.

A triplet, (X,τ_1,τ_2) where X is a non-empty set, τ_1 and τ_2 are topologies defined on X is called a bitopological space. Kelly (1963) initiated the study of such spaces. Maheswari and Prasad (1977) introduced semi-open sets in bitopological spaces and further properties of this notion were studied by Bose (1981).

Fukutake (1986) introduced the concepts of g-closed sets in bitopological spaces and after that several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. The concepts of gpr-closed, g^*-closed, αg-closed, gs-closed, g^*s-closed and g^*p-closed sets were developed in bitopological spaces by Fukutake (2002), Sheik John, et.al.,(2004), Tantawy (2005), Khedr,et.al.,(2009), Pushpalathet.al.,(2011) and Vadivel.et.al.,(2012) respectively.

Sheik John et al (2004) introduced $^{*}T_{1/2}$-spaces and g^{*}-continuity for bitopological spaces and investigated some of their properties. Lellis Thivagar et al (2006) generalized the concept of semi-generalized closed sets to (1, 2)*- semi-generalized closed sets and obtained various bitopological properties.