



Murugan

Avinashilingam Institute for Home Science and Higher Education for Women
 Deemed to be University Estd.u/s 3 of UGC Act 1956, Category A by MHRD
 Re-accredited with 'A++' Grade by NAAC.CGPA 3.65/4, Category I by UGC
 Colmbatore-641 043, Tamil Nadu, India

Continuous Internal Assessment Test I- August 2024
 Semester- III

Class:II PG
 Major:Mathematics

Time: 2Hrs
 Max.Marks:60

23MMAC14 -ComplexAnalysis

CourseOutcomes:

- CO1 : Use Poisson formula and Mean-value property in Harmonic functions.CO2: Expand Taylor's series and Laurent's series for a given function.
 CO3: Convert various functions into canonical product form.CO4: Identify elliptic functions.
 CO5: Apply Weierstrass functions in Brownian motion.

Part-A
 Answer all the questions

6x1=6

1. Laplace's equation in polar co-ordinates is _____ CO1K2

a. $r \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{\partial u}{\partial \theta} = 0$

b. $r \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{\partial^2 u}{\partial \theta^2} = 0$

c. $r \frac{\partial}{\partial r} \left(\frac{\partial u}{\partial r} \right) + \frac{\partial^2 u}{\partial \theta^2} = 0$

d. $\frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{\partial^2 u}{\partial \theta^2} = 0$

2. The sum of two _____ functions is harmonic CO1K1

- a. harmonic b. continuous c. analytic d. entire

3. Expand $\frac{1}{z}$ as Taylor's series about $z=1$ is CO2K2

a. $1 + (z-1) + (z-1)^2 + (z-1)^3 + \dots$

b. $1 - (z-1) - (z-1)^2 - (z-1)^3 - \dots$

c. $1 - (z-1) + (z-1)^2 - (z-1)^3 + \dots$

d. $1 + (z+1) + (z+1)^2 + (z+1)^3 + \dots$

4. If u is a harmonic function and c is a constant then cu is _____ CO2K1

- a. Harmonic function b. constant function
 c. single valued function d. entire function

5. A function that is holomorphic on all of domain except for a set of isolated points, which are poles of the function, is called _____ CO3K2

- a. Harmonic function b. constant function
 c. meromorphic function d. entire function

6. The value of $\prod_{n=2}^{\infty} \left(1 - \frac{1}{n^2}\right)$ is

- a. 1 b. $\frac{1}{2}$ c. $\frac{3}{2}$ d. $-\frac{1}{2}$ CO3K2

Part B

Answer ALL questions

3 x 6 = 18

7. a. Prove that any harmonic function u which depends only on r is of the form $u = a \log r + b$ CO1 K4
(or)

7. b. If u is harmonic in Ω , then show that CO1 K3

(i) $\int_{\gamma} \frac{\partial u}{\partial n} |dz|$ (ii) $\int_{\gamma} \frac{\partial u}{\partial n} |dz| = 0$, where γ is regular curve homologous to zero in Ω .

8. a. State and prove Hurwitz theorem. CO2 K3

(or)

8. b. State and prove Taylor's Series. CO2 K3

9. a. Prove that, the necessary and sufficient condition for the absolute convergence of the

product $\prod_{n=1}^{\infty} (1 + a_n)$ for the convergence of the series $\sum_{n=1}^{\infty} |a_n|$. CO3 K4

(or)

9. b. Show that $\pi \cot \pi z = \frac{1}{z} + \sum_{n \neq 0} \left(\frac{1}{z-n} + \frac{1}{n} \right)$ CO3 K3

Part C

Answer ALL questions

3 x 12 = 36

10. a. State and prove mean value property. CO1 K3
(or)

10. b. State and prove Poisson's formula. CO1 K3

11. a. State and prove Weierstrass theorem. CO2 K3
(or)

11. b. State and prove Laurent's series theorem. CO2 K3

12. a. State and prove Mittag-Leffler theorem. CO3 K3
(or)

12. b. Prove that the infinite product $\prod_{n=1}^{\infty} (1 + a_n)$ with $(1 + a_n) \neq 0$ converges simultaneously with CO3 K4

the series $\sum_{n=1}^{\infty} \log (1 + a_n)$ whose terms represent the values of the principal branch of the logarithm

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