Avinashilingam Institute for Home Science and Higher Education for Women
(Deemed to be University), Coimbatore – 641 043

Master's Degree Examination - November 2018
Semester – I

Class : I PG | **Time : 3 Hours**
Major: Mathematics | **Max Marks :60**

17MMAC02 REAL ANALYSIS

Part A $10x½ = 5$

Choose the Correct answer

1. A partition P' of [a,b] is said to be finer than p if -----------------------

   a. $p' \subset p$ b. $p \subseteq p'$ c. $p' \subseteq p$ d. p= p'

2. If $\propto \uparrow$ is constant through [a,b] the integral $\int_a^b f d\propto$ exists and has value

   a. 0 b. 1 c. -1 d. ∞

3. The value of [-4.8]

   a. -4 b. -5 c. 4 d. 5

4. Step function provide connecting link between Riemann-Stieltjes integrals and -----------------------------------

   a. Infinite sum b. finite sum c. partition d. integral

5. Assume that $\alpha \uparrow$ on [a,b] . If $f \in R(\alpha)$ on[a,b] then $f^2 \in R(\alpha)$ on ------------

   a. (a ,b) b. [a, b] c. (a,b] d. [a,b)

6. Assume $f \in R(\alpha)$, $g \in R(\alpha)$ on [a,b] where $\alpha \uparrow$ on [a,b) is also valid if α is of bounded variation on ---------------------------

   a. (a ,b) b. [a, b] c. (a,b] d. [a,b)

7. If a sequence functions $\{f_n\}$ converges pointwise to a limit function f on [a ,b] then $\lim_{n \to \infty} \int_a^b f_n(x)dx =$ ________

   a. $\int_a^\infty f(x)dx$ b. $\int_a^b f(x)dx$ c. $\int_{-\infty}^\infty f(x)dx$ d. $\int_0^\infty f(x)dx$

8. The Riemann integral of S over [a,b] is $\int_a^b s(x)dx =$ _____

   a. $\sum C_k(x_k - x_{k-1})$ b. $\sum C_k(x_{k-2} - x_{k-1})$

   c. $\sum C_0(x_k - x_{k-1})$ d. $\sum C_1(x_k - x_{k-1})$

9. Let f be defined on I . If f=0 almost everywhere on I , then $f \in L(I)$ and $\int_I f =$ ----------

   a. ∞ b. 1 c. -1 d. 0

10. If f is real – valued function, the relation between f^+ and f^- is given by

    a. $f = f^+ + f^-$ b. $f = f^+ - f^-$ c. $|f| = f^+ - f^-$ d. $|f| = f^- - f^+$

Part B **5X 4 =20**

Answer all questions

11.a. If $f \in R(\alpha)$ and if $g \in R(\alpha)$ on [a,b] then $C_1 f + C_2 g \in R(\alpha)$ on [a,b]

for any two constants c_1 and c_2 we have

$$\int_a^b (C_1 f + C_2 g)\, d\alpha = C_1 \int_a^b f\, d\alpha + C_2 \int_a^b g\, d\alpha$$

(OR)

b. If $f \in R(\alpha)$ on [a,b] , then $\alpha \in R(f)$ on [a,b] and we have

$$\int_a^b f(x)\,d\alpha(x) + \int_a^b \alpha(x)\,df(x) = f(b)\,\alpha(b) - f(a)\,\alpha(a)$$

12. a. Assume that $\alpha \uparrow$ on [a,b] then

i) If P' is finer than P we have $U(P',f,\alpha) \le U(P,f,\alpha)$, and $L(P',f,\alpha) \ge L(P,f,\alpha)$

ii) For any two partitions P_1 and P_2 we have $L(P_1, f, \alpha) \le U(P_2, f, \alpha)$

(OR)

b. Assume that $\alpha \uparrow$ on [a,b] Then $\underline{I}(f,\alpha) \le \bar{I}(f,\alpha)$

13. a. Assume that $\alpha \uparrow$ on [a,b] . If $f \in R(\alpha)$ and if $g \in R(\alpha)$ on [a,b] and if

$f(x) \le g(x)$ for all in [a,b] then we have,

$$\int_a^b f(x)\,d\alpha(x) \le \int_a^b g(x)\,d\alpha(x)$$

(OR)

b. State and prove first mean value theorem for Riemann- Stieltjes integerals

14. a. Assume that $f \in U(I)$ and let $\{s_n\}$ and $\{t_m\}$ be two sequences generating f.

Then prove that $\lim_{n\to\infty} \int_I s_n = \lim_{m\to\infty} \int_I t_m$

(OR)

b. If $f \in U(I)$ and $g \in U(I)$ then show that $\max(f, g) \in U(I)$ and $\min(f,g) \in U(I)$

15.a. If f and g are in L(I) then so are the functions f^+ , f^- , $|f|$, max (f, g) and min (f, g) .

Moreover we have $|\int f| \le \int_I |f|$

(OR)

b. Assume $f \in U(I)$ and $g \in U(I)$ Then prove that $\int_I f \le \int_I g$

if $f(x) \le g(x)$ almost everywhere on I

Part C **5x7=35**

Answer the following

16.a. If $c \in (a,b)$, and if two of the three integrals in the following equation exist, then the third also exist and we have

$$\int_a^c f\,d\alpha + \int_c^b f\,d\alpha = \int_a^b f\,d\alpha$$

(OR)

b. Assume f$\in$ R(α) on [a,b] and assume that α has a continuous derivatives α' on [a,b]

$\int_a^b f(x)\alpha^1(x)dx$ exists ,then prove that the Riemann integral $\int_a^b f(x)d\alpha(x) =$

$\int_a^b f(x) \propto^I (x)dx$

17.a. Demonstrate Eulers summation formulae.

(OR)

b. Assume that $\alpha \uparrow$ on [a,b] then show that the following three statements are equivalent

i) f$\in$ R(α) on [a,b]

ii) f satisfies Riemann's condition with respect to on [a,b]

iii) $\underline{I}(f,\alpha) = \bar{I}(f,\alpha)$

18. a. Assume that α is of bounded variation on [a,b] . Let V(x) denote the total Variation of α on [a,x] if $a < x \leq b$, and let V(a) =0. Let f be defined and bounded On [a,b]. If f$\in$R(α) on [a,b] then f $\in$R(V) on [a,b].

(OR)

b. If f is continuous on [a,b] and if α is of bounded variation on [a,b] then f$\in$R(α) on [a,b]. Also prove that if α (x) = x, each of the following condition is sufficient for the existence of the Rieman integral $\int_a^b f(x)dx$:

a) If f is continuous on [a,b]

b) f is of bounded variation on [a,b]

19. a. Let f be defined and bounded on a compact interval [a,b] and assume that f is continuous almost everywhere on [a,b] . Then show that f $\in$U([a,b]) and the integral of f as a function in U([a,b]) is equal to the Riemann integral $\int_a^b f(x)\, dx$

(OR)

b. Assume that f$\in$ L(I) and let $\in$ >0 be given. Then prove that

i) There exist function u and v in U(I) such that f = u-v where v is

non-negative a.e on I and $\int_I^v < \epsilon$

ii) There exists a step function s and a function g in L(I) such that f = s + g

where $\int_I |g| < \in$

20.a. State and prove Levi theorem for step functions

(OR)

b. State and prove Levi theorem for upper functions

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