Review of Literature

REVIEW OF LITERATURE

Zadeh [40] introduced the concepts of fuzzy sets in 1965. Based on Zadeh's concept, Chang developed the theory of fuzzy topological spaces. From then on quiet a number of research papers have been published on generalizations of fuzzy sets and generalizations of fuzzy topological spaces, one of which, developed by Sostak [39], used the idea of degree of openness. This type of generalizations of a fuzzy topological space was later rephrased by Chattopadhyay, Hazra, and Samanta [9] in 1992, and by Ramadan [30] in 1992, namely gradation of openness: fuzzy topology.

In 1983, Atanassov [5] introduced the concept of intuitionistic fuzzy sets. Using this type of generalized fuzzy set, Coker [12] defined intuitionistic fuzzy topological spaces. Mondal and Samanta [28] introduced the concept of intuitionistic gradation of openness which is a generalization of the concept of gradation of openness defined by Chattopadhay.

Here we shall give a brief survey of some of the articles published on gradation of openness and on separation axioms in intuitionistic fuzzy topological spaces:

1) **Separation axioms in fuzzy topological spaces**

[Bruce Hutton and Iven Reilly, 1980] [7]

In this article, the authors have provided a hierarchy of separation properties for fuzzy topological spaces which are generalizations of the standard topological notions.

2) **The Hausdorff separation axiom for fuzzy topological spaces**

[S.E. Rodabaugh, 1980] [34]

In this article, the author has introduced a theory of α–Hausdorff fuzzy topological spaces which is compatible with α-compactness and fuzzy continuity, and for α, a certain type of member of a given lattice he has obtained a characterization of the α–Hausdorff subspaces of the fuzzy unit interval, the fuzzy open unit interval, and the fuzzy real line. Here, the author has given an easy proof of the fuzzy Tychonov theorem for α–compactness and extended the theory of one-point α–compactifications.

3) **On separation axioms and T_i- fuzzy continuity**

[S.Ganguly and S.Sara, 1985] [15]

In this article, the authors have introduced and generalized the concepts of T_i- continuous functions in a fuzzy setting and studied them in connection with fuzzy continuity and fuzzy separation

axioms. Some fuzzy topological properties are also investigated under these functions.

4) **On separation axioms and separations of connected sets in fuzzy topological spaces**

[S.Ganguly and S.Saha, 1987] [16]

In this article, some separation axioms based on the concept of quasi coincidence between a fuzzy point and a fuzzy set have been defined. The authors have defined the concept of connectedness and discussed some separations of connected sets in fuzzy setting.

5) **Strong separation and strong countability in fuzzy topological spaces**

[M.Macho Stadler and M.A. De Prada Vicente, 1991] [27]

In this article, given a fuzzy topological space (X, δ), the authors have introduced new notions of fuzzy separation and fuzzy countability axioms, using the family of its level-topologies: $\{l_t(\sigma) : t \in [0, 1)\}$. They have well-defined fuzzy topological concepts and compared them with the analogous fuzzy ones introduced in the literature. The authors have proved that these notions are not equivalent, and they have given a large number of examples which illustrate this fact.

6) **Separation axioms in fuzzy topology characterized by bitopologies**

[Riza Ertiirk, 1993] [33]

In this article, the author has associated a bitopological space with a fuzzy topological space, and examined the relation between various fuzzy and bitopological separation axioms.

7) **Some strong separation in fuzzy topological spaces**

[K.M. Abd El-Hakeim,1993] [3]

In 1991, Macho stadler et. al. introduced the notion of fuzzy separation using the family of its level-topologies $\{I_t(T) : t \in T_1\}$. Here, the author has introduced and studied the operations on R_i-fuzzy topological spaces, i = 0, 1. Moreover, the notion of fuzzy separation which is well-defined as a fuzzy topological concept is described. A comparison between the above notions with the analogous fuzzy ones are discussed.

8) **Separation axioms in ordered fuzzy topological spaces**

[A.K. Chaudhuri and P.Das, 1993] [11]

In this article, the authors have introduced the concept of T_i (i = 1, 2, 3) ordered separation axioms in ordered fuzzy topological spaces and the validity of some results analogous to those in the crisp case are examined.

9) **Fuzzy Separation axioms and fuzzy continuity in fuzzy bitopological spaces**

[Abu Safiya, Fora and Warner, 1994] [4]

In this article, the authors have shown goodness of extension of separation axioms for fuzzy bitopological spaces, and proved the space (X, T_1, T_2) is P-T_i, P-T_{iw}, P-regular, and P-normal iff the fuzzy bitopological space $(X, w(T_1), w(T_2))$ is P-T_i, P-T_{iw}, P-regular, and P-normal where $w(T_i)$ $(i = 1, 2)$ be the set of all lower semi continuous functions defined from X into the closed unit interval [0, 1].

10) **On separation axioms in a newly defined fuzzy topology**

[Rekha Srivastava, 1994] [31]

In 1992, Hazra et. al. defined a fuzzy topology on a non-empty set in a new fashion. Here, the author has introduced separation axioms and some allied notions in new set-up and proved several results related to these notions.

11) **Gradation of uniformity and gradation of proximity**

[M.H.Ghanim, O.A .Tantawy, Fawzia M.Selim, 1996] [17]

In 1992, Samanta et. al. gave a new definition of a fuzzy topological spaces by introducing the concept of a gradation of openness. In this article, the authors have given a new definition of a fuzzy uniform (resp. proximity) space by introducing the concept of

a gradation of a uniformity (resp. proximity) on a nonempty set X. Construction of a gradation of openness induced by a gradation of uniformity (resp. proximity) is explained. The connection between gradation of proximity and gradation of uniformity are investigated.

12) On fuzzy continuity in intuitionistic fuzzy topological spaces [H.Gurcay, D.Coker and A.H.Es, 1997] [20]

In this article, the authors have extended the notion of fuzzy semiopen sets and fuzzy semiclosed sets due to Azad to intuitionistic fuzzy topological spaces. Further they have discussed the concepts of intuitionistic fuzzy regular open sets and intuitionistic fuzzy regular closed sets and obtained the interrelationship between them.

13) Separation axioms in fuzzy topological ordered spaces [K.El-Saady and M.Y.Bakeir, 1998] [13]

Since the fuzzy topological space (X, τ) may be considered as a fuzzy topological ordered space when it is realized that the non-empty set X is partially ordered by agreeing that $x \leq y$ in X if and only if $x = y$. Then the study of the fuzzy topological ordered spaces not only induces the study of the abstract fuzzy topological spaces but also reveals many generalizations of well-known results concerning the abstract fuzzy topological spaces. In this article, the author have provided a certain number of separation axioms for fuzzy topological ordered spaces, which they label FT_i-order

separation axioms [for i = 1, 2, 3, 4]. The relationships between some of the FT_i-order separation axioms are studied.

14) Strong and ultra Separation axioms of fuzzy bitopological spaces [Kandil, Nouh, El-Sheikh, 1999] [23]

In this article, the authors have introduced a new notion of fuzzy pairwise separation axioms by using the family of its level bitopologies $I_\alpha(\tau_1)$, $I_\alpha(\tau_2)$, $\alpha \in [0,1)$ for a given fuzzy bitopological space (X, τ_1, τ_2) and proved that these concepts are extensions and compared them with its corresponding FPT_i and FPT_i^* (i = 0,1,2,3,4) respectively. The authors have shown that these notions are not equivalent and gave a number of examples which illustrates this fact.

15) A note on connectedness in intuitionistic fuzzy special topological spaces

[Selma Ozcag and Dogan coker, 2000] [36]

In this article, the authors have defined different types of connectedness with respect to intuitionistic fuzzy topological spaces and have obtained the interrelationships between them.

16) Gradation of openness Chang's fuzzy topologies

[V.Gregori, A.Vidal, 2000] [19]

In this article, the author have proved that each gradation of openness δ is the supremum (infimum) of strictly increasing

(decreasing) sequence of gradation of openness which are equivalent to δ. Further characterizations of those fuzzy topological spaces (X, T) with the property that there exists a gradation of openness σ from I^X onto I such that $G \in T$ iff $\sigma(G) > 0$ are given.

17) Gradation of Supra-Openness

[M.H. Ghanim, O.A. Tantaway, Fawzia M.Selim, 2000] [18]

In 1992, Chattopadhyay et. al. introduced the concept of a gradation of openness. In this article, the authors have defined fuzzy supra-topological (resp. supra-proximity, supra-uniform) space using the concept of a gradation of supra-openness (resp. supra-proximity, supra-uniformity) on a non-empty set X. The construction of a gradation of supra-openness induced by a gradation of supra-proximity (resp. supra-uniformity) is given and the connection between gradation of supra-proximity and gradation of a supra-uniformity is discussed.

18) Separation axioms in L-fuzzy topological spaces (I): T_0 and T_1^*

[Sheng-Gang Li, 2000] [38]

In this article, using the notion of remote neighbourhood the author has defined the separation axioms T_0 and T_1^* in L-fuzzy topological spaces (L-fts). The relations between these concepts due to Hutton and Reilly's and Wang's are discussed and the separations of Hutton's fuzzy unit interval and Gantner's fuzzy real

line are examined. Characterizations of these L-fts properties are investigated. Moreover, some results on minimal T_0 L-fts and T_1 L-fts are established.

19) On separation axioms in fuzzifying topology

[F.H. Khedr, F.M. Zeyada, O.R. Sayed, 2001] [24]

In this article, the authors have introduced R_0-and R_1-seperation axioms in fuzzifying topology and studied their relations with T_1- and T_2-seperation axioms, respectively. Furthermore, they have introduced and studied semi-T_0-, semi-R_0-, semi- T_1-, semi- R_1-, semi- T_2 (semi-Hausdorff)-, semi- T_3- (semi-regularlity)-, and semi- T_4-(semi-normality)- separation axioms in fuzzifying topology and given some of their characterizations and also the relations of these axioms and other separation axioms in fuzzifying topology.

20) On separability of intuitionistic fuzzy sets

|[K.T.Atanassov, Janusz Kacprzyk, Eulalia Szmidst et. al, 2003] [6]

In this article, the authors have introduced the separability of two intuitionistic fuzzy sets and have obtained results which are relevant for decision making and classification.

21) Hausdorffness in intuitionistic fuzzy topological spaces

[F.G.Lupianez, 2003] [26]

Using the basic concepts of intuitionistic fuzzy topological spaces introduced by Coker, the author has introduced a new notion of Hausdorffness in intuitionistic fuzzy topological spaces and some new properties on convergences are obtained.

22) Intuitionistic fuzzy proximity spaces

[Seok Jong Lee and Eun Pyo Lee, 2004] [37]

In this article, the authors have generalized the concept of fuzzy proximity to intuitionistic fuzzy sets and have defined intuitionistic fuzzy proximity. The relations between intuitionistic fuzzy proximity, fuzzy proximity and intuitionistic fuzzy topology are investigated.

23) Intuitionistic Supra fuzzy topological spaces

[S.E. Abbas, 2004] [1]

In this article, the author has introduced an intuitionistic supra fuzzy closure space and investigated the relationship between intuitionistic supra fuzzy topological spaces and intuitionistic supra fuzzy closure spaces. Moreover, the author has obtained intuitionistic supra fuzzy topological space induced by an intuitionistic fuzzy bitopological space. Also he has studied the relationship between intuitionistic supra fuzzy closure space and the intuitionistic

supra fuzzy topological space induced by an intutionistic fuzzy bitopological space.

24) On the intuitionistic fuzzy topological (metric and normed) spaces

[Reza, Saadati, 2005] [32]

In this article, the authors have defined precompact set in intuitionistic fuzzy metric spaces and proved that any subset of an intuitionistic fuzzy proximity spaces is compact if it is precompact and complete. Further, they have defined topologically complete intuitionistic fuzzy mertizable spaces and proved that any set in a complete intuitionistic fuzzy metric space is topologically complete intuitionistic fuzzy metrizable spaces and vice-versa. The authors have introduced intuitionistic fuzzy normed space and fuzzy boundedness for linear operators and proved that every finite dimensional intuitionistic fuzzy normed space is complete.

25) Pairwise separation axioms in intuitionistic topological spaces

[Sad K. Bayhan and Dogan Coker, 2005] [35]

In this article, the authors have proved that the existence of natural factors between the category of intuitionistic topological spaces and continuous mappings and the category of bitopological spaces and pairwise continuous mappings. These factors are then

used to generalize bitopological notions of separation to intuitionistic topological spaces.

26) Two Separation axioms in L-fuzzy topological spaces [Ji-Shu Cheng, 2006] [22]

In this article, $T_{1\frac{1}{2}}$ and T_2 spaces properties are studied. The $T_{1\frac{1}{2}}$ and T_2 separation axioms are described with molecular nets and ideals. The author has given an applications of some weakly continuous order homomorphism in separation axioms.

27) A note on "On separation axioms in I-fuzzy topological spaces" [Fu-Gui-Shi, Hong-Yan Li, 2007] [14]

As an extension of T_0'' separation axiom in general topology, RT_0'' axiom and $s-T_0''$ axiom were introduced in I-fuzzy topological space by Yue and Fang in 2006. In this article, the authors have proved that the RT_0'' axiom and $s-T_0''$ axiom are equivalent to each other.

28) Some separation axioms in I-fuzzy topological spaces [Hong-Yan Li, Fu-Gui Shi, 2008] [21]

In this article, some separation axioms such as the $L-T_2''$, L-Urysohn, and L-completely Hausdorff axioms in Shi's sense are

generalized from L-topologies to I-fuzzy topologies. The authors have discussed their properties and the relations among them.

29) Intuitionistic Supra Gradation of Openness

[A.M. Zahran, S.E. Abbas and E.El-Sanousy, 2008] [41]

In this article, the authors have introduced the concept of intuitionistic supra gradation of openness that was created from an intuitionistic fuzzy bitopological spaces to study the concept of continuity, some kinds of separation axioms and compactness.

30) Sum intuitionistic fuzzy closure spaces

[A.M. Zahran, M.Azab Abd-Allah, Kamal El-Saady and A.Ghareeb, 2008] [42]

The authors have proved the existence of final intuitionistic fuzzy topological spaces and final intuitionistic fuzzy closure spaces. From this fact, intuitionistic quotient spaces of their spaces and sum of intuitionistic fuzzy closure spaces are introduced and properties of intuitionistic fuzzy closure spaces are studied.