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Avinashilingam Institute for Home Science and Higher Education for Women

(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)
Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B
Coimbatore - 641 043, Tamil Nadu, India

Master's Degree Examination – June 2021
IV Semester

Class : II M.Sc. **Time: 3 Hours**
Major : Mathematics **Max. Marks: 100**

17MMAC21 Topology- II

Part A **10x1=10**
Choose the Correct Answer

1. A subset A of a topological space X is dense in X if
   a. Int(A) = X b. $\overline{A} = X$ c. $\overline{A} =$ d. Int (A) =

2. A regular space is
   a. Hausdorff space b. Non - Hausdorff space
   c. Null space d. None of the above

3. If every pair of disjoint closed sets in a topological space can be separated by disjoint open sets, then each such pair can be separated by a
   a. Discontinuous function b. Variable function
   c. continuous function d. None of the above

4. A normal space is a
   a. Completely regular space b. Completely non- regular space
   c. Non - Hausdorff space d. None of the above

5. An arbitrary product of compact spaces is .... in the product topology.
   a. non-compact b. Connected c. compact d. None of the above

6. A compactification of a space X is .a compact Hausdorff space Y containing X as a subspace such that
   a. $\overline{X} = Y$ b. $\overline{X} \supset Y$ c. $\overline{X} \neq Y$ d. $Y \subset X$

7. A metric space X is complete if every Cauchy sequence in X has a
   a. Divergent subsequence b. Oscillating subsequence
   c. Convergent subsequence d. None of the above

8. A function $f : X \rightarrow Y$ is bounded if its imag $f(X)$ is a............ subset of the metric space (Y, d)
   a. Unbounded b. bounded c. Dense d. None of the above

9. A space X is compactly generated if it satisfies the following condition : a set A is open in X if $A \cap C$ is .... in C for each compact subspace C of X.
   a. closed b. open c. d. X

10. A sequence $f_n : X \rightarrow Y$ of functions converges to the function f in the topology of compact convergence if and only if for each compact subspace C of X , the sequence f_n/C .... to f/C
    a. converges uniformly b. Converges c. diverges d. None of the above.

Part B $5\times 6 = 30$

Answer ALL questions

11.a. If X is a topological space in which one point -sets are closed, then prove that X is regular *if and only if* given a point x of X and a neighbourhood U of x, there is a neighbourhood V of x such that $\bar{V} \subset$ U.

(or)

11 b. Establish that every metrizable space is normal.

12.a. Establish that a subspace of a completely regular space is completely regular.

(or)

12.b. State and prove the Urysohn lemma.

13.a. State and prove the Tychonoff theorem on compact spaces.

(or)

13.b. If $f : A \to Z$ is a continuous map of A into the Hausdorff space Z, where A $\subset$ X, then establish that there is atmost one extension of f to a continuous function $g: \bar{A} \to$ Z.

14.a. If (X,d) is a metric space, then prove that there is an isometric imbedding of X into a complete metric space.

(or)

14.b. Establish that a metric space (X, d) is compact *if and only if* it is complete and totally bounded

15.a. If X is locally compact ,or if X satisfies the first countability axiom, then prove that X is compactly generated.

(or)

15.b. If X is a compactly generated space and (Y,d) is a metric space, then prove that C (X,Y) is closed in Y^X in the topology of compact convergence.

Part C $5 \times 12 = 60$

Answer ALL questions

16.a. Establish that every regular space with a countable basis is normal.

(or)

16.b. [i]. Explain : A subspace of the Hausdorff space is Hausdorff.
[ii]. Establish that every compact Hausdorff space is normal.

17.a. State and prove the Tietze Extension Theorem.

(or)

17.b. State and prove the Urysohn Metrization Theorem.

18.a. State and prove the Tychonoff theorem on compact spaces.

(or)

18.b. If X is a completely regular space, then prove that there exists a compactification Y of X having the property that every bounded continuous map $f: X \to R$ extends uniquely to a continuous map of Y into R.

19.a. If the space Y is complete in the metric d, then prove that the space Y^J is complete in the uniform metric $\bar{\rho}$ corresponding to d.

(or)

19.b. Let X be a topological space and (Y,d) be a metric space. Assume that X and Y are compact. If the subset F of C(X,Y) is equicontinuous under d, then prove that F is totally bounded under the uniform and sup metrics corresponding to d.

20.a. If X is a topological space and (Y,d) is a metric space, then establish that on the set C (X,Y), the compact - open topology and the topology of compact convergence coincide.

(or)

20.b. State and prove the Ascoli’s theorem.

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