
Summary and Conclusion

In 1966, Imai and Iseki [25,26] introduced two new classes of abstract algebras: BCK-algebras and BCI-algebras. These algebras have been extensively studied since their introduction. In 2017, Chandramouleeswaran et al.[18] introduced the concept of Z-algebras as a new structure of algebra based on propositional calculus. The Z-algebra is not a generalization of BCK/BCI-algebras. In 1965, Zadeh[73] introduced the fundamental concept of a fuzzy set which is a generalization of an ordinary set. The fuzzy set theories developed by Zadeh and others are found many applications in the domain of mathematics and elsewhere. In 1971, Rosenfeld [65] introduced the notion of fuzzy groups. In 1991, following the idea of fuzzy groups, Xi[71] introduced the notion of fuzzy BCK-algebras.

In 1994, Jun and Meng [32] introduced the notion of fuzzy p-ideals and in 1999, Khalid and Ahmad [40] introduced the concept of fuzzy H-ideals in BCI-algebras and studied their properties. In 1997, Meng et al. [49] and Mostafa [50] fuzzified the concept of implicative ideals in BCK-algebras, independently. In the year 2009, fuzzy translations and fuzzy multiplications in BCK/BCI-algebras have been discussed by Lee et al.[44].

In 1986, the idea of “intuitionistic fuzzy set” was first published by Atanassov [8], as a generalization of the notion of fuzzy set. In 1984, intuitionistic L-fuzzy set was introduced by Atanassov and Stoeva [11] as a generalization of L-fuzzy set. In 1975, Zadeh [74] introduced the notion of interval-valued fuzzy sets as an extension of fuzzy sets [73]. In 1989, K. T. Atanassov and G. Gargov [10] proposed interval-valued intuitionistic fuzzy set based on the comparative analysis of interval-valued fuzzy sets and intuitionistic fuzzy sets. Intuitively, the extension of intuitionistic fuzzy sets to interval-valued intuitionistic fuzzy sets furnishes additional capability to handle the vague information. In 2012, Jun et al. [36] have introduced a remarkable theory, namely, the theory of cubic sets. This structure is comprised of an interval-valued fuzzy set and a fuzzy set. Meanwhile, in 2010, Jun et al. [35] introduced the notion of cubic subalgebras/ideals in BCK/BCI algebras and investigated several properties. In 2011, Jun et al. [34] introduced the notion of cubic subgroups. They also studied how the images

or inverse images of cubic subgroups become cubic subgroup.

We have reviewed the existing literature of fuzzy algebra and Z -algebra. Motivated by the study, we have extended the concepts of fuzzy sets, intuitionistic fuzzy sets, intuitionistic L -fuzzy sets, interval-valued fuzzy sets, interval-valued intuitionistic fuzzy sets and cubic sets to Z -algebras. Many interesting and important results pertaining to level subset, Z -homomorphism and Cartesian product of Z -Subalgebras and Z -Ideals in Z -Algebras.

In Chapter 1, brief advancement in the theory of fuzzy sets, intuitionistic fuzzy sets, intuitionistic L -fuzzy sets, interval-valued fuzzy sets, interval-valued intuitionistic fuzzy sets and cubic sets are furnished. Further, we have recalled the definitions from algebras that arise from propositional calculi such as BCK-algebras, BCI-algebras and so on. In particular, we have presented the basic definitions and results in Z -algebras.

In Chapter 2, we have initiated fuzzy Z -Subalgebras and fuzzy Z -ideals in Z -algebras and obtained some characterization theorems using level subsets. We have also proved some theorems using the concepts of strongest fuzzy relation, Z -homomorphism and cartesian product on fuzzy Z -Subalgebras and fuzzy Z -ideals in Z -algebras.

In Chapter 3, we have defined H -ideal, fuzzy H -ideal, p -ideal, fuzzy p -ideal, implicative ideal, fuzzy implicative ideal, fuzzy characteristic, sub-implicative ideal, fuzzy sub-implicative ideal in Z -algebras. Also, the Artinian Z -algebras and Noetherian Z -algebras and characterized them by fuzzy p -ideals in Z -algebras. Further, the relationship between fuzzy Z -ideal, fuzzy H -ideal, fuzzy p -ideal, fuzzy implicative ideal and fuzzy sub-implicative ideals in Z -algebras are also explored.

In Chapter 4, we have defined fuzzy α -translations, fuzzy extensions and fuzzy β -multiplications of fuzzy Z -Subalgebras (fuzzy Z -ideals) of Z -algebras and obtained relations among them. Moreover, we have also discussed Z -homomorphism and cartesian product on fuzzy α -translations and fuzzy β -multiplications of Z -algebras and proved some interesting results.

In Chapter 5, the notions of intuitionistic fuzzy Z -Subalgebras and intuitionistic fuzzy Z -ideals in Z -algebras are defined and some of their properties are investigated. Using the

concepts of upper level and lower level subsets of intuitionistic fuzzy set, we have proved some characterization theorems on these notions. Finally, we have proved some theorems under Z -homomorphism and cartesian product on intuitionistic fuzzy Z -Subalgebras and intuitionistic fuzzy Z -ideals in Z -algebras.

In Chapter 6, the notions of intuitionistic L -fuzzy Z -Subalgebras and intuitionistic L -fuzzy Z -ideals in Z -algebras are introduced and some of their properties are discussed.

In Chapter 7, the notions of interval-valued fuzzy Z -Subalgebras and interval-valued fuzzy Z -ideals in Z -algebras are introduced. Moreover, how the Z -homomorphic images and inverse images of interval-valued fuzzy Z -Subalgebras (interval-valued fuzzy Z -ideals) become interval-valued fuzzy Z -Subalgebras (interval-valued fuzzy Z -ideals) are also analysed. Further, we have proved that the Cartesian product of interval-valued fuzzy Z -Subalgebras (interval-valued fuzzy Z -ideals) is an interval-valued fuzzy Z -Subalgebra (interval-valued fuzzy Z -ideal) in Z -algebras.

In Chapter 8, the notions of interval-valued intuitionistic fuzzy Z -Subalgebras and interval-valued intuitionistic fuzzy Z -ideals in Z -algebras are introduced and some fundamental properties are discussed. We have proved that “Any Z -Subalgebra of a Z -algebra X can be realized as both interval-valued upper level and interval-valued lower level Z -Subalgebras of some interval-valued intuitionistic fuzzy Z -Subalgebra of X ”. The Z -homomorphic properties of interval-valued intuitionistic fuzzy Z -Subalgebras and interval-valued intuitionistic fuzzy Z -ideals in Z -algebras are provided. Moreover, the cartesian product of interval-valued intuitionistic fuzzy Z -Subalgebras and interval-valued intuitionistic fuzzy Z -ideals in Z -algebras are discussed.

In Chapter 9, the concepts of cubic sets in Z -Subalgebras and Z -ideals in Z -algebras are introduced. The Z -homomorphic image and the inverse image of the cubic Z -subalgebras and the cubic Z -ideals are studied. Finally, the Cartesian product of cubic Z -Subalgebras and cubic Z -ideals in Z -algebras are investigated and some related properties are analysed.

For future study, one can apply our results to various algebraic structures and also in the fields of graph theory and soft set theory.