A HYBRID GROUP ACCEPTANCE SAMPLING PLANS FOR LIFETIMES BASED ON WEIBULL DISTRIBUTION

Article · July 2012

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International Journal of Applied Mathematical Analysis and Applications
January-June 2012, Volume 7, No. 1, pp. 73-80

ISSN: 0973-3868

A HYBRID GROUP ACCEPTANCE SAMPLING PLANS FOR LIFETIMES BASED ON WEIBULL DISTRIBUTION

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ABSTRACT: In this paper we have developed a hybrid group acceptance sampling plan for a truncated life test when the lifetime of an item follows Weibull distribution. The minimum number of testers and acceptance number are determined when the consumer's risk and the test termination time and group size are specified. The operating characteristic values according to various quality levels are also obtained.

Keywords: Weibull distribution, Group acceptance sampling plan, Consumer's risk, Producer's risk, Operating characteristic, Truncated life tests.

1. Introduction

An acceptance sampling plan involves quality contracting on product orders between the producers and the consumers. It is an essential tool in the Statistical Quality Control. In most of the statistical quality control experiment, it is not possible to perform hundred percent inspection, due to various reasons. The acceptance sampling plan was first applied in the US Military for testing the bullets during World War II. For example, if every bullet was tested in advance, no bullets were available for shipment, and on the other hand if no bullets were tested, then disaster might occur in the battle field at the crucial time. Acceptance sampling plan is a 'middle path' between hundred percent inspection and no inspection at all. In a time – truncated sampling plan, a random sample is selected from a lot of products and put on the test where the number of failures is recorded until the pre – specified time. If the number of failures observed is not greater than the specified acceptance number, then the lot will be accepted. Two risks are always attached to an acceptance sampling. The probability of rejecting a good lot is known as the producer's risk and the probability of accepting a bad lot is called the consumer's risk This process is called Lot Acceptance Sampling or just Acceptance Sampling. In most acceptance sampling plans for a truncated life test, the major issue is to determine the sample size from a lot under consideration. It is implicitly assumed in the usual sampling plan that only a single item is put in a tester. However, testers accommodating a multiple number of items at a time are used in practice because testing time and cost can be saved by testing those items simultaneously. Sudden death testing is frequently adopted by using this type of testers (Pascual and Meeker, 1998; Vlcek et al., 2003; Jun et al., 2006). For this type of testers the number of items to be equipped in a tester is given by the specification. The acceptance sampling plan under this type of testers will be called a group acceptance sampling plan. When designing a group sampling plan, determining the sample size is equivalent to determining the number of groups as the group size is already given. The items in a group are tested independently, identically and simultaneously on the different testers for a pre-assigned time. The experiment is truncated if more than the acceptable number of failures occurred in any group during the experiment time. The method of determining the minimum number of testers for a predetermined number of groups is called as Hybrid Group Acceptance Sampling Plan (HGASP). If the HGASP is used in conjunction with truncated life tests, it is called a HGASP based on truncated life test assuming that the lifetime of product follows a certain probability distribution.

Acceptance sampling based on truncated life tests were discussed by many authors. Aslam M. and C. H., have studied, a group acceptance sampling plans for truncated life tests based on the inverse Rayleigh and log-logistic distributions. Baklizi A., and El Masri A. E. K., (2004) have studied acceptance sampling plans based on truncated life tests in the Birnbaum Saunders model. Gupta R. D., and Kunda D., (2003) have studied, discriminating between the Weibull and GE distributions. Gupta, S. S. and Groll P. A., (1961), Gamma distribution in acceptance sampling based on life tests. Muhammad Aslam, Chi-Hyuck Jun, Munir Ahmad (2009) studied a group acceptance plan based on truncated life test for Gamma distribution. Again Muhammad Aslam, Chi-Hyuck Jun, Munir Ahmad along with Mujahid Rasool (2011) have studied Improved group sampling plans based on time – truncated life tests. Srinivasa Rao, (2009), have studied a group acceptance sampling plans for lifetimes following a generalized exponential distribution. Srinivasa Rao G., (2010) have studied a group acceptance sampling plans for truncated life tests for Marshall-Olkin extended Lomax distribution. Also Srinivasa Rao G., (2011) have studied a hybrid group acceptance sampling plans for lifetimes based on generalized exponential distribution.The purpose of this study is to propose a HGASP based on truncated life tests when the lifetime of a product follows the Weibull distribution.

2. Weibull Distribution

The cumulative distribution function (cdf) of the Weibull distribution is given by

$$F(t, \sigma) = 1 - e^{-\left(\frac{t}{\sigma}\right)^m} \tag{1}$$

where σ is a scale parameter. If some other parameters are involved, then they are assumed to be known, for an example, if shape parameter of a distribution is unknown it is very difficult to design the acceptance sampling plan. In quality control analysis, the scale parameter is often called the quality parameter or characteristics parameter. Therefore, it is assumed that the distribution function depends on time only through the ratio of t/σ.

The failure probability of an item by time t_0 is given by

$$p = F(t_0 : \sigma). \tag{2}$$

The quality of an item is usually represented by its true mean lifetime although some other options such as median lifetime or B_{10} life are sometimes used. Let us assume that the true mean μ can be represented by the scale parameter. In addition, it is convenient to specify the test time as a multiple of the specified life so that $a\mu_0$ and the quality of an item as a ratio of the true mean to the specified life (μ/μ_0).

Then we can rewrite (2) as a function of 'a' (termination time) and the ratio μ/μ_0

$$p = F\,(a\mu_0 : \mu/\mu_0) \tag{3}$$

Here the underlying distribution is the Weibull distribution having known shape parameter and unknown scale parameter σ.

Then the true mean life of a product under the Weibull distribution is given by

$$\mu = \gamma\sigma \tag{4}$$

and

$$p = 1 - e^{-\left(\frac{ba}{\mu/\mu_0}\right)^m} \tag{5}$$

3. DESIGN OF THE PROPOSED SAMPLING PLAN

We are interested in designing a group sampling plan in order to assure that the mean life of an item in a lot (μ, say) is greater than the specified life μ_0, say under the assumption that the life time of an item follows Weibull distribution with known shape parameter. A lot of products or items are considered to be "good" if the true average life ì is greater than the specified life μ_0. We will accept the lot if $\mu \geq \mu_0$ at a certain level of consumer's risk. Otherwise, we have to reject the lot. The following hybrid group acceptance sampling plan based on the truncated life test is proposed:

1. Select the number of testers, r and assign the r items to each predefined groups g so that the sample size for a lot will be n = gr.
2. Pre-fix the acceptance number, c for each group and the experiment time t_0.
3. Accept the lot if less than or equal to c failures occurs in each of all groups.
4. Terminate the experiment if more than c failures occur in any group and reject the lot.

The proposed sampling plan is an extension of the ordinary sampling plan available in literature such as in Kantam et al., (2001) and Rosaiah and Kantam (2005), for which r = 1. We are interested in determining the number of testers required for each of two distributions under study, whereas the various values of acceptance number c and the termination time t_0 are assumed to be specified. Since it is convenient to set the termination time as a multiple of the specified life μ_0, we will consider $t_0 = a\mu_0$ for a specified constant a (termination ratio).

The probability of rejecting a good lot is called the producer's risk, whereas theprobability of accepting a bad lot is known as the consumer's risk. When determining the parameters of the proposed sampling plan, we will use the consumer's risk. Often, the consumer's risk is expressed by the consumer's confidence level. If the confidence level is p^*, then the consumer's risk will be $\beta = 1 - p^*$. We will determine the number of testers in the proposed sampling plan so that the consumer's risk does not exceed β. According to the HGASP, the lot of products is accepted only if there are less than or equal to c failures observed in each of the g groups.

Table 1
Consumer's Risk (β), Truncated Time (a), Group Size (g) and Acceptance Number (c)

β	a	g	c
0.25	0.7	2	0
0.10	0.8	3	1
0.05	1.0	4	2
0.01	1.2	5	3
	1.5	6	4
	2.0	7	5
		8	6
		9	7
		10	8

The HGASP is characterized by the three parameters. So, the lot acceptance probability will be

$$L(p) = \left(\sum_{i=0}^{c} \binom{r}{i} p^i (1-p)^{r-1} \right)^g \tag{6}$$

where p is the probability that an item in a group fails before the termination time $t_0 = a\mu_0$.

The probability p for the Weibull distribution is given by

$$p = 1 - e^{-\left(\frac{ba}{\mu/\mu_0}\right)^m} \tag{7}$$

where $b = (\Gamma(1/m)/m)^m$. The minimum number of testers required can be determined by considering the consumer's risk when the true mean life equals the specified mean life ($\mu = \mu_0$) (worst case) by means of the following inequality:

$$L(p) \leq \beta \tag{8}$$

where p_0 is the failure probability at $\mu = \mu_0$, and it is given by

$$p_0 = 1 - e^{-(ba)^m}. \tag{9}$$

4. OPERATING CHARACTERISTIC FUNCTIONS

The probability of acceptance can be regarded as a function of the deviation of the specified value μ_0 of the mean from its true value μ. This function is called Operating Charateristic (OC) function of the sampling plan. Once the number of testers are obtained, one may be interested to find the probability of acceptance of a lot when the quality (or reliability) of the product is sufficiently good. As mentioned earlier, the product is considered to be good if $\mu \geq \mu_0$. For $\gamma = 2$ the probabilities of acceptance are displayed in Table 3 for various values of the mean ratios μ/μ_0, producer's risks β and time multiplier a.

5. NOTATION

- g – Number of groups
- r – Number of items in a group
- n – Sample size
- d – Number of defectives
- c – Acceptance number
- t_0 – Termination time
- a – Test termination time multiplier
- m – Shape parameter
- σ – Scale parameter
- β – Consumer's risk
- p – Failure probability
- L (p) – Probability of acceptance
- p* – Minimum probability
- μ – Mean life
- μ_0 – Specified life

6. DESCRIPTION OF RABLES AND EXAMPLES

The design parameters of HGASP are found at the various values of the consumer's risk and the test termination time multiplier in Table 2. It should be noted that if one needs the minimum sample size, it can be obtained by n = rg. Table 2 indicates that, as the test termination time multiplier a increases, the number of testers r decrease, i.e., a smaller number of testers is needed, if the test termination time multiplier increases at a fixed number of groups. For an example, from Table 2, if $\beta = 0.10$, g = 4, c = 2 and a changes from 0.7 to 0.8, the required values of design parameters of HGASP remains r = 5. However, this trend is not monotonic since it depends on the acceptance number as well. The probability of acceptance for the lot at the mean ratio corresponding to the producer's risk is also given in Table 3.

Table 2
Minimum Number of Testers (r) the Proposed Plan for Weibull Distribution

			μ/μ_0					
P*	g	c	0.7	0.8	1.0	1.2	1.5	2.0
0.75	2	0	1	1	1	1	1	1
	3	1	4	3	3	2	2	2
	4	2	4	4	4	3	3	3
	5	3	6	5	5	5	4	4
	6	4	7	7	6	6	5	5
	7	5	9	8	7	7	6	6
	8	6	10	10	9	8	7	7
	9	7	12	11	10	9	9	8
	10	8	13	12	11	10	10	9
0.90	2	0	2	2	2	1	1	1
	3	1	4	4	3	3	2	2
	4	2	5	5	4	4	3	3
	5	3	7	6	5	5	5	4
	6	4	8	8	7	6	6	5
	7	5	10	9	8	8	7	6
	8	6	11	10	9	8	8	7
	9	7	13	12	11	10	9	8
	10	8	14	13	12	11	10	9
0.95	2	0	3	2	2	2	1	1
	3	1	5	4	3	3	2	2
	4	2	6	5	4	4	4	3
	5	3	7	7	6	5	5	4
	6	4	9	8	7	6	6	5
	7	5	10	9	8	8	7	6
	8	6	12	11	10	9	8	7
	9	7	13	12	11	10	9	8
	10	8	15	14	12	11	10	9
0.99	2	0	4	3	3	2	2	2
	3	1	7	6	4	4	3	2
	4	2	7	6	5	5	4	4
	5	3	8	7	6	6	5	5
	6	4	10	9	8	7	6	6
	7	5	11	10	9	8	7	7
	8	6	13	12	10	9	9	8
	9	7	14	13	12	11	10	9
	10	8	16	15	13	12	11	10

Suppose that the lifetime of a product follows the weibull distributions with b = 1 and m = 1. It is desired to design a HGASP to test if the median is greater than 1,000 hrs based on a testing time of 700 hrs and using 4 groups. It is assumed that c = 2 and β = 0.10. This leads to the termination multiplier a = 0.700. From Table 2, the minimum number of testers required is r = 5. Thus, we will draw a random sample of size 20 items and allocate 5 items to each of 4 groups to put on test for 700 hrs. This indicates that 20 products are needed and that 5 items are allocated to each of 4 groups. We will accept the lot if no more than 2 failure occurs before 700 hrs in each of 4 groups. We truncate the experiment as soon as the 3rd failure occurs before the 700th hr. For this proposed sampling plan the probability of acceptance is 0.877819 when the true mean is 4,000 hrs. This shows that, if the true mean life is 4 times of 1000 hrs, the producer's risk is 0.122181.

Table 3
Operating Characteristics Values of the Hybrid Group Sampling Plan with g = 4 and c = 2 for Weibull Distribution

			μ/μ_0					
P*	r	a	2	4	6	8	10	12
0.75	4	0.7	0.715772	0.943026	0.980543	0.991211	0.995315	0.997216
	4	0.8	0.633393	0.920164	0.972090	0.987257	0.993165	0.995922
	4	1.0	0.470573	0.863196	0.949751	0.976539	0.987257	0.992334
	3	1.2	0.680194	0.932155	0.976387	0.989233	0.994229	0.996557
	3	1.5	0.529684	0.883180	0.957405	0.980159	0.989233	0.993526
	3	2.0	0.312074	0.777710	0.911954	0.957405	0.976387	0.985606
0.90	5	0.7	0.505182	0.877819	0.955775	0.979493	0.988906	0.993344
	5	0.8	0.400474	0.833388	0.937557	0.970592	0.983951	0.990317
	4	1.0	0.470573	0.863196	0.949751	0.976539	0.987257	0.992334
	4	1.2	0.328260	0.793771	0.920164	0.961835	0.978994	0.987257
	3	1.5	0.529684	0.88318	0.957405	0.980159	0.989233	0.993526
	3	2.0	0.312074	0.777710	0.911954	0.957405	0.976387	0.985606
0.95	6	0.7	0.321404	0.792337	0.919908	0.961801	0.979008	0.987279
	5	0.8	0.400474	0.833388	0.937557	0.970592	0.983951	0.990317
	4	1.0	0.470573	0.863196	0.949751	0.976539	0.987257	0.992334
	4	1.2	0.328260	0.793771	0.920164	0.961835	0.978994	0.987257
	4	1.5	0.173021	0.674873	0.863196	0.980159	0.961835	0.976539
	3	2.0	0.312074	0.777710	0.911954	0.957405	0.976387	0.985606
0.99	7	0.7	0.187095	0.693762	0.873616	0.937878	0.965290	0.978744
	6	0.8	0.223509	0.725060	0.888847	0.945860	0.969899	0.981624
	5	1.0	0.231682	0.730521	0.891292	0.947089	0.970592	0.982050
	5	1.2	0.121949	0.617790	0.833388	0.916008	0.952403	0.970592
	4	1.5	0.173021	0.674873	0.863196	0.932092	0.961835	0.976539
	4	2.0	0.048244	0.470573	0.742465	0.863196	0.920164	0.949751

7. CONCLUSION

We here proposed a hybrid group acceptance sampling plan from the truncated life test, the number of testers and the acceptance number was determined for Weibull distributions when the consumer's risk (β) and the other plan parameters are specified. It can be observed that the minimum number of testers required decreases as test termination time multiplier increases and also the operating characteristics values increases more rapidly as the quality improves. This HGASP can be used when a multiple number of items at a time are adopted for a life test and it would be beneficial in terms of test time and cost because a group of items will be tested simultaneously.

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