Avinashilingam Institute for Home Science and Higher Education for Women
(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)
Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B
Coimbatore - 641 043, Tamil Nadu, India

Bachelor's Degree Examination – January 2021
V Semester

Class : III UG **Time : 3 Hours**
Major : Mathematics and Spl. Education and Mathematics **Max. Marks : 100**

18BMAC14 / 18BSMC11 Real Analysis - I

PART A **10 x 1 = 10**
Choose the Correct Answer

1. The cardinal number of the set S={0, 1, 2,…, n} is
a.n b. n+1 c. n-1 d. n-2

2.The set of all rational numbers is
a. uncountable b. countable c. bounded d. finite

3. A set S is open if and only if
a. $S \neq \bar{S}$ b. $S = \bar{S}$ c. $S = int\ S$ d. $S \neq int\ S$

4. The set of all accumulation points of a set S is called
a. open set b. closed set c. derived set d. convex set

5. If the real line $\mathbf{R^1}$is covered by the collection of all open intervals (a,b) then it is
a. countable b. uncountable c. bounded d. unbounded

6. A set S in $\mathbf{R^n}$lies entirely within an n-ball $B(\boldsymbol{a};\boldsymbol{r})$ for some **r**>0 and some **a** in $\mathbf{R^n}$, is said to be
a. unbounded b. closed c. bounded d. open

7. Every open covering of a set S contains a finite sub-cover if and only if
a. S is compact b. S is closed c. S is open d. S is not compact

8. Every ball $B_M(\boldsymbol{a};\boldsymbol{r})$ in a metric space M is _____ in M.
a. closed b. bounded c. open d. boundary

9. The subspace T=(0,1] of a complete space $\mathbf{R^1}$is _____
a. complete b. not complete c. metric d. open

10. Which one of the following function is continuous?
a. f(z)= constant b.$f(z) = \frac{1}{z},\ z \in C$ c.$f(z) = \tan z$ d. none

Part B 5 x 6 = 30

Answer ALL questions
Each answer should not exceed 400 words or two pages

11.a. Prove that every infinite set S contains a proper subset similar to S.

(or)

11.b. Show that the set of all real numbers is uncountable.

12.a. Show that the union of any collection of open sets is an open set.

(or)

12.b. Prove that a set S is $\mathbf{R^n}$ is closed if and only if it contains all its adherent points.

13.a. Let $\{Q_1, Q_2, Q_3, \ldots\}$ be a countable collection of nonempty sets $\mathbf{R^n}$ such that:
i. $Q_{k+1} \subseteq Q_k \quad (k = 1, 2, 3, \ldots)$; ii. Each set Q_k is closed and nonempty. Then prove that $\cap_{k=1}^{\infty} Q_k \neq \emptyset$

(or)

13.b. Give an example of a set S which is closed but not bounded and exhibit a countable open covering F such that no finite subset of F covers S.

14.a. Define Metric space. Give any three examples.

(or)

14.b. Let X be a closed subset of a compact metric space. Then show that X is compact.

15.a. A sequence $\{x_n\}$ is converges to p in a metric space (S, d) if and only if every subsequence $\{x_{k(n)}\}$ converges to p.

(or)

15.b. Consider $f(x,y) = \frac{x^2-y^2}{x^2+y^2}$ *if* $(x,y) \neq (0,0)$, $f(0,0) = 0$. Evaluate $\lim_{(x,y)\to(0,0)} f(x,y)$

Part C 5 x 12 = 60

Answer ALL questions
Each answer should not exceed 800 words or fourpages

16.a. Prove that every subset of a countable set is countable.

(or)

16.b. If F is a countable collection of disjoint sets $F = \{A_1, A_2, A_3, \ldots\}$, such that each set A_n is countable then show that the union $\cup_{k=1}^{\infty} A_k$ is also countable.

17.a. Prove that every non-empty open set S in $\mathbf{R^1}$ is the union of a countable collection of disjoint component interval of S

(or)

17.b. Show that the union of finite collection of closed sets is a closed set and the intersection of an arbitrary collection is open.

18.a. State and prove the Bolzano-Weierstrass theorem.

(or)

18.b. State and prove the Lindelof covering theorem.

19.a. State and prove the Heine-Borel covering theorem.

(or)

19.b. For any subset S of M prove that the following statements are equivalent.
i. S is closed in M.
ii. S contains all its adherent points.
iii. S contains all its accumulation points.
iv. $S = \bar{S}$.

20.a. Prove that every Cauchy sequence in Euclidean space $\mathbf{R^k}$ is convergent.

(or)

20.b. Define complete metric space. Also prove every compact subset T in any metric space (S, d) is complete.

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