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MULTI CRITERIA DECISION MAKING METHODS ON PRIORITY WEIGHTED NEUTROSOPHIC SOFT SET

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MULTI CRITERIA DECISION MAKING METHODS ON PRIORITY WEIGHTED NEUTROSOPHIC SOFT SET

INTRODUCTION

In 1965, Lotfi A-Zadeh [21] introduced fuzzy set as an extension of crisp set or non-fuzzy set. A fuzzy set is a class of objects with a continuum of grade of membership. Such a set is characterized by a membership ranging between zero and one. The intuitionistic fuzzy set are sets whose elements have degrees of membership and non-membership. Intuitionistic fuzzy sets was introduced by Krassimir Atanassov [19] (1983) as an extension of Lotfi Zadeh's notation of fuzzy set, which itself extends the classical notation of a set.

The intuitionistic fuzzy sets can only handle the incomplete information considering both the truth-membership (or simply membership) and falsity-membership (or non-membership) values. It does not handle the indeterminate and inconsistent information which exist in belief system. Smarandache [12] introduced the concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. The neutrosophic components T, I, F which represents the membership, indeterminacy, and non-membership values respectively, where $]^{-}0\,,1^{+}[$ is the non-standard unit interval, and thus defines the neutrosophic set. Later, Salama and Alblow [1] defined generalized neutrosophic sets (GNSs), where the triplet functions satisfy the condition $T \wedge I \wedge F \leq 0.5$.

In 1999, Molodtsov [7] introduced the notion of soft set theory for dealing with complicated problems and various types of uncertainties and the concept has been applied diverse practical fields such as decision making [33, 36, 38, 39], data analysis [46], forecasting [47], optimization [8], etc. Several researchers have incorporated different mathematical hybrid structures such as fuzzy soft sets [29, 30, 34], intuitionistic fuzzy soft set theory [25, 26, 35], possibility fuzzy soft set [41], generalized fuzzy soft sets [13, 37], generalized intuitionistic fuzzy soft [20], possibility intuitionistic fuzzy soft set [23], vague soft set [45], possibility vague soft set [17], neutrosophic soft sets [38], weighted neutrosophic soft sets [36], etc by generalizing and extending classical soft set theory of Molodtsov [7].

Maji [38] combined the concept of soft sets and neutrosophic set together by introducing a new concept called neutrosophic soft set and gave application of neutrosophic soft set in decision making problem. Recently, the properties and applications on the neutrosophic sets have studied increasingly [14, 15, 43, 44]. Recently, Broumi [44] studied generalized neutrosophic soft sets (GNSSs) and provided some definitions and operations of the concept. He also provided an application of GNSSs in decision making problems. Sahin, and Kucuk [40] discussed a method to find out similarity measures of two GNSSs and provided an application of GNSSs in decision making problem. In [32] Maji et al. introduced several operators for soft set theory: equality of two soft sets, subsets and superset of soft sets, complement of soft set, null soft sets and absolute soft sets. But some of these definitions and their properties have few gaps, which have been pointed out by Ali et al.[24] and yang [3]. In 2010, Cagman and Enginoglo [27] made some modifications the operations of soft sets and filled in these gap. In 2014, Cagman [28] redefined soft sets using the single parameter set and compared definitions with those defined before.

Multi criteria decision-making (MCDM) is considered as a complex decision-making (DM) tool involving both qualitative and quantitative factors. In recent years, severs MCDM techniques and approaches have been suggested for choosing the best probable options. De et al. [9] studied the Sanchez’s approach for medical diagnosis and also they extended this concept which is a generalization of fuzzy set theory with the notion of intuitionistic fuzzy set theory. TOPSIS (Technique for order preference by similarity to an ideal solution) method which is one of the most favorable and effective MCDM methods to solve MCDM problems and most used classical MCDM methods has developed by Hwang and Yoon [4]. The fundamental idea of TOPSIS is that the chosen alternative should have the shortest distance from the positive-ideal solution and the farthest distance from the negative-ideal solution. In classical MCDM methods, the attribute values and weights are determined precisely. Then the proposed set theories have provided the different multi-criteria decision making methods. To deal with problems consisting of incomplete and vague information, in 2000 Chen [5] conferred the fuzzy version of TOPSIS method for the first time. Chung and Chu [6] presented fuzzy TOPSIS method under group decision for facility location selection problems. The Boran [2] combined TOPSIS method with intuitionistic fuzzy set. They proposed a method to select best supplier in group decision making environment. Then the TOPSIS method for MCDM problems has extended in interval valued intuitionistic fuzzy sets by Ye [16].

Liu and Wang [22] presented new methods in an intuitionistic fuzzy environment for solving multi-criteria decision-making problems. Firstly, they defined an evaluation function for the decision-making problem and then introduced operators which will reduce the degree of uncertainly of the elements corresponding to an intuitionistic fuzzy set. Biswas et al. [31] extended the notion of TOPSIS method for MAGDM problems under single valued neutrosophic environment.

In this study, TOPSIS method merged with Prioritized Neutrosophic Soft set is used to select best college in group decision making environment. Finally, analytical example based on selection process is stated to illustrate the application of Prioritized Neutrosophic Soft set TOPSIS method.

REVIEW OF LITRATURE:

Zadeh (1965) introduced the concept of Fuzzy set. It gave an opportunity for the people to deal with non-statistical vague matters (concepts). It had also shown its importance and gained the interests of researchers in various fields such as medicine, engineering, political science, artificial intelligence, robotics, signal processing, network systems and attempted to quantify the same.

Atanassov (1986 and 1989) introduced the Intuitionistic fuzzy sets and its applications were found in various disciplines of research. Some degree of hesitation in an information had been captured and studied under Intuitionistic fuzzy sets which was the generalization of a fuzzy set.

The neutrosophic set (NS) was introduced by F. Smarandache who introduced the degree of indeterminacy (i) as indepedent component in his 1995 manuscris that was published in 1998. Neutrosophic set is a novel tool to deal with vagueness considering the truth-membership T, indeterminacy-membership I and falsity-membership F satisfying the condition $0 \leq T + I + F \leq 3$. It can be used to characterize the uncertain information more sufficiently and accurately than intuitionistic fuzzy set. Neutrosophic set has attracted great attention of many scholars that have been extended to new types and these extensions have been used in many areas such as aggregation operators, decision making, image processing, information measures, graph and algebraic structures. Because of such a growth, we present an overview on neutrosophic set with the aim of offering a clear perspective on the different concepts, tools and trends related to their extensions. A total of 137 neutrosophic set publication records from Web of Science are analyzed. The term "neutrosophic" because "neutrosophic" etymologically comes from "neutrosophy"[French neuter, Latin neuter, neutral, and Greek Sophia, skill/wisdom] which meansknowledge of neutral thought, and this third/neutral represents the main distinction between "fuzzy"/ "intuitionistic fuzzy" logic/setand „neutrosophic” logic/set, i.e. the included middle component (Lupasco-Nicolescu’s logicin philosophy), (i.e.) the neutral/indeterminate/unknown part (besides the "truth"/"membership" and "falsehood"/"non-membership" components that both appear in fuzzy logic/set).

Molodtsov introduced the theory of soft sets, which can be seen as a new mathematical approach to vagueness. Maji et al. presented the concept of the fuzzy soft sets (fs-sets) by embedding the ideas of fuzzy sets. By using this definition of fs-sets many interesting applications of soft set theory have been expanded by some researchers. Roy and Maji gave some applications of fs-sets. Som defined soft relation and fuzzy soft relation on the theory of soft sets. Aktas and Cagman compared soft sets with the related concepts of fuzzy sets and rough sets. Yang et al. defined the operations on fuzzy soft sets which are based on three fuzzy logic operators: negation, triangular norm and triangular conorm. Zou and Xiao introduced the soft set and fuzzy soft set into the incomplete environment. Xiao et al. used forecasting accuracy as the criterion of fuzzy membership function, and purposed a combined forecasting approached based on fs-sets. Yang et al. presented the combination of interval-valued fuzzy set and soft set. Kong et al. defined the normal parameter reduction in the fs-sets, and showed that Roy and Maji's algorithm is not convenient in general cases. Naim Çağman, Filiz Çıtak and Serdar Enginoğlu we give definition of fuzzy parameterized fuzzy soft (fpfs) sets and their operations. We then define fpfs-aggregation operator to form fpfs-decision making method that allows constructing more efficient decision processes.

Maji et al. presented the concept of the intuitionistic fuzzy soft set theory by combining the intuitionistic fuzzy set with the soft set. Possibility intuitionistic fuzzy soft set and its operations are introduced by Maruah Bashir, Abdul Razak Salleh and Shawkat Alkhazaleh. An application of possibility intuitionistic fuzzy soft sets in decision making and a similarity measure of two possibility intuitionistic fuzzy soft sets and an application of this similarity measure in medical diagnosis had been shown by Maruah Bashir, Abdul Razak Salleh and Shawkat Alkhazaleh

Based soft set and neutrosophic sets a hybrid structure ‘neutrosophic soft sets’ had been initiated by PK Maji. Some neutrosophic soft set definitions, operations and some properties of this concept had been established by PK Maji . Maji introduced the concept of neutrosophic soft set. The parameters considered here are neutrosophic in nature.

Imposing the weights on the parameters (may be in a particular parameter also) aweighted neutrosophic soft sets had been introduced by Pabitra Kumar Maji.

CHAPTER-I

PRELIMINARIES

DEFINITION 1.1

If X is a collection of objects denoted generically by *x*, then a fuzzy set Ã in X is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$$

$\mu_{\tilde{A}}(x)$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of *x* in Ã that maps X to the membership space M (When M contains only the two points 0 and 1, Ã is non fuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of a nonfuzzy set). The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite. Elements with a zero degree of membership are normally not listed.

DEFINITION 1.2

An intuitionistic fuzzy set A in U is given by

$A = \{\langle u, \mu_A(u), \vartheta_A(u)\rangle \mid u \in U\}$

Where

$$\mu_A : U \rightarrow [0, 1], \vartheta_A : U \rightarrow [0, 1]$$

And

$$0 \leq \mu_A(u) + \vartheta_A(u) \leq 1 \; \forall u \in U$$

For each u, the numbers $\mu_A(u)$ and $\vartheta_A(u)$ are the degree of membership and degree of non membership of u to A, respectively.

DEFINITION 1.3

A neutrosophic set A on the universe of discourse X is defined as A = $\{< x, T_A(x), I_A(x), F_A(x) >, x \in X\}$, where T, I, F: X → $]-0, 1+[$ and $-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3+$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]-0, 1+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]-0, 1+[$. Hence we consider the neutrosophic set which takes the value from the subset of [0,1].

DEFINITION 1.4

A neutrosophic set A is contained in another neutrosophic set B i.e. A ⊆ B if $\forall x \in$ X , $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, $F_A(x) \geq F_B(x)$.

EXAMPLE 1.5

Assume that the universe of discourse X = $\{x_1, x_2, x_3\}$, where x_1 characterises the capability, x_2 characterises the trustworthiness and x_3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in [0, 1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is a Neutrosophic Set (NS) of X, such that, A = $\{< x_1, 0.4, 0.5, 0.3 >, < x_2, 0.7, 0.2, 0.4 >, < x_3, 0.8, 0.3, 0.4 >\}$, where the degree of goodness of capability is 0.4, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.3 etc.

DEFINITION 1.6

Let U be an initial universe set and E be a set of parameters or attributes with respect to U. Let P(U) denotes the power set of U. Consider a nonempty set A, A ⊂ E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A → P(U).

EXAMPLE 1.7

Assume that U = $\{h_1, h_2, h_3, h_4, h_5, h_6\}$ be a universal set consisting of a set of six houses under consideration, E = $\{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters with respect to U, where each parameter e_i, i = 1, 2, ⋯, 5 stands for ‘expansive’, ‘beautiful’, ‘cheap’, ‘modern’, ‘wooden’, respectively and A = $\{e_1, e_2, e_3\} \subset$ E. Suppose a soft set (F, A) describes the attractions of the houses, such that F(e_1) = $\{h_2,\ h_4\}$, F(e_2) = $\{h_1,\ h_3,\ h_5\}$ and F(e_3) = $\{h_3,\ h_4,\ h_5\}$. Then the soft set (F, A) is a parameterized family $\{\mathrm{F}(e_i) : i = 1, 2, 3\}$ of subset of U defined as (F, A) = $\{\mathrm{F}(e_1), \mathrm{F}(e_2), \mathrm{F}(e_3)\}$, i.e., (F, A) = $\{\{h_2,\ h_4\}, \{h_1,\ h_3,\ h_5\}, \{h_3,\ h_4,\ h_5\}\}$. The soft set (F, A) can also be represented as a set of ordered pairs as follows:

(F, A) = $\{(e_1, F(e_1)),\ (e_2, F(e_2)),\ (e_3, F(e_3))\}$ i.e.,

(F, A) = $\{(e_1, \{h_2, h_4\}),\ (e_2, \{h_1,\ h_3,\ h_5\}),\ (e_3, \{h_3,\ h_4,\ h_5\})\}$ other notations for (F, A) are F_A or $(F_A,\ E)$.

NEUTROSOPHIC SOFT SET

DEFINITION 1.8

Let U be an initial universe set and E be a set of parameters. Consider A ⊂ E. Let P(U) denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the soft neutrosophic set over U, where F is a mapping given by F: A → P(U). For illustration we consider an example.

EXAMPLE 1.9

Let U be the universal set of houses under consideration and E is the set of parameters. Each parameter is a neutrosophic word or sentence involving neutrosophic words. Consider E = { beautiful, wooden, costly, very costly, moderate, green surroundings, in good repair, in bad repair, cheap, expensive }. In this case, to define a neutrosophic soft set means to point out beautiful houses, wooden houses, houses in the green surroundings and so on. Suppose that, there are five houses in the universe U given by, U = $\{h_1, h_2, h_3, h_4, h_5\}$ and the set of parameters A = $\{e_1,\ e_2,\ e_3, e_4\}$, where e_1 stands for the parameter 'beautiful', e_2 stands for the parameter 'wooden', e_3 stands for the parameter 'costly'' and the parameter e_4 stands for 'moderate'. Suppose that,

F (beautiful) = $\{< h_1,\ 0.5, 0.6, 0.3 >, < h_2, 0.4,\ 0.7, 0.6 >, < h_3,\ 0.6, 0.2, 0.3 >, < h_4, 0.7, 0.3, 0.2 >, < h_5, 0.8, 0.2, 0.3 >\}$,

F (wooden) = $\{< h_1, 0.6, 0.3, 0.5 >, < h_2, 0.7,\ 0.4, 0.3 >, < h_3, 0.8, 0.1, 0.2 >, < h_4, 0.7,\ 0.1, 0.3 >, < h_5, 0.8, 0.3, 0.6 >\}$,

F (costly) = $\{< h_1, 0.7, 0.4, 0.3 >, < h_2, 0.6,\ 0.7, 0.2 >, < h_3, 0.7, 0.2, 0.5 >, < h_4, 0.5, 0.2, 0.6 >, < h_5, 0.7, 0.3, 0.4 >\}$, F (moderate) = $\{< h_1, 0.8, 0.6, 0.4 >, < h_2, 0.7, 0.9, 0.6 >, < h_3, 0.7, 0.6, 0.4 >, < h_4, 0.7, 0.8, 0.6 >, < h_5, 0.9, 0.5, 0.7 >\}$.

The neutrosophic soft set (NSS) (F, E) is a parametrized family $\{F(e_i),\ i = 1 \cdots 10\}$ of all neutrosophic sets of U and describes a collection of approximation of an object. The mapping F here is 'houses (.)', where dot (.) is to be filled up by a parameter e ∈ E. Therefore, F(e_1) means 'houses(beautiful)' whose functional-value is the neutrosophic set $\{< h_1, 0.5, 0.6, 0.3 >, < h_2, 0.4, 0.7, 0.6 >, < h_3, 0.6, 0.2, 0.3 >, < h_4, 0.7, 0.3, 0.2 >, < h_5, 0.8, 0.2, 0.3 >\}$

Thus we can view the neutrosophic soft set (NSS) (F, A) as a collection of approximation as below:

(F, A) = { beautiful houses = $\{< h_1, 0.5, 0.6, 0.3 >, < h_2, 0.4, 0.7, 0.6 >, h_3, 0.6, 0.2, 0.3 >, < h_4, 0.7, 0.3, 0.2 >, < h_5, 0.8, 0.2, 0.3 >\}$,

wooden houses= $\{< h_1, 0.6, 0.3, 0.5 >, < h_2, 0.7, 0.4, 0.3 >, < h_3, 0.8, 0.1, 0.2 >, < h_4, 0.7, 0.1, 0.3 >, < h_5, 0.8, 0.3, 0.6 >\}$,

costly houses = $\{ < h_1, 0.8, 0.6, 0.4 >, < h_2, 0.6, 0.7, 0.2 >, < h_3, 0.7, 0.2, 0.5 >, < h_4, 0.5, 0.2, 0.6 >, < h_5, 0.7, 0.3, 0.4 > \}$,

moderate houses= $\{ < h_1, 0.7, 0.4, 0.3 >, < h_2, 0.7, 0.9, 0.6 >, < h_3, 0.7, 0.6, 0.4 >, < h_4, 0.7, 0.8, 0.6 >, < h_5, 0.9, 0.5, 0.7 > \}$.

where each approximation has two parts:

(i) A predicate p

(ii) An approximate value-set v (or simply to be called value-set v).

FOR EXAMPLE,

For the approximation 'beautiful houses =$\{ < h_1, 0.5, 0.6, 0.3 >, < h_2, 0.4, 0.7, 0.6 >, < h_3, 0.6, 0.2, 0.3 >, < h_4, 0.7, 0.3, 0.2 >, < h_5, 0.8, 0.2, 0.3 > \}$,

we have (i) the predicate name `beautiful houses', and (ii) the approximate value-set is $\{ < h_1, 0.5, 0.6, 0.3 >, < h_2, 0.4, 0.7, 0.6 >, < h_3, 0.6, 0.2, 0.3 >, < h_4, 0.7, 0.3, 0.2 >, < h_5, 0.8, 0.2, 0.3 > \}$. Thus, a neutrosophic soft set (F, E) can be viewed as a collection of approximation like (F, E) = $\{p_1 = v_1, p_2 = v_2, \cdots p_{10} = v_{10}\}$. For the purpose of storing a neutrosophic soft set in a computer, we could represent it in the form of a table as shown below (corresponding to the neutrosophic soft set in the above Example). In this table, the entries are c_{ij} corresponding to the house h_i and the parameter e_j, where c_{ij} = (true-membership value of

h_i, indeterminacy-membership value of h_i, falsity-membership value of h_i) in F(e_j). The tabular representation of the neutrosophic soft set (F, A) is as follow:

U	**beautiful**	**wooden**	**costly**	**moderate**
$\boldsymbol{h_1}$	(0.5, 0.6, 0.3)	(0.6, 0.3, 0.5)	(0.7, 0.4, 0.3)	(0.8, 0.6, 0.4)
$\boldsymbol{h_2}$	(0.4, 0.7, 0.6)	(0.7, 0.4, 0.3)	(0.6, 0.7, 0.2)	(0.7, 0.9, 0.6)
$\boldsymbol{h_3}$	(0.6, 0.2, 0.3)	(0.8, 0.1, 0.2)	(0.7, 0.2, 0.5)	(0.7, 0.6, 0.4)
$\boldsymbol{h_4}$	(0.7, 0.3, 0.2)	(0.7, 0.1, 0.3)	(0.5, 0.2, 0.6)	(0.7, 0.8, 0.6)
$\boldsymbol{h_5}$	(0.8, 0.2, 0.3)	(0.8, 0.3, 0.6)	(0.7, 0.3, 0.4)	(0.9, 0.5, 0.7)

Table 1: Tabular form of the NSS (F, A).

DEFINITION 1.10

The class of all value sets of a neutrosophic soft set (F, E) is called value-class of the neutrosophic soft set and is denoted by $C_{(F,E)}$. For the above Example, $C_{(F,E)} = \{ v_1 , v_2 , \cdots \; v_{10}\}$. Clearly, $C_{(F,E)} \subset P(U)$.

DEFINITION 1.11

Let (F, A) and (G, B) be two neutrosophic soft sets over the common universe U. (F, A) is said to be neutrosophic soft subset of (G, B) if A ⊂ B and $T_{F(e)}(x) \leq T_{G(e)}(x)$,$I_{F(e)}(x) \leq I_{G(e)}(x)$, $F_{F(e)}(x) \geq F_{G(e)}(x)$, $\forall\, e \in A , x \in U$. We denote it by (F, A) ⊆ (G, B). (F, A) is said to be neutrosophic soft super set of (G, B) if (G, B)is a neutrosophic soft subset of (F, A). We denote it by (F, A) ⊇ (G, B).

EXAMPLE 1.12

Consider the two NSSs (F, A) and (G, B) over the common universe U = $\{\sigma_1 , \sigma_2 , \sigma_3 , \sigma_4 , \sigma_5\}$ .The NSS (F, A) describes the sizes of the objects whereas the NSS (G, B) describes its surface textures. Consider the tabular representation of the NSS (F, A) is as follows:

U	**small**	**large**	**moderate**
$\boldsymbol{\sigma_1}$	(0.4, 0.3, 0.6)	(0.6, 0.1, 0.7)	(0.5, 0.7, 0.5)
$\boldsymbol{\sigma_2}$	(0.3, 0.1, 0.4)	(0.6, 0.7, 0.8)	(0.6, 0.3, 0.6)
$\boldsymbol{\sigma_3}$	(0.6, 0.2, 0.7)	(0.3, 0.1, 0.6)	(0.5, 0.3, 0.8)
$\boldsymbol{\sigma_4}$	(0.7, 0.1, 0.6)	(0.1, 0.5, 0.7)	(0.7, 0.5, 0.7)
$\boldsymbol{\sigma_5}$	(0.3, 0.2, 0.4)	(0.6, 0.1, 0.6)	(0.3, 0.2, 0.3)

Table 2: Tabular form of the NSS (F, A).

The tabular representation of the NSS (G, B) is as follows:

U	**Small**	**large**	**moderate**	**very smooth**
$\boldsymbol{\sigma_1}$	(0.6, 0.4, 0.3)	(0.7, 0.2, 0.5)	(0.6, 0.7, 0.4)	(0.6, 0.8, 0.7)
$\boldsymbol{\sigma_2}$	(0.7, 0.5, 0.2)	(0.6, 0.7, 0.6)	(0.7, 0.3, 0.5)	(0.5, 0.7, 0.6)
$\boldsymbol{\sigma_3}$	(0.6, 0.3, 0.5)	(0.7, 0.2, 0.4)	(0.6, 0.4, 0.3)	(0.7, 0.9, 0.4)
$\boldsymbol{\sigma_4}$	(0.7, 0.1, 0.6)	(0.3, 0.6, 0.4)	(0.7, 0.5, 0.6)	(0.7, 0.3, 0.5)
$\boldsymbol{\sigma_5}$	(0.5, 0.4, 0.2)	(0.6, 0.6, 0.5)	(0.6, 0.4, 0.3)	(0.5, 0.8, 0.3)

Table 3: Tabular form of the NSS (G, B).

Here (F, A) ⊂ (G, B).

DEFINITION 1.13

Equality of two neutrosophic soft sets. Two NSSs (F, A) and (G, B) over the common universe U are said to be equal if (F, A) is neutrosophic soft subset of (G, B) and (G, B) is neutrosophic soft subset of (F, A). We denote it by (F, A) = (G, B).

DEFINITION 1.14

NOT set of a set of parameters. Let E =$\{e_1 , e_2, \cdots\ e_n\}$ be a set of parameters. The NOT set of E is denoted by ⌉E is defined by ⌉ E = { ⌉e_1 , ⌉e_2 , ⋯ ⌉e_n } where ⌉e_i = not $e_i, \forall i$ (it may be noted that ⌉ and ⌉ are different operators).

EXAMPLE 1.15

Consider the Example 1.9. Here $\urcorner E$ = { not beautiful, not wooden, not costly, not moderate }.

DEFINITION 1.16

Complement of a neutrosophic soft set. The complement of a neutrosophic soft set (F, A) denoted by $(\mathrm{F}, \mathrm{A})^{c}$ and is defined as $(\mathrm{F}, \mathrm{A})^{c}$ = $(\mathrm{F}^{c}, \urcorner\mathrm{A})$; where $\mathrm{F}^{c} : \urcorner\mathrm{A} \rightarrow \mathrm{P(U)}$ is a mapping given by $\mathrm{F}^{c}(\alpha)$ = neutrosophic soft complement with $T_{\mathrm{F}^{c}(x)} = F_{F(x)}$, $I_{\mathrm{F}^{c}(x)} = I_{F(x)}$ and $F_{\mathrm{F}^{c}(x)} = T_{F(x)}$.

EXAMPLE 1.17

Consider the Example 1.9. Then $(\mathrm{F}, \mathrm{A})^{c}$ describes the 'not attractiveness of the houses'. We have

F (not beautiful) = { $< h_1, 0.3, 0.6, 0.5 >, < h_2, 0.6, 0.7, 0.4 >,$
$< h_3, 0.3, 0.2, 0.6 >, < h_4, 0.2, 0.3, 0.7 >, h_5, 0.3, 0.2, 0.8 >$ }

F (not wooden) = { $< h_1, 0.5, 0.3, 0.6 >, < h_2, 0.3, 0.4, 0.7 >,$
$< h_3, 0.2, 0.1, 0.8 >, < h_4, 0.3, 0.1, 0.7 >, h_5, 0.6, 0.3, 0.8 >$ }

F (not costly) = { $< h_1, 0.3, 0.4, 0.7 >, < h_2, 0.2, 0.7, 0.6 >,$
$< h_3, 0.5, 0.2, 0.7 >, < h_4, 0.6, 0.2, 0.5 >, h_5, 0.4, 0.3, 0.7 >$ }

F (not moderate) = { $< h_1, 0.4, 0.6, 0.8 >, < h_2, 0.6, 0.9, 0.7 >,$
$< h_3, 0.4, 0.6, 0.7 >, < h_4, 0.6, 0.8, 0.7 >, h_5, 0.7, 0.5, 0.9 >$ }.

DEFINITION 1.18

Empty or Null neutrosophic soft set with respect to a parameter. A neutrosophic soft set (H, A) over the universe U is termed to be empty or null neutrosophic soft set with respect to the parameter A if $T_{H(e)}(m) = 0$, $F_{H(e)}(m) = 0$ and $I_{H(e)}(m) = 0$, $\forall m \in U$, $\forall e \in A$ In this case the null neutrosophic soft set (NNSS) is denoted by Φ_A.

EXAMPLE 1.19

Let U = $\{h_1, h_2, h_3, h_4, h_5\}$ the set of five houses be considered as the universal set and A = { beautiful, wooden, in the green surroundings } be the set of parameters that characterizes the houses. Consider the neutrosophic soft set (H, A) which describes the cost of the houses and

H(beautiful) = { $< h_1, 0, 0, 0 >, < h_2, 0, 0, 0 >, < h_3, 0, 0, 0 >, < h_4, 0, 0, 0 >$
$, < h_5, 0, 0, 0 >$ }

H(wooden) = { $< h_1, 0, 0, 0 >, < h_2, 0, 0, 0 >, < h_3, 0, 0, 0 >, < h_4, 0, 0, 0 >$
$, < h_5, 0, 0, 0 >$ }

H(in the green surroundings) = $\{ < h_1, 0, 0, 0 \quad >, < h_2, 0,\ 0, 0 >, \quad , < h_3, 0, 0, 0 >$
$< h_4, 0.5, 0, 0 > \ , < h_5, 0, 0, 0 > \}$

Here the NSS (H, A) is the null neutrosophic soft set.

DEFINITION 1.20

Union of two neutrosophic soft sets. Let (H, A) and (G, B) be two NSSs over the common universe U. Then the union of (H, A) and (G, B) is denoted by '(H, A) ∪ (G,B)',and is defined by (H, A) ∪ (G, B) = (K, C), where C = A ∪ B and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:

$T_{k(e)}(m)$ = $T_{H(e)}(m)$, if $e \in A - B$

= $T_{G(e)}(m)$, if $e \in B - A$

= $max\ \left(T_{H(e)}(m)\ , T_{G(e)}(m)\ \right)$, if $e \in A \cap B$.

$I_{k(e)}(m)$ = $I_{H(e)}(m)$, if $e \in A - B$

= $I_{G(e)}(m)$, if $e \in B - A$

= $\frac{I_{H(e)}(m) + I_{G(e)}(m)}{2}$, if $e \in A \cap B$.

$F_{k(e)}(m)$ = $F_{H(e)}(m)$, if $e \in A - B$

= $F_{G(e)}(m)$, if $e \in B - A$

= $min\ \left(F_{H(e)}(m)\ , F_{G(e)}(m)\ \right)$, if $e \in A \cap B$.

EXAMPLE 1.21

Let (H, A) and (G, B) be two NSSs over the common universe U. Consider the tabular representation of the NSS (H, A) is as follow:

U	beautiful	wooden	moderate
h_1	(0.6, 0.3, 0.7)	(0.7, 0.3, 0.5)	(0.6, 0.4, 0.5)
h_2	(0.5, 0.4, 0.5)	(0.6, 0.7, 0.3)	(0.6, 0.5, 0.4)
h_3	(0.7, 0.4, 0.3)	(0.7, 0.3, 0.5)	(0.7, 0.4, 0.5)
h_4	(0.8, 0.4, 0.7)	(0.6, 0.3, 0.6)	(0.7, 0.5, 0.6)
h_5	(0.6, 0.7, 0.2)	(0.7, 0.3, 0.4)	(0.8, 0.6, 0.5)

Table 4: Tabular form of the NSS (H, A).

The tabular representation of the NSS (G, B) is as follow:

U	costly	moderate
h_1	(0.7, 0.6, 0.6)	(0.7, 0.8, 0.6)
h_2	(0.8, 0.4, 0.5)	(0.8, 0.8, 0.3)
h_3	(0.7, 0.4, 0.6)	(0.5, 0.6, 0.7)
h_4	(0.6, 0.3, 0.5)	(0.8, 0.5, 0.6)
h_5	(0.8, 0.5, 0.4)	(0.6, 0.3, 0.5)

Table 5: Tabular form of the NSS (G, B).

Then the union of (H, A) and (G, B) is (K, C) whose tabular representation is as:

U	beautiful	wooden	moderate	costly
h_1	(0.6, 0.3, 0.7)	(0.7, 0.3, 0.5)	(0.7, 0.6, 0.5)	(0.7, 0.6, 0.6)
h_2	(0.5, 0.4, 0.5)	(0.6, 0.7, 0.3)	(0.8, 0.65, 0.3)	(0.8, 0.4, 0.5)
h_3	(0.7, 0.4, 0.3)	(0.7, 0.3, 0.5)	(0.7, 0.5, 0.5)	(0.7, 0.4, 0.6)
h_4	(0.8, 0.4, 0.7)	(0.6, 0.3, 0.6)	(0.8, 0.5, 0.6)	(0.6, 0.3, 0.5)
h_5	(0.6, 0.7, 0.2)	(0.7, 0.3, 0.4)	(0.8, 0.45, 0.5)	(0.8, 0.5, 0.4)

Table 6: Tabular form of the NSS (K, C).

DEFINITION 1.22

Intersection of two neutrosophic soft sets. Let (H, A) and (G, B) be two NSSs over the same universe U. Then the intersection of (H, A) and (G, B) is denoted by (H, A) ∩ (G,B) and is defined by (H, A) ∩ (G, B) = (K, C), where C = A ∩ B and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:

$T_{k(e)}(m) = min\ \left(T_{H(e)}(m)\ , T_{G(e)}(m)\ \right)$,

$I_{k(e)}(m) = \frac{I_{H(e)}(m) + I_{G(e)}(m)}{2}$ and

$F_{k(e)}(m) = max\ \left(F_{H(e)}(m)\ , F_{G(e)}(m)\ \right), \forall\ e\ \in\ C$.

EXAMPLE 1.23

Consider the above Example 1.21. Then that tabular representation of (H, A) ∩ (G, B) is as follow:

U	moderate
h_1	(0.6, 0.6, 0.6)
h_2	(0.6, 0.65, 0.4)
h_3	(0.5, 0.5, 0.7)
h_4	(0.7, 0.5, 0.6)
h_5	(0.6, 0.45, 0.5)

Table 7: Tabular form of the NSS (K, C).

For any two NSSs (H, A) and (G, B) over the same universe U and on the basis of the operations defined above, we have the following propositions:

PROPOSITION 1.24

(1) (H, A) ∪ (H, A) = (H, A).

(2) (H, A) ∪ (G, B) = (G, B) ∪ (H, A).

(3) (H, A) ∩ (H, A) = (H, A).

(4) (H, A) ∩ (G, B) = (G, B) ∩ (H, A).

(5) (H, A) ∪ **Φ** = (H, A).

(6) (H, A) ∩ **Φ = Φ**.

(7) $[(H, A)^{c}]^{c}$ = (H, A).

Proof. The proof of the Proposition (1.24) 1 to 7 are obvious.

For any three NSSs (H, A), (G, B) and (K, C) over the same universe U, we have the following propositions:

PROPOSITION 1.25

(1) (H, A) $\cup [(\mathrm{G,B}) \cup (\mathrm{K,C})] = [(\mathrm{H,A}) \cup (\mathrm{G,B})] \cup (\mathrm{K,C})$

(2) (H, A) $\cap [(\mathrm{G,B}) \cap (\mathrm{K,C})] = [(\mathrm{H,A}) \cap (\mathrm{G,B})] \cap (\mathrm{K,C})$

(3) (H, A) $\cup [(\mathrm{G,B}) \cap (\mathrm{G,B})] = [(\mathrm{H,A}) \cup (\mathrm{G,B})] \cap [(\mathrm{H,A}) \cup (\mathrm{K,C})]$

(4) (H, A) $\cap [(\mathrm{G,B}) \cup (\mathrm{K,C})] = [(\mathrm{H,A}) \cap (\mathrm{G,B})] \cup [(\mathrm{H,A}) \cap (\mathrm{K,C})]$

Proof. Proofs are simple and thus omitted.

DEFINITION 1.26

AND operation on two neutrosophic soft sets. Let (H, A) and (G, B) be two NSSs over the same universe U. Then 'AND' operation on them is denoted by '(H, A) ∧ (G, B)' and is defined by (H, A) ∧ (G, B) = (K, A × B), where the truth-membership, indeterminacy-membership and falsity-membership of (K, A × B) are as follows:

$$T_{K(\alpha,\beta)}(m) = min\left(T_{H(\alpha)}(m), T_{G(\beta)}(m)\right), I_{K(\alpha,\beta)}(m) = \frac{I_{H(\alpha)}(m)+I_{G(\beta)}(m)}{2},$$

$$F_{K(\alpha,\beta)}(m) = max\left(F_{H(\alpha)}(m), F_{G(\beta)}(m)\right), \forall\, \alpha \in \mathrm{A}, \forall\, \beta \in \mathrm{B}.$$

EXAMPLE 1.27

Consider the same Example 1.21 above. Then the tabular representation of (H, A) AND (G, B) is as follow:

U	**(beautiful, costly)**	**(beautiful, moderate)**	**(wooden, costly)**
h_1	(0.6, 0.45, 0.7)	(0.6, 0.55, 0.7)	(0.7, 0.45, 0.6)
h_2	(0.5, 0.4, 0.5)	(0.5, 0.6, 0.5)	(0.6, 0.55, 0.5)
h_3	(0.7, 0.4, 0.6)	(0.5, 0.5, 0.7)	(0.7, 0.35, 0.6)
h_4	(0.6, 0.35, 0.7)	(0.8, 0.45, 0.7)	(0.6, 0.3, 0.6)
h_5	(0.6, 0.6, 0.4)	(0.6, 0.5, 0.5)	(0.7, 0.4, 0.4)

U	**(wooden, moderate)**	**(moderate, costly)**	**(moderate, moderate)**

h_1	(0.7, 0.55, 0.6)	(0.6, 0.5, 0.6)	(0.6, 0.6, 0.6)
h_2	(0.6, 0.75, 0.3)	(0.6, 0.45, 0.5)	(0.6, 0.65, 0.4)
h_3	(0.5, 0.45, 0.7)	(0.7, 0.4, 0.6)	(0.5, 0.5, 0.7)
h_4	(0.6, 0.4, 0.6)	(0.6, 0.4, 0.6)	(0.7, 0.5, 0.6)
h_5	(0.6, 0.3, 0.5)	(0.8, 0.55, 0.5)	(0.6, 0.45, 0.5)

Table 8: Tabular representation of the NSS (K, $A \times B$).

DEFINITION 1.28

If (F, A) and (G, B) be two NSSs over the common universe U then '(F, A) OR (G, B)' denoted by (F, A) ∨ (G, B) is defined by (F, A) ∨ (G, B) = (O, A × B), where, the truth-membership, indeterminacy- membership and falsity-membership of O(α, β) are given as follows:

$$T_{O(\alpha,\beta)}(m) = max\left(T_{H(\alpha)}(m), T_{G(\beta)}(m)\right),$$

$$I_{O(\alpha,\beta)}(m) = \frac{I_{H(\alpha)}(m) + I_{G(\beta)}(m)}{2},$$

$$F_{O(\alpha,\beta)}(m) = min\left(F_{H(\alpha)}(m), F_{G(\beta)}(m)\right), \forall\, \alpha \in \text{A}, \forall\, \beta \in \text{B}.$$

EXAMPLE 1.29

Consider the same Example 1.21 above. Then the tabular representation of (H, A) OR (G, B) is as follow:

U	(beautiful, costly)	(beautiful, moderate)	(wooden, costly)
h_1	(0.7, 0.45, 0.6)	(0.7, 0.55, 0.6)	(0.7, 0.45, 0.5)
h_2	(0.8, 0.4, 0.5)	(0.8, 0.6, 0.3)	(0.8, 0.55, 0.3)
h_3	(0.7, 0.4, 0.3)	(0.7, 0.5, 0.3)	(0.7, 0.35, 0.5)
h_4	(0.8, 0.35, 0.5)	(0.8, 0.45, 0.6)	(0.6, 0.3, 0.5)
h_5	(0.8, 0.6, 0.2)	(0.8, 0.5, 0.2)	(0.8, 0.4, 0.4)

U	**(wooden, moderate)**	**(moderate, costly)**	**(moderate, moderate)**
$\boldsymbol{h_1}$	(0.7, 0.55, 0.5)	(0.7, 0.5, 0.5)	(0.7, 0.6, 0.5)
$\boldsymbol{h_2}$	(0.8, 0.75, 0.3)	(0.8, 0.45, 0.4)	(0.8, 0.65, 0.3)
$\boldsymbol{h_3}$	(0.7, 0.45, 0.5)	(0.7, 0.4, 0.5)	(0.7, 0.5, 0.5)
$\boldsymbol{h_4}$	(0.8, 0.4, 0.6)	(0.7, 0.4, 0.5)	(0.8, 0.5, 0.6)
$\boldsymbol{h_5}$	(0.7, 0.3, 0.4)	(0.8, 0.55, 0.4)	(0.8, 0.45, 0.5)

Table 9: Tabular representation of the NSS (O, $A \times B$).

For any two NSSs (H, A) and (G, B) over the common universe U, the De Morgan's types of results are true.

CHAPTER-II

PRIORITY WEIGHTED NEUTROSOPHIC SOFT SETS

In this section, we first introduce a new type of set called priority weighted neutrosophic soft set (PWNSS), and further construct a priority weighted neutrosophic soft decision making method.

DEFINITION 2.1

Let U be an initial universe, E be a parameter set, N(U) be the collection of all neutrosophic sets of U and F(U) is collection of all fuzzy subset of U. A priority weighted neutrosophic soft set (PWNS-set) $P_{[\alpha,\beta]}$ over U is a set of triple defined by

$$P_{[\alpha,\beta]} = \left\{\left(e_{k\,,}\left\{\left(\frac{u_j}{P(e_k)(u_j)}\,,\alpha(e_k)\big(u_j\big)\right)\right\},\beta(e_k)\right) : e_k \in E\right\}$$

or a mapping defined by $P_{[\alpha,\beta]} : E \rightarrow$ N(U) × F(U) × F(U), where, i ∈ $\wedge_1$ and k ∈ $\wedge_2$, P is a mapping given by P : E ⟶ N(U) and $\alpha\ (e_k)$, $\beta(e_k)$ is a fuzzy set such that α : E ⟶ F(U) and β : E → F(U).

For each parameter

$$e_k \in \mathrm{E},\ \mathrm{P}(e_k) = \{\langle u_j, T_{P(e_k)}\big(u_j\big), I_{P(e_k)}\big(u_j\big)\,, F_{P(e_k)}\big(u_j\big)\rangle : u_j\ \in U\ \}$$

Indicates neutrosophic value set of parameter e_k and where T, I, F are the truth, indeterminacy and falsity values respectively of the element $u_i \in$ U. For each $u_i \in$ U and $e_k \in$ E, $0 \leq T_{P(e_k)}\big(u_j\big) + I_{P(e_k)}\big(u_j\big) + F_{P(e_k)}\big(u_j\big) \leq 3$. Also $\alpha\ (\ e_k\ )$, degrees of priority given for the belongingness of elements of U in P(e_k) and $\beta(e_k)$ the weightage given to the parameters by the experts. So we can write

$$P_{[\alpha,\beta]}\ (e_k) = \left\{\left(\frac{u_1}{P(e_k)(u_1)}\,,\alpha(e_k)(u_1)\right)\left(\frac{u_2}{P(e_k)(u_2)}\,,\alpha(e_k)(u_2)\right),\ldots,\left(\frac{u_n}{P(e_k)(u_n)}\,,\alpha(e_k)(u_n)\right),\beta(e_k)\right\}$$

EXAMPLE 2.2

Let U = $\{u_{1,}u_{2,}u_3\}$ be a set of three restaurants. Let E =$\{e_{1,}e_{2,}e_3\}$ be a set of qualities where e_1=Taste, e_2 = Variety,e_3 =Service and let α : E →F(U) and β : E →F(U). We can define a function $P_{[\alpha,\beta]} : E \rightarrow$ N(U) × F(U) × F(U) as follow:

$$P_{[\alpha,\beta]} = \left\{\begin{matrix} P_{[\alpha,\beta]}(e_1) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.4}\,,\mathbf{0.1}\,,\mathbf{0.5})}\,,0.7\right),\left(\frac{u_2}{(\mathbf{0.6}\,,\mathbf{0.2}\,,\mathbf{0.4})}\,,0.3\right),\left(\frac{u_3}{(\mathbf{0.3}\,,\mathbf{0.4}\,,\mathbf{0.7})}\,,0.6\right)\right]\,,0.8\right\} \\ P_{[\alpha,\beta]}(e_2) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.7}\,,\mathbf{0.3}\,,\mathbf{0.4})}\,,0.5\right),\left(\frac{u_2}{(\mathbf{0.4}\,,\mathbf{0.6}\,,\mathbf{0.1})}\,,0.7\right),\left(\frac{u_3}{(\mathbf{0.6}\,,\mathbf{0.2}\,,\mathbf{0.8})}\,,0.3\right)\right]\,,0.7\right\} \\ P_{[\alpha,\beta]}(e_3) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.5}\,,\mathbf{0.6}\,,\mathbf{0.7})}\,,0.2\right),\left(\frac{u_2}{(\mathbf{0.4}\,,\mathbf{0.2}\,,\mathbf{0.6})}\,,0.5\right),\left(\frac{u_3}{(\mathbf{0.5}\,,\mathbf{0.4}\,,\mathbf{0.3})}\,,0.4\right)\right]\,,0.6\right\} \end{matrix}\right\}$$

For the purpose of storing a PWNS in a computer, we can use matrix notation of priority weighted neutrosophic soft set $P_{[\alpha,\beta]}$. For Example, matrix notation of PWNS $P_{[\alpha,\beta]}$ can be written as follows: for m, n ∈ ∧,

$$P_{[\alpha,\beta]} = \begin{pmatrix} (\langle 0.4\,,0.1\,,0.5\rangle\,,0.7)(\langle 0.6\,,0.2\,,0.4\rangle\,,0.3)(\langle 0.3\,,0.4\,,0.7\rangle\,,0.6)(0.8) \\ (\langle 0.7\,,0.3\,,0.4\rangle\,,0.5)(\langle 0.4\,,0.6\,,0.1\rangle\,,0.7)(\langle 0.6\,,0.2\,,0.8\rangle\,,0.3)(0.7) \\ (\langle 0.5\,,0.5\,,0.4\rangle\,,0.1)(\langle 0.4\,,0.2\,,0.6\rangle\,,0.5)(\langle 0.5\,,0.4\,,0.3\rangle\,,0.4)(0.6) \end{pmatrix}$$

where the m-th row vector shows P(e_m) and n-th column vector shows u_n

DEFINITION 2.3

Let $P_{[\alpha,\beta]}$, $Q_{[\gamma,\delta]}$ ∈ PWN (U, E). Then, $P_{[\alpha,\beta]}$ is said to be a priority weighted neutrosophic soft subset (PWNS-subset) of $Q_{[\gamma,\delta]}$, and denoted by $P_{[\alpha,\beta]} \subseteq Q_{[\gamma,\delta]}$ if

1. α(e) and β(e) are a fuzzy subset of γ(e) and δ(e), for all e ∈ E
2. P is a neutrosophic subset of.

EXAMPLE 2.4

Let U = $\{u_{1,}u_{2,}u_3\}$ be a set of three broadband service, and let E =$\{e_{1,}e_{2,}e_3\}$ be a set of qualities where e_1= Stability, e_2= Security, e_3 = price. Let $P_{[\alpha,\beta]}$ be a PWNS-set define as follows:

$$P_{[\alpha,\beta]} = \begin{cases} P_{[\alpha,\beta]}(e_1) = \left\{\left[\left(\frac{u_1}{(0.4,0.7,0.6)}\ ,0.8\right),\left(\frac{u_2}{(\mathbf{0.8,0.6,0.8})}\ ,0.3\right),\left(\frac{u_3}{(\mathbf{0.4,0.5,0.8})}\ ,0.7\right)\right]\ ,0.8\right\} \\ P_{[\alpha,\beta]}(e_2) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.7,0.4,0.6})}\ ,0.6\right),\left(\frac{u_2}{(\mathbf{0.5,0.7,0.5})}\ ,0.7\right),\left(\frac{u_3}{(\mathbf{0.7,0.4,0.8})}\ ,0.5\right)\right]\ ,0.7\right\} \\ P_{[\alpha,\beta]}(e_3) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.5,0.6,0.7})}\ ,0.2\right),\left(\frac{u_2}{(\mathbf{0.6,0.5,0.8})}\ ,0.5\right),\left(\frac{u_3}{(\mathbf{0.6,0.5,0.7})}\ ,0.4\right)\right]\ ,0.6\right\} \end{cases}$$

Also we can define a function $Q_{[\gamma\ ,\delta]}$: E ⟶ N(U) × F(U) as follows:

$$Q_{[\gamma\ ,\delta]} = \begin{cases} Q_{[\gamma\ ,\delta]}(e_1) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.4,0.7,0.6})}\ ,0.8\right),\left(\frac{u_2}{(\mathbf{0.8,0.6,0.8})}\ ,0.3\right),\left(\frac{u_3}{(\mathbf{0.4,0.5,0.8})}\ ,0.7\right)\right]\ ,0.8\right\} \\ Q_{[\gamma\ ,\delta]}(e_2) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.8,0.6,0.5})}\ ,0.7\right),\left(\frac{u_2}{(\mathbf{0.6,0.8,0.3})}\ ,0.8\right),\left(\frac{u_3}{(\mathbf{0.8,0.5,0.6})}\ ,0.5\right)\right]\ ,0.7\right\} \\ Q_{[\gamma\ ,\delta]}(e_3) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.6,0.7,0.5})}\ ,0.4\right),\left(\frac{u_2}{(\mathbf{0.7,0.6,0.7})}\ ,0.7\right),\left(\frac{u_3}{(\mathbf{0.8,0.6,0.4})}\ ,0.6\right)\right]\ ,0.6\right\} \end{cases}$$

It is clear that $P_{[\alpha,\beta]}$ is PWNS –subset of $Q_{[\gamma\ ,\delta]}$.

DEFINITION 2.5

Let $P_{[\alpha,\beta]}\ , Q_{[\gamma,\delta]} \in$ PWN (U, E). Then, $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]}$ are called priority weighted neutrosophic soft equal set and denote by$P_{[\alpha,\beta]} = \ Q_{[\gamma,\delta]}$ if $P_{[\alpha,\beta]} \subseteq Q_{[\gamma,\delta]}$ and $P_{[\alpha,\beta]} \supseteq Q_{[\gamma,\delta]}$.

DEFINITION 2.6

Let$P_{[\alpha,\beta]} \in$ PWN (U, E). Then, $P_{[\alpha,\beta]}$ is said to be priority weighted neutrosophic soft null set, denoted by$\phi_{[\alpha,\beta]}$, if $\forall e\ \in$, $\phi_{[\alpha,\beta]} : E \to$ N(U) × F(U) × F(U) such that $\phi_{[\alpha,\beta]}(e)$= $\left\{\left[\frac{u}{\phi(e)(u)}\ , \alpha(e)(u) : u \in U\right] \beta(e)\right\}$where ϕ(e) $=\{\langle u\ , 0\ , 0\ , 1\rangle : u \in U\}$ and $\alpha(e) = \{(u\ , 0) : u \in U\ \}$ and $\beta(e) = \{(u\ , 0) : u \in U\}$.

DEFINITION 2.7

Let$P_{[\alpha,\beta]} \in$ PWN(U,E) .Then ,$P_{[\alpha,\beta]}$ is said to be priority weighted neutrosophic soft universal set, denoted by$U_{[\alpha,\beta]}$, if $\forall e\ \in E, U_{[\alpha,\beta]} : E \to$ N(U) × F(U) such that $U_{[\alpha,\beta]}(e) = \left\{\left[\frac{u}{U(e)(u)}\ , \alpha(e)(u) : u \in U\right] \beta(e)\right\}$where U(e) = $\{\langle u\ , 1\ , 1\ , 0\rangle : u \in U\}$and $\alpha(e) = \{(u\ , 1) : u \in U\ \}$ and $\beta(e) = \{(u\ , 1) : u\ \in U\ \}$

PROPOSITION 2.8

Let $P_{[\alpha,\beta]}$, $Q_{[\gamma,\delta]}$ and $H_{[\theta,\phi]}$ ∈ PWN (U, E). Then,

1. $\phi_{[\alpha,\beta]} \subseteq P_{[\alpha,\beta]}$
2. $P_{[\alpha,\beta]} \subseteq U_{[\alpha,\beta]}$
3. $P_{[\alpha,\beta]} \subseteq \phi_{[\alpha,\beta]}$ and $\phi_{[\alpha,\beta]} \subseteq H_{[\theta,\phi]} \rightarrow P_{[\alpha,\beta]} \subseteq H_{[\theta,\phi]}$

***Proof* .** The proof follows from the Definitions (2.5) - (2.7)

DEFINITION 2.9

Let $P_{[\alpha,\beta]}$ ∈ PWN (U,E), where

$P_{[\alpha,\beta]}(e_k)$ = $\{([P(e_k)(u_i), \alpha(e_k)(u_i)], \beta(e_k)): e_k \in E, u_i \in U\}$

$P(e_k)$ = $\{\langle u, T_{P(e_k)}(u_i), I_{P(e_k)}(u_i), F_{P(e_k)}(u_i)\rangle\}\ \forall e_k \in E, u \in U.$

Then for $e_k \in E$ and $u_i \in U$, and

1. $P^T_{[\alpha,\beta]}$ is said to be truth- membership part of $P_{[\alpha,\beta]}$

$P^T_{[\alpha,\beta]}$ = $\{(P^T_{kj}(e_k), \alpha_{kj}(e_k))\}$

and $P^T_{kj}(e_k)$ = $\{(u_j, T_{P(e_k)}(u_j))\}$, $\alpha_{kj}(e_k)$ = $\{(u_j, \alpha(e_k)(u_j))\}$

2. $P^I_{[\alpha,\beta]}$ is said to be indeterminacy - membership part of $P_{[\alpha,\beta]}$

$P^I_{[\alpha,\beta]}$ = $\{(P^I_{kj}(e_k), \alpha_{kj}(e_k))\}$

and $P^I_{kj}(e_k)$ = $\{(u_j, I_{P(e_k)}(u_j))\}$, $\alpha_{kj}(e_k)$ = $\{(u_j, \alpha(e_k)(u_j))\}$

3. $P^F_{[\alpha,\beta]}$ is said to be falsity- membership part of $P_{[\alpha,\beta]}$

$P^F_{[\alpha,\beta]}$ = $\{(P^F_{kj}(e_k), \alpha_{kj}(e_k))\}$

and $P^F_{kj}(e_k)$ = $\{(u_j, F_{P(e_k)}(u_j))\}$, $\alpha_{kj}(e_k)$ = $\{(u_j, \alpha(e_k)(u_j))\}$

We can write a PWNS in form $P_{[\alpha,\beta]} = \left(P^T_{[\alpha,\beta]}, P^I_{[\alpha,\beta]}, P^F_{[\alpha,\beta]}\right)$.

A PWNS can be expressed in matrix form.

Let us consider priority weighted neutrosophic soft set $P_{[\alpha,\beta]}$ given in Example 2.4. Then priority weighted neutrosophic soft set $P_{[\alpha,\beta]}$ can be expressed in

matrix form as follows :

$$P^T_{[\alpha,\beta]} = \begin{pmatrix} (0.4\,,0.8)(0.8\,,0.3)(0.4\,,0.7)(0.8) \\ (0.7\,,0.6)(0.5\,,0.7)(0.7\,,0.5)(0.7) \\ (0.5\,,0.2)(0.6\,,0.5)(0.6\,,0.4)(0.6) \end{pmatrix}$$

$$P^I_{[\alpha,\beta]} = \begin{pmatrix} (0.7\,,0.8)(0.6\,,0.3)(0.5\,,0.7)(0.8) \\ (0.4\,,0.6)(0.7\,,0.7)(0.4\,,0.5)(0.7) \\ (0.6\,,0.2)(0.5\,,0.5)(0.5\,,0.4)(0.6) \end{pmatrix}$$

$$P^F_{[\alpha,\beta]} = \begin{pmatrix} (0.6\,,0.8)(0.8\,,0.3)(0.8\,,0.7)(0.8) \\ (0.6\,,0.6)(0.5\,,0.7)(0.8\,,0.5)(0.7) \\ (0.7\,,0.2)(0.8\,,0.5)(0.7\,,0.4)(0.6) \end{pmatrix}$$

DEFINITION 2.10

Let $P_{[\alpha,\beta]}$, $Q_{[\gamma,\delta]} \in$ PWN (U, E). The union of two PWNSs $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]}$ over U, denote by $P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}$ is defined by

$$P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]} = \left\{\left(e_k, \left\{\left(\rho, \alpha_{kj}(e_k) \oplus \gamma_{kj}(e_k)\right)\right\}, \beta_{kj}(e_k) \oplus \delta_{kj}(e_k)\right) : e_k \in E\right\}$$

Where

$$\rho = \frac{u_j}{\left(P^T_{kj}(e_k) \oplus Q^T_{kj}(e_k), P^I_{kj}(e_k) \oplus Q^I_{kj}(e_k), P^F_{kj}(e_k) \otimes Q^F_{kj}(e_k)\right)}$$

and $\oplus$ represents n-conorm and $\otimes$ represents n-norm functions respectively.

DEFINITION 2.11

Let $P_{[\alpha,\beta]}$, $Q_{[\gamma,\delta]} \in$ PWN (U, E). The intersection of two PWNSs

$P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]}$ over U , denote by $P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}$ is defined by

$$P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]} = \left\{\left(e_k, \left\{\left(\theta, \alpha_{kj}(e_k) \otimes \gamma_{kj}(e_k)\right)\right\}, \beta_{kj}(e_k) \otimes \delta_{kj}(e_k)\right) : e_k \in E\right\}$$

Where

$$\theta = \frac{u_j}{\left(P^T_{kj}(e_k)\otimes Q^T_{kj}(e_k),\ P^I_{kj}(e_k)\otimes Q^I_{kj}(e_k),\ P^F_{kj}(e_k)\oplus Q^F_{kj}(e_k)\right)}$$

and $\oplus$ represents n-conorm and $\otimes$ represents n-norm functions respectively .

EXAMPLE 2.12

Let us consider the PWNSs $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]}$ Defined as in 2.4. Let us suppose that n-norm is defined by a $\oplus$ b = $min\,\{a, b\}$ and the n-conorm is defined by a+b = $max\{a, b\}$ for a, b ∈ [0, 1]

$P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}$ =

$$\left\{\begin{array}{l}
\left(P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}\right)(e_1) = \left\{\left[\left(\frac{u_1}{\mathbf{(0.4,0.7,0.6)}}, 0.8\right), \left(\frac{u_2}{\mathbf{(0.8,0.6,0.8)}}, 0.3\right), \left(\frac{u_3}{\mathbf{(0.4,0.5,0.8)}}, 0.7\right)\right], 0.8\right\} \\
\left(P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}\right)(e_2) = \left\{\left[\left(\frac{u_1}{\mathbf{(0.8,0.6,0.5)}}, 0.7\right), \left(\frac{u_2}{\mathbf{(0.6,0.8,0.3)}}, 0.8\right), \left(\frac{u_3}{\mathbf{(0.8,0.5,0.6)}}, 0.5\right)\right], 0.7\right\} \\
\left(P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}\right)(e_3) = \left\{\left[\left(\frac{u_1}{\mathbf{(0.6,0.7,0.5)}}, 0.4\right), \left(\frac{u_2}{\mathbf{(0.7,0.6,0.7)}}, 0.7\right), \left(\frac{u_3}{\mathbf{(0.8,0.6,0.4)}}, 0.6\right)\right], 0.6\right\}
\end{array}\right\}$$

And

$P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}$ =

$$\left\{\begin{array}{l}
\left(P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}\right)(e_1) = \left\{\left[\left(\frac{u_1}{\mathbf{(0.4,0.7,0.6)}}, 0.8\right), \left(\frac{u_2}{\mathbf{(0.8,0.6,0.8)}}, 0.3\right), \left(\frac{u_3}{\mathbf{(0.4,0.5,0.8)}}, 0.7\right)\right], 0.8\right\} \\
\left(P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}\right)(e_2) = \left\{\left[\left(\frac{u_1}{\mathbf{(0.8,0.6,0.5)}}, 0.7\right), \left(\frac{u_2}{\mathbf{(0.6,0.8,0.3)}}, 0.8\right), \left(\frac{u_3}{\mathbf{(0.8,0.5,0.6)}}, 0.5\right)\right], 0.7\right\} \\
\left(P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}\right)(e_3) = \left\{\left[\left(\frac{u_1}{\mathbf{(0.6,0.7,0.5)}}, 0.4\right), \left(\frac{u_2}{\mathbf{(0.7,0.6,0.7)}}, 0.7\right), \left(\frac{u_3}{\mathbf{(0.8,0.6,0.4)}}, 0.6\right)\right], 0.6\right\}
\end{array}\right\}$$

PROPOSITION 2.13

Let $P_{[\alpha,\beta]}$, $Q_{[\gamma,\delta]}$ and $H_{[\theta,\phi]}$ ∈ PWN (U, E). Then,

1. $P_{[\alpha,\beta]} \cap \phi_{\alpha} = \phi_{\alpha}$ and $P_{[\alpha,\beta]} \cap U_{\alpha} = P_{[\alpha,\beta]}$

2. $P_{[\alpha,\beta]} \cup \phi \ = P_{[\alpha,\beta]}$and $P_{[\alpha,\beta]} \cup U_{\alpha} = U_{\alpha}$
3. $P_{[\alpha,\beta]} \cap \ (\ Q_{[\gamma,\delta]} \cap H_{[\theta,\phi]}) = \ (\ P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]} \cap H_{[\theta,\phi]}$ and $P_{[\alpha,\beta]} \cup (Q_{[\gamma,\delta]} \cup H_{[\theta,\phi]}) = (P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}) \cap H_{[\theta,\phi]}$
4. $P_{[\alpha,\beta]} \cap \ (\ Q_{[\gamma,\delta]} \cap H_{[\theta,\phi]}) = (\ P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}) \cup (P_{[\alpha,\beta]} \cap H_{[\theta,\phi]})$ and $P_{[\alpha,\beta]} \cup (Q_{[\gamma,\delta]} \cap H_{[\theta,\phi]}) = (P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}) \cap (P_{[\alpha,\beta]} \cup H_{[\theta,\phi]})$

$proof.$ The proof can be obtained from Definitions 2.10 and 2.11

DEFINITION 2.14

Let $\mathrm{P}_{[\alpha,\beta]} \epsilon$ PWN (U, E). Complement of PWNS $\mathrm{P}_{[\alpha,\beta]}$, denoted by $P^{c}_{[\alpha,\beta]}$, is defined by

$$P^{c}_{[\alpha,\beta]} \ = \left\{ \left(e \ , \left\{ \left(\frac{u_j}{n(P(e_k))} \ , \mathrm{N} \left(\alpha_{kj}(e_k)(\ u_j) \right) \right) , \mathrm{N}(\beta_{kj}(e_k)(\ u_j) \right\} \right) , \mathrm{e}\epsilon\mathrm{E} \right\}$$

Where,

$\left(\sim(\mathrm{P}_{kj})(e_k) \ \right)=\left(\sim \ (P^{T}_{kj}(e_k)), \sim(P^{I}_{kj}(e_k)), \sim(P^{F}_{kj}(e_k))\right), \forall$ k$\epsilon\wedge_1$, j$\epsilon\wedge_2$.

EXAMPLE 2.15

Let us consider the PWNS $\mathrm{P}_{[\alpha,\beta]}$ define in Example 2.4. Suppose that the negation is defined by $\left(\sim P^{T}_{kj}(e_k)\right) \ = \ P^{F}_{kj}(e_k \ ), \sim(\mathrm{P}^{\mathrm{F}}{}_{\mathrm{kj}}(e_k \ )) \ = P^{T}_{kj}(e_k \ ), \ \sim \ (P^{I}_{kj}(e_k \ )) \ = \ 1\text{-} \ P^{I}_{kj}(e_k \ )$ and $\sim\left(\alpha_{ij}(e_k) \ \right)= 1\text{-} \ \alpha_{kj}(e_k)$ and $\sim(\beta_{ij}(e_k))= 1\text{-}\beta_{ij}(e_k)$ respectively.

Then, $\sim\mathrm{P}_{[\alpha,\beta]}$ is defined as follow:

$$\sim\mathrm{P}_{[\alpha,\beta]} = \begin{cases} \sim\mathrm{P}_{[\alpha,\beta]}(e_1) = \left\{ \left[\left(\frac{u_1}{(0.6,0.3,0.4)} \ ,0.2 \right), \left(\frac{u_2}{(0.8,0.6,0.8)} \ ,0.7 \right), \left(\frac{u_3}{(0.8,0.5,0.4)} \ ,0.2 \right) \right] \ ,0.2 \right\} \\ \sim\mathrm{P}_{[\alpha,\beta]}(e_2) = \left\{ \left[\left(\frac{u_1}{(0.6,0.6,0.7)} \ ,0.4 \right), \left(\frac{u_2}{(0.5,0.3,0.5)} \ ,0.3 \right), \left(\frac{u_3}{(0.8,0.6,0.7)} \ ,0.3 \right) \right] \ ,0.3 \right\} \\ \sim\mathrm{P}_{[\alpha,\beta]}(e_3) = \left\{ \left[\left(\frac{u_1}{(0.7,0.4,0.5)} \ ,0.8 \right), \left(\frac{u_2}{(0.8,0.6,0.6)} \ ,0.5 \right), \left(\frac{u_3}{(0.7,0.5,0.6)} \ ,0.4 \right) \right] \ ,0.4 \right\} \end{cases}$$

PROPOSITION 2.16

Let $\mathrm{P}_{[\alpha,\beta]} \epsilon$ PWN (U, E). Then

1. $\sim\phi_{[\alpha,\beta]} = U_{[\alpha,\beta]}$

2. $\sim U_{[\alpha,\beta]} = \phi_{[\alpha,\beta]}$
3. $\sim(\sim P_{[\alpha,\beta]} = P_{[\alpha,\beta]}$.

Proof. It is clear from Definition 2.14.

PROPOSITION 2.17

Let $P_{[\alpha,\beta]}$, $Q_{[\gamma,\delta]} \in$ PW N(U,E). Then De Morgans law is valid.

1. $\sim(P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}) = \sim P_{[\alpha,\beta]} \cap \sim Q_{[\gamma,\delta]}$.
2. $\sim(P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}) = \sim P_{[\alpha,\beta]} \cup \sim Q_{[\gamma,\delta]}$.

Proof:

1. Let i, j ϵ A, $\sim\left(P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}\right)$

$$= \sim\left\{\left(e_k,\left\{\left(\left(\frac{u_j}{\left(P^T_{kj}(e_k) \oplus Q^T_{kj}(e_k),\ P^I_{kj}(e_k) \oplus Q^I_{kj}(e_k),\ P^F_{kj}(e_k) \otimes Q^F_{kj}(e_k)\right)}, \alpha_{kj}(e_k) \oplus \gamma_{kj}(e_k)\right), \beta_{kj}(e_k) \oplus \delta_{kj}(e_k)\right\}\right)\right) : u_j \in U, e_k \in E\right\}$$

$$= \left\{\left(e_k,\left\{\left(\left(\frac{u_j}{\left(P^F_{kj}(e_k) \otimes Q^F_{kj}(e_k),\ \sim\left(P^I_{kj}(e_k) \oplus Q^I_{kj}(e_k)\right),\ P^T_{kj}(e_k) \oplus Q^T_{kj}(e_k)\right)}, \sim(\alpha_{kj}(e_k) \oplus \gamma_{kj}(e_k))\right), \sim\left((\beta_{kj}(e_k) \oplus \delta_{kj}(e_k)\right)\right\}\right)\right) : u_j \in U, e_k \in E\right\}$$

$$= \left\{\left(e_k,\left\{\left(\left(\frac{u_j}{\left(P^F_{kj}(e_k) \otimes Q^F_{kj}(e_k),\ \sim\left(P^I_{kj}(e_k)\right) \oplus \sim\left(Q^I_{kj}(e_k)\right), P^T_{kj}(e_k) \oplus Q^T_{kj}(e_k)\right)}, \sim(\alpha_{kj}(e_k)) \otimes \sim\left(\gamma_{kj}(e_k)\right)\right), \sim\left((\beta_{kj}(e_k)\right) \otimes \sim\left(\delta_{kj}(e_k)\right)\right\}\right)\right) : u_j \in U, e_k \in E\right\}$$

$$= \left\{\left(e_k, \left\{\left(\frac{u_j}{\left(P_{kj}^F(e_k),\ \sim\left(P_{kj}^I(e_k)\right),\ P_{kj}^T(e_k)\right)}{}'\ \sim(\alpha_{kj}(e_k))\right), \sim\left(\beta_{kj}(e_k)\right)\right\}\right): u_j \in \mathrm{U}, e_k \in \mathrm{E}\ \right\}$$

$$= \left\{\left(e_k, \left\{\left(\frac{u_j}{\left(P_{kj}^T(e_k), \left(P_{kj}^I(e_k)\right), P_{kj}^F(e_k)\right)}{}'\ (\alpha_{kj}(e_k))\right), \left(\beta_{kj}(e_k)\right)\right\}\right): u_j \in \mathrm{U}, e_k \in \mathrm{E}\right\}^c$$

$$\cap \left\{\left(e_k, \left\{\left(\frac{u_j}{Q_{kj}^T(e_k),\ Q_{kj}^I(e_k),\ Q_{kj}^F(e_k)}{}'\ \gamma_{kj}(e_k)\right), \left(\delta_{kj}(e_k)\right)\right\}\right): u_j \in \mathrm{U}, e_k \in \mathrm{E}\right\}^c$$

$$= \sim P_{[\alpha,\beta]}^c \cap \sim Q_{[\gamma,\delta]}^c$$

2. The techniques used to prove (2) are similar to those used for (1), therefore we skip this proof.

DEFINITION 2.18

Let $\mathrm{P}_{[\alpha,\beta]}$ and $\mathrm{Q}_{[\gamma,\delta]} \in$ PW N(U, E). Then ‘AND’ product of PWNS set $\mathrm{P}_{[\alpha,\beta]}$ and $\mathrm{Q}_{[\gamma,\delta]}$ denoted by $\mathrm{P}_{[\alpha,\beta]} \wedge \mathrm{Q}_{[\gamma,\delta]}$ id defined as follows:

$$\mathrm{P}_{[\alpha,\beta]} \wedge \mathrm{Q}_{[\gamma,\delta\ ]} =$$

$$\left\{\left((e_k, e_1), P_{kj}^T(e_k) \wedge g_{lj}^T(e_1), P_{kj}^I(e_k) \wedge g_{lj}^I(e_1), P_{kj}^F(e_k) \vee g_{lj}^F(e_1)), \alpha_{kj}(e_k) \wedge \gamma_{lj}(e_1)), \beta_{kj}(e_k) \wedge \delta_{lj}(e_1)\right) : (e_k, e_1) \in \mathrm{E} \times \mathrm{E}, \mathrm{j}, \mathrm{k}, \mathrm{l} \in \wedge\right\}$$

DEFINITION 2.19

Let $\mathrm{P}_{[\alpha,\beta]}$ and $\mathrm{Q}_{[\gamma,\delta]} \in$ PWN (U, E). Then ‘OR’ product of PWNS set $\mathrm{P}_{[\alpha,\beta]}$ and $\mathrm{Q}_{[\gamma,\delta]}$ denoted by $\mathrm{P}_{[\alpha,\beta]} \vee \mathrm{Q}_{[\gamma,\delta]}$ id defined as follows:

$$\mathrm{P}_{[\alpha,\beta]} \wedge \mathrm{Q}_{[\gamma,\delta]} =$$

$$\left\{\left((e_k, e_1), P_{kj}^T(e_k) \vee Q_{lj}^T(e_1), P_{kj}^I(e_k) \vee Q_{lj}^I(e_1), P_{kj}^F(e_k) \wedge Q_{lj}^F(e_1)), \alpha_{kj}(e_k) \vee \gamma_{lj}(e_1)), \beta_{kj}(e_k) \vee \delta_{lj}(e_1)\right) : (e_k, e_1) \in \mathrm{E} \times \mathrm{E}, \mathrm{j}, \mathrm{k}, \mathrm{l} \in \wedge\right\}$$

APPLICATION OF PWNSS IN DECISION MAKING

DEFINITION 2.20

Let $Q_{[\gamma,\delta]}$,$L_{[\theta,\phi]}\epsilon$ PWN (U, E), $P_{[\alpha,\beta]}$ = $Q_{[\gamma,\delta]} \wedge L_{[\theta,\phi]}$ and $P^{T}_{[\alpha,\beta]}$, $P^{I}_{[\alpha,\beta]}$ and $P^{F}_{[\alpha,\beta]}$ be the truth ,indeterminacy and falsity matrices of ∧ −product matrix, respectively. Then, weighted matrices of $\left(P_{[\alpha,\beta]}\right)^{T}$, $\left(P_{[\alpha,\beta]}\right)^{I}$ and $\left(P_{[\alpha,\beta]}\right)^{F}$denoted $\wedge^{T,}$ $\wedge^{I}$and $\wedge^{F}$ are defined as follows:

- $\wedge^{T}(e_{kj},u_r) = T_{Q_{[\gamma,\delta]} \wedge L_{[\theta,\phi]}(e_{kj})}(u_r)+ (\gamma_{kr}(e_k) \wedge \theta_{jr}(e_j)) - T_{Q_{[\gamma,\delta]} \wedge L_{[\theta,\phi]}(e_{kj})}(u_r)\times(\alpha_{kr}(e_k) \wedge (\gamma_{jr}(e_j)),\beta(e_k)$
- $\wedge^{I}(e_{kj}, u_r) =I_{Q_{[\gamma,\delta]} \wedge L_{[\theta,\phi]}(e_{kj})}(u_r)+ (\gamma_{kr}(e_k) \wedge \theta_{jr}(e_j)) - I_{Q_{[\gamma,\delta]} \wedge L_{[\theta,\phi]}(e_{kj})}(u_r) \times (\alpha_{kr}(e_k) \wedge (\gamma_{jr}(e_j)),\beta(e_k)$
- $\wedge^{F}(e_{kj}, u_r) =P_{Q_{[\gamma,\delta]} \wedge L_{[\theta,\phi]}(e_{kj})}(u_r)\times(\gamma_{kr}(e_k) \wedge \theta_{jr}(e_j)),\beta(e_k)$

DEFINITION 2.21

Let $Q_{[\gamma,\delta]}$, $L_{[\theta,\phi]}\epsilon$ PWN(U, E), $P_{[\alpha,\beta]}$ =$Q_{[\gamma,\delta]} \wedge L_{[\theta,\phi]}$ and let $\wedge^{T,}$ $\wedge^{I}$and $\wedge^{F}$ be the weighted matrices of $P_{[\alpha,\beta]})^{T}$, $P_{[\alpha,\beta]})^{I}$ and $P_{[\alpha,\beta]})^{F}$ respectively. Then, in the weighted matrices $\wedge^{T,}$ $\wedge^{I}$and $\wedge^{F}$scores of $u_n \epsilon$ U denoted by $s^{T}(u_n)$, $s^{I}(u_n)$ and $s^{F}(u_n)$ are defined as follows:

$$s^{T}(u_n) = \sum_{k,j\,\epsilon\wedge}(\pi^{T}_{kj}(u_n) \times \beta(e_k))$$

$$s^{I}(u_n) = \sum_{k,j\,\epsilon\wedge}(\pi^{I}_{kj}(u_n) \times \beta(e_k))$$

$$s^{F}(u_n)= \sum_{k,j\,\epsilon\wedge}(\pi^{F}_{kj}(u_n) \times \beta(e_k))$$

Where $$\pi^{T}_{kj}(u_m) = \begin{cases} \wedge^{T}(e_{kJ},u_n), & \wedge^{T}(e_{kJ},u_n) = \max\{ \wedge^{T}(e_{kJ},u_m) : u_m \epsilon \text{ U} \} \\ 0, & otherwise \end{cases}$$

$$\pi^{I}_{kj}(u_m) = \begin{cases} \wedge^{I}(e_{kJ},u_n), & \wedge^{I}(e_{kJ},u_n) = \max\{ \wedge^{I}(e_{kJ},u_m) : u_m \epsilon \text{ U} \} \\ 0, & otherwise \end{cases}$$

$$\pi^{F}_{kj}(u_m) = \begin{cases} \vee^{\boldsymbol{F}}(e_{kJ},u_n), & \vee^{\boldsymbol{F}}(e_{kJ},u_n) = \max\{ \vee^{\boldsymbol{F}}(e_{kJ},u_m) : u_m \epsilon \text{ U} \} \\ 0, & otherwise \end{cases}$$

DEFINITION 2.22

Let $s^T(u_n)$, $s^I(u_n)$ and $s^F(u_n)$be scores of $u_n \epsilon$ U in the weighted matrices $\wedge^{T,}$ $\wedge^{I}$ and $\wedge^{F}$.Then, decision score of $u_n \epsilon$ U, denoted by ds(u_n), is defined by ds(u_n) = $s^T(u_n)$ + $s^I(u_n)$ - $s^F(u_n)$. Now, we construct a PWNS –decision making method by the following algorithm:

ALGORITHM

Step 1: Input the PWNSs,

Step 2: Construct the ∧-product matrix,

Step3: Construct the truth, indeterminacy and falsity matrices of the ∧-product matrix,

Step4: Construct the weighted matrices $\wedge^T$, $\wedge^I$ and$\wedge^F$.

Step5: Compute score of $u_t \in U$,for each of the weighted matrices,

Step6: Compute decision score, for all $u_t \in U$,

Step7: The optimal decision is to select $u_t = maxds(u_i)$

Example

Assume that U = $\{C_{1,}C_{2,}C_3\}$ is a set of three colleges and E = $\{e_{1,}e_{2,}e_3\}$ = {Academics, Sports, Fine Arts} is a set of parameters which is best college among the three, α-represents priority given to the parameters by the college and β-represents the weightage given to the parameters by the experts.

Suppose that a committee wants to award a particular college.

Step 1: Based on the choice parameters, PWNSs $Q_{[\gamma,\delta]}$ and $L_{[\theta,\phi]}$, constructed by two experts are as follows:

$$Q_{[\gamma,\delta]} = \begin{Bmatrix} Q_{[\gamma,\delta]}(e_1) = \left\{\left(\frac{u_1}{(\mathbf{0.6,0.5,0.6})},0.7\right),\left(\frac{u_2}{(\mathbf{0.7,0.3,0.5})},0.3\right),\left(\frac{u_3}{(\mathbf{0.8,0.7,0.6})},0.5\right),0.9\right\} \\ Q_{[\gamma,\delta]}(e_2) = \left\{\left(\frac{u_1}{(\mathbf{0.4,0.3,0.6})},0.5\right),\left(\frac{u_2}{(\mathbf{0.8,0.9,0.4})},0.6\right),\left(\frac{u_3}{(\mathbf{0.3,0.5,0.5})},0.7\right),0.8\right\} \\ Q_{[\gamma,\delta]}(e_3) = \left\{\left(\frac{u_1}{(\mathbf{0.8,0.7,0.6})},0.5\right),\left(\frac{u_2}{(\mathbf{0.5,0.6,0.3})},0.4\right),\left(\frac{u_3}{(\mathbf{0.6,0.4,0.6})},0.3\right),0.7\right\} \end{Bmatrix}$$

also we can define a function $Q_{[\gamma\,,\delta]}$: E ⟶ N(U) × F(U) as follows:

$$L_{[\theta,\phi]} = \left\{ \begin{array}{l} L_{[\theta,\phi]}(e_1) = \left\{ \left(\frac{u_1}{\mathbf{(0.4,0.5,0.6)}}\ ,0.7 \right), \left(\frac{u_2}{\mathbf{(0.8,0.4,0.5)}}\ ,0.3 \right), \left(\frac{u_3}{\mathbf{(0.5,0.6,0.3)}}\ ,0.5 \right)\ ,0.7 \right\} \\ L_{[\theta,\phi]}(e_2) = \left\{ \left(\frac{u_1}{\mathbf{(0.5,0.7,0.6)}}\ ,0.5 \right), \left(\frac{u_2}{\mathbf{(0.3,0.6,0.4)}}\ ,0.8 \right), \left(\frac{u_3}{\mathbf{(0.5,0.7,0.3)}}\ ,0.9 \right)\ ,0.8 \right\} \\ L_{[\theta,\phi]}(e_3) = \left\{ \left(\frac{u_1}{\mathbf{(0.3,0.2,0.7)}}\ ,0.8 \right), \left(\frac{u_2}{\mathbf{(0.9,0.5,0.6)}}\ ,0.4 \right), \left(\frac{u_3}{\mathbf{(0.7,0.5,0.4)}}\ ,0.5 \right)\ ,0.6 \right\} \end{array} \right\}$$

Step 2: Let us consider PWNS ∧-product $P_{[\alpha,\beta]} = Q_{[\gamma\,,\delta]} \wedge L_{[\theta,\phi]}$ which is the mapping ∧ **:** E × E ⟶ N(U) × F(U) × F(U) given as follows:

$$\left(\begin{array}{c|cccc} \wedge & u_1\,,\alpha & u_2\,,\alpha & u_3\,,\alpha & \beta \\ \hline e_{11} & (\langle 0.4,0.5,0.6\rangle, 0.7) & (\langle 0.7,0.3,0.5\rangle, 0.3) & (\langle 0.5,0.6,0.6\rangle, 0.5) & (0.7) \\ e_{12} & (\langle 0.5,0.5,0.6\rangle, 0.5) & (\langle 0.3,0.3,0.5\rangle, 0.3) & (\langle 0.5,0.7,0.6\rangle, 0.5) & (0.8) \\ e_{13} & (\langle 0.3,0.2,0.7\rangle, 0.6) & (\langle 0.7,0.3,0.6\rangle, 0.2) & (\langle 0.7,0.5,0.6\rangle, 0.6) & (0.6) \\ e_{21} & (\langle 0.4,0.3,0.6\rangle, 0.5) & (\langle 0.8,0.4,0.5\rangle, 0.3) & (\langle 0.3,0.5,0.5\rangle, 0.5) & (0.7) \\ e_{22} & (\langle 0.4,0.3,0.6\rangle, 0.5) & (\langle 0.3,0.6,0.4\rangle, 0.6) & (\langle 0.3,0.5,0.5\rangle, 0.7) & (0.8) \\ e_{23} & (\langle 0.3,0.2,0.7\rangle, 0.5) & (\langle 0.8,0.5,0.6\rangle, 0.5) & (\langle 0.3,0.5,0.5\rangle, 0.5) & (0.6) \\ e_{31} & (\langle 0.4,0.3,0.6\rangle, 0.5) & (\langle 0.5,0.4,0.5\rangle, 0.3) & (\langle 0.5,0.4,0.6\rangle, 0.3) & (0.7) \\ e_{32} & (\langle 0.5,0.3,0.6\rangle, 0.5) & (\langle 0.3,0.6,0.4\rangle, 0.4) & (\langle 0.5,0.4,0.6\rangle, 0.3) & (0.7) \\ e_{33} & (\langle 0.3,0.2,0.6\rangle, 0.5) & (\langle 0.5,0.5,0.6\rangle, 0.4) & (\langle 0.6,0.4,0.6\rangle, 0.3) & (0.6) \end{array} \right)$$

Step 3: We construct matrices $P^{T}_{[\alpha,\beta]}$, $P^{I}_{[\alpha,\beta]}$ and $P^{F}_{[\alpha,\beta]}$ as follows:

$$\left(\begin{array}{c|cccc} \wedge & u_1\,,\alpha & u_2\,,\alpha & u_3\,,\alpha & \beta \\ \hline e_{11} & (0.4,0.7) & (0.7,0.3) & (0.5,0.5) & (0.7) \\ e_{12} & (0.5,0.5) & (0.3,0.3) & (0.5,0.5) & (0.8) \\ e_{13} & (0.3,0.7) & (0.7,0.3) & (0.7,0.6) & (0.6) \\ e_{21} & (0.4,0.5) & (0.8,0.3) & (0.3,0.5) & (0.7) \\ e_{22} & (0.4,0.5) & (0.3,0.6) & (0.3,0.7) & (0.8) \\ e_{23} & (0.3,0.5) & (0.8,0.5) & (0.3,0.5) & (0.6) \\ e_{31} & (0.4,0.5) & (0.5,0.3) & (0.5,0.3) & (0.7) \\ e_{32} & (0.5,0.5) & (0.4,0.4) & (0.5,0.3) & (0.7) \\ e_{33} & (0.3,0.5) & (0.5,0.4) & (0.6,0.3) & (0.6) \end{array} \right)$$

$$\left(\begin{array}{c|cccc}
\wedge & u_1,\alpha & u_2,\alpha & u_3,\alpha & \beta \\
\hline
e_{11} & (0.4,0.7) & (0.3,0.3) & (0.6,0.5) & (0.7) \\
e_{12} & (0.4,0.5) & (0.3,0.3) & (0.7,0.5) & (0.8) \\
e_{13} & (0.2,0.7) & (0.3,0.3) & (0.5,0.6) & (0.6) \\
e_{21} & (0.3,0.5) & (0.4,0.3) & (0.5,0.5) & (0.7) \\
e_{22} & (0.3,0.5) & (0.6,0.6) & (0.5,0.7) & (0.8) \\
e_{23} & (0.2,0.5) & (0.5,0.5) & (0.5,0.5) & (0.6) \\
e_{31} & (0.3,0.5) & (0.4,0.3) & (0.4,0.3) & (0.7) \\
e_{32} & (0.3,0.5) & (0.6,0.4) & (0.4,0.3) & (0.7) \\
e_{33} & (0.2,0.5) & (0.5,0.4) & (0.4,0.3) & (0.6)
\end{array}\right)$$

$$\left(\begin{array}{c|cccc}
\wedge & u_1,\alpha & u_2,\alpha & u_3,\alpha & \beta \\
\hline
e_{11} & (0.6,0.7) & (0.5,0.3) & (0.6,0.5) & (0.7) \\
e_{12} & (0.6,0.5) & (0.5,0.3) & (0.6,0.5) & (0.8) \\
e_{13} & (0.7,0.7) & (0.6,0.3) & (0.6,0.6) & (0.6) \\
e_{21} & (0.6,0.5) & (0.5,0.3) & (0.5,0.5) & (0.7) \\
e_{22} & (0.6,0.5) & (0.4,0.6) & (0.5,0.7) & (0.8) \\
e_{23} & (0.7,0.5) & (0.6,0.5) & (0.5,0.5) & (0.6) \\
e_{31} & (0.6,0.5) & (0.5,0.3) & (0.6,0.3) & (0.7) \\
e_{32} & (0.6,0.5) & (0.4,0.4) & (0.6,0.3) & (0.7) \\
e_{33} & (0.7,0.5) & (0.6,0.4) & (0.6,0.3) & (0.6)
\end{array}\right)$$

Matrix $P^F_{[\alpha,\beta]}$ of $\wedge$ -product

Step 4: We obtain weighted matrices $\wedge^T$, $\wedge^I$ and $\wedge^F$ by using Definition 2.21 as follws:

$$\left(\begin{array}{c|cccc}
\wedge^T & u_1,\alpha & u_2,\alpha & u_3,\alpha & \beta \\
\hline
e_{11} & \underline{0.82} & 0.79 & 0.75 & 0.7 \\
e_{12} & \underline{0.75} & 0.51 & 0.70 & 0.8 \\
e_{13} & 0.79 & 0.79 & \underline{0.88} & 0.6 \\
e_{21} & 0.70 & \underline{0.86} & 0.65 & 0.7 \\
e_{22} & 0.70 & 0.72 & \underline{0.79} & 0.8 \\
e_{23} & 0.65 & \underline{0.90} & 0.65 & 0.6 \\
e_{31} & \underline{0.70} & 0.65 & 0.65 & 0.7 \\
e_{32} & \underline{0.75} & 0.58 & 0.65 & 0.7 \\
e_{33} & 0.65 & 0.7 & \underline{0.72} & 0.6
\end{array}\right),$$

$$\left(\begin{array}{c|cccc}
\wedge^{I} & u_1,\alpha & u_2,\alpha & u_3,\alpha & \beta \\
\hline
e_{11} & \underline{0.85} & 0.51 & 0.80 & 0.7 \\
e_{12} & 0.70 & 0.51 & \underline{0.85} & 0.8 \\
e_{13} & 0.76 & 0.51 & \underline{0.80} & 0.6 \\
e_{21} & 0.65 & 0.58 & \underline{0.75} & 0.7 \\
e_{22} & 0.65 & 0.84 & \underline{0.85} & 0.8 \\
e_{23} & 0.60 & 0.75 & \underline{0.75} & 0.6 \\
e_{31} & \underline{0.65} & 0.58 & 058 & 0.7 \\
e_{32} & 0.65 & \underline{0.76} & 0.58 & 0.7 \\
e_{33} & 0.60 & \underline{0.70} & 0.58 & 0.6
\end{array}\right),$$

$$\left(\begin{array}{c|cccc}
\wedge^{F} & u_1,\alpha & u_2,\alpha & u_3,\alpha & \beta \\
\hline
e_{11} & \underline{0.42} & 0.15 & 0.15 & 0.7 \\
e_{12} & \underline{0.15} & 0.12 & \underline{0.15} & 0.8 \\
e_{13} & \underline{0.49} & 0.18 & 0.24 & 0.6 \\
e_{21} & 0.30 & 0.12 & \underline{0.15} & 0.7 \\
e_{22} & 0.15 & \underline{0.24} & 0.21 & 0.8 \\
e_{23} & 0.35 & 0.20 & 0.20 & 0.6 \\
e_{31} & 0.30 & 0.09 & 0.09 & 0.7 \\
e_{32} & \underline{0.15} & 0.12 & 0.09 & 0.7 \\
e_{33} & \underline{0.30} & 0.12 & 0.12 & 0.6
\end{array}\right)$$

Matrix $P^{T}_{[\alpha,\beta]}$, $P^{I}_{[\alpha,\beta]}$ and $P^{F}_{[\alpha,\beta]}$ from left to right, respectively

Step 5: For all $u \in U$, we find a scores by using Definition 2.22 as follows:

$s^{T}(u_1)$ = 2.19, $s^{T}(u_2)$ = 1.045, $s^{T}(u_3)$ = 0.7

$s^{I}(u_1)$ = 2.10, $s^{I}(u_2)$ = 0.952, $s^{I}(u_3)$ = 0.26

$s^{F}(u_1)$ = 1.59, $s^{F}(u_3)$ = 2.815, $s^{F}(u_3)$ = 0.29

Step 6: For all $u \in U$, we find a scores by using Definition 2.22 as follows:

$ds(u_1)$ = 2.19 + 1.045 – 0.7 = 2.535

$ds(u_2)$ = 2.19 + 0.952 – 0.26 = 2.792

$ds(u_3)$ = 1.59 + 2.815 – 0.29 =3.815

Step 7: Then the optimal selection of the committee is the college

CHAPTER-III

A PRIORITIZED NEUTROSOPHIC SOFT MADM BASED ON TOPSIS METHOD

Let us consider U = $\{C_1, C_2, \cdots, C_n\}$, $(n \geq 2)$ a set of alternatives in a MADM problems. Let E = $\{e_1, e_2, \cdots, e_n\}$, $(n \geq 2)$ be the set of parameters involved in the problem. The technique based on TOPSIS method for solving priority weighted neutrosophic soft MCDM problem is defined as follows:

Step 1: Input the priority weighted neutrosophic soft set (PWNS-set) $P_{[\alpha,\beta]}$ over U is a set of triple defined by

$$P_{[\alpha,\beta]} = \left\{\left(e_k, \left\{\left(\frac{u_j}{P(e_k)(u_j)}, \alpha(e_k)(u_j)\right)\right\}, \beta(e_k)\right) : e_k \in E\right\}$$

or a mapping defined by $P_{[\alpha,\beta]} : E \rightarrow$ N(U) × F(U) × F(U), where, i ∈ $\wedge_1$ and k ∈ $\wedge_2$, P is defined by P : E → N(U) and α (e_k), $\beta(e_k)$ are such as α : E → F(U) and β : E → F(U), where F(U) is a fuzzy set.

For each parameter

$$e_k \in \text{E, P}(e_k) = \{\langle u_j, T_{P(e_k)}(u_j), I_{P(e_k)}(u_j), F_{P(e_k)}(u_j)\rangle : u_j \in U\}$$

Represents the neutrosophic estimate of the parameter e_k and T, I, F indicates the truth value, indeterminacy value and falsity values of the element u_i ∈ U. For each u_i ∈ U and e_k ∈ E, $0 \leq T_{P(e_k)}(u_j) + I_{P(e_k)}(u_j) + F_{P(e_k)}(u_j) \leq 3$. Also α (e_k), degrees of priority given for the association of the alternatives of U in P(e_k) and $\beta(e_k)$ the weightage given to the parameters by the experts . So we can write

$P_{[\alpha,\beta]}$ (e_k) =

$$\left\{\left(\frac{u_1}{P(e_k)(u_1)}, \alpha(e_k)(u_1)\right)\left(\frac{u_2}{P(e_k)(u_2)}, \alpha(e_k)(u_2)\right), \ldots, \left(\frac{u_n}{P(e_k)(u_n)}, \alpha(e_k)(u_n)\right), \beta(e_k)\right\}$$

Step 2: Compute the ∧-product matrix

Let $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]} \epsilon$ PWN (U, E). Then the product ('AND') of PWNS set $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]}$, $P_{[\alpha,\beta]} \wedge Q_{[\gamma,\delta]}$ is obtained as follows:

$$P_{[\alpha,\beta]} \wedge Q_{[\gamma,\delta\ ]} = D_G =$$
$$\left\{\left((e_k, e_1), P^T_{kj}(e_k) \wedge g^T_{lj}(e_1), P^I_{kj}(e_k) \wedge g^I_{lj}(e_1), P^F_{kj}(e_k) \vee g^F_{lj}(e_1)), \alpha_{kj}(e_k) \wedge \gamma_{lj}(e_1)), \beta_{kj}(e_k) \wedge \delta_{lj}(e_1)\right) : (e_k, e_1) \epsilon \mathrm{E} \times \mathrm{E}, \mathrm{j}, \mathrm{k}, \mathrm{l} \in \wedge \right\}$$

Step 3: Determination of weights of the choice parameters.

The specialist selects the parameters in the DM situation. To be specific, choice parameters are such that their weights dissimilar and unknown to the estimator. In this work, we utilize the information entropy technique to achieve the weights of the choice of parameters. The entropy measure H_j with respect to j-th attribute is determined as:

$$H_j = 1 - \frac{1}{r}\left\{\sum_{i=1}^{n}\left[\left(T_{ij}(x_i) + F_{ij}(x_i)\right)\left|I_{ij}(x_i) - I^c_{ij}(x_i)\right|\right]\alpha_{ij}(x_i)\right\}\beta_j, \text{ j = 1, 2, } \quad \text{, r}$$

Here, 0 $\quad H_j \quad$ 1 and the entropy weight of the j-th attribute is obtained
from the Eq. as given below.

$$w_j = \frac{1-H_j}{\sum_{j=1}^{r}(1-H_j)} \text{ , with 0 } \quad w_j \quad \text{ 1 and } \sum_{j=1}^{r} w_j = 1.$$

Step 4: Description of weighted decision matrix

Aggregated weighted decision matrix is calculated by multiplying the weights (w_j) of the parameters and $\langle d_{ij}\rangle_{p\times r}$ the aggregated decision matrix.

$$D_G^w = D_G \otimes w = \langle d_{ij}\rangle_{n\times r} \otimes w_i \ \ \langle d_{ij}^{w_i}\rangle_{n\times r} = \begin{bmatrix} d_{11}^{w_1} & d_{12}^{w_1} & \ldots d_{1r}^{w_1} \\ d_{21}^{w_2} & d_{22}^{w_2} & \cdots d_{2r}^{w_2} \\ \cdot & \cdot & \ldots \ \cdot \\ \cdot & \cdot & \ldots \ \cdot \\ d_{n1}^{w_n} & d_{n2}^{w_n} & \cdots d_{nr}^{w_n} \end{bmatrix}$$

Here, $d_{ij}^{w_j} = \langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j}\rangle$ where $T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j}$ [0, 1] and 0 $T_{ij}^{w_j}$ $I_{ij}^{w_j}$ $F_{ij}^{w_j}$ 3, i = 1, 2, ,n; j = 1, 2, , r.

Step 5: Computation of relative positive ideal and relative negative ideal solution RPID and RNIS

In real physical DMs, the attributes are categorized into two types: benefit type (J_1) and cost type (J_2) attributes. Assume R_G^{w+} as the relative positive ideal solution and R_G^{w-} as relative negative ideal solution.

Next, R_G^{w+} and R_G^{w-} can be described by:

$$R_G^{w+} = \left(\langle T_1^{w_1+}, I_1^{w_1+}, F_1^{w_1+}\rangle, \langle T_2^{w_2+}, I_2^{w_2+}, F_2^{w_2+}\rangle, \cdots, \langle T_r^{w_r+}, I_r^{w_r+}, F_r^{w_r+}\rangle\right)$$

$$R_G^{w-} = \left(\langle T_1^{w_1-}, I_1^{w_1-}, F_1^{w_1-}\rangle, \langle T_2^{w_2-}, I_2^{w_2-}, F_2^{w_2-}\rangle, \cdots, \langle T_r^{w_r-}, I_r^{w_r-}, F_r^{w_r-}\rangle\right)$$

Where

$$\langle T_j^{w_j+}, I_j^{w_j+}, F_j^{w_j+}\rangle = \left[\left\{\underset{i}{Max}\left(T_{ij}^{w_j}\right) \mid j \in J_1\right\}; \left\{\underset{i}{Min}\left(T_{ij}^{w_j}\right) \mid j \in J_2\right\}\right], \left[\left\{\underset{i}{Min}\left(I_{ij}^{w_j}\right) \mid j \in J_1\right\}; \left\{\underset{i}{Max}\left(I_{ij}^{w_j}\right) \mid j \in J_2\right\}\right], \left[\left\{\underset{i}{Min}\left(F_{ij}^{w_j}\right) \mid j \in J_1\right\}; \left\{\underset{i}{Max}\left(F_{ij}^{w_j}\right) \mid j \in J_1\right\}\right],$$

j = 1, 2, ⋯ , r .

$$\langle T_j^{w_j-}, I_j^{w_j-}, F_j^{w_j-}\rangle = \left[\left\{\underset{i}{Min}\left(T_{ij}^{w_j}\right) \mid j \in J_1\right\}; \left\{\underset{i}{Max}\left(T_{ij}^{w_j}\right) \mid j \in J_2\right\}\right], \left[\left\{\underset{i}{Max}\left(I_{ij}^{w_j}\right) \mid j \in J_1\right\}; \left\{\underset{i}{Min}\left(I_{ij}^{w_j}\right) \mid j \in J_2\right\}\right], \left[\left\{\underset{i}{Max}\left(F_{ij}^{w_j}\right) \mid j \in J_1\right\}; \left\{\underset{i}{Min}\left(F_{ij}^{w_j}\right) \mid j \in J_1\right\}\right], \ j = 1, 2, \cdots, r.$$

Step 6: Computation of distance measure from RPIS and RNIS

The normalized Euclidean distance of the alternative $\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j}\rangle$ from the RPIS $\langle T_j^{w_j+}, I_j^{w_j+}, F_j^{w_j+}\rangle$ for i = 1, 2, ⋯ , n; j = 1, 2, ⋯ , r is given by:

$D_{Euc}^{i+}\left(d_{ij}^{w_j}, d_j^{w_j+}\right) =$

$$\sqrt{\frac{1}{3r}\sum_{j=1}^{r}\left\{\left(T_{ij}^{w_j}(x_j) - T_j^{w_j+}(x_j)\right)^2 + \left(I_{ij}^{w_j}(x_j) - I_j^{w_j+}(x_j)\right)^2 + \left(F_{ij}^{w_j}(x_j) - F_j^{w_j+}(x_j)\right)^2\right\}}$$

Similarly, normalized Euclidean distance of each alternative $\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j}\rangle$ from the RNIS $\langle T_j^{w_j-}, I_j^{w_j-}, F_j^{w_j-}\rangle$ for i = 1, 2, ⋯ , n; j = 1, 2, ⋯ , r can be written as follows:

$D_{Euc}^{i-}\left(d_{ij}^{w_j}, d_j^{w_j-}\right) =$

$$\sqrt{\frac{1}{3r}\sum_{j=1}^{r}\left\{\left(T_{ij}^{w_j}(x_j) - T_j^{w_j-}(x_j)\right)^2 + \left(I_{ij}^{w_j}(x_j) - I_j^{w_j-}(x_j)\right)^2 + \left(F_{ij}^{w_j}(x_j) - F_j^{w_j-}(x_j)\right)^2\right\}}$$

Step 7: Finding out the relative closeness co-efficient with respect to the ideal solution

The relative closeness co-efficient of RPIS is taken as:

$$\rho_i^* = \frac{D_{Euc}^{i-}\left(d_{ij}^{w_j}, d_j^{w_j-}\right)}{D_{Euc}^{i+}\left(d_{ij}^{w_j}, d_j^{w_j+}\right) + D_{Euc}^{i-}\left(d_{ij}^{w_j}, d_j^{w_j-}\right)}$$

Where, $0 \leq \rho_i^* \leq 1$.

Step 8: Ranking the alternatives

Ranking the alternatives is done according to the values of ρ_i^*, i = 1, 2, ⋯ , n; j = 1, 2, ⋯ , p and bigger value of ρ_i^*, j = 1, 2, ⋯ , p depicts the better alternative.

PROPOSED TOPSIS ALGORITHM FOR MADM PROBLEMS

TOPSIS algorithm for prioritized neutrosophic soft MAGDM problems

Step 1: Get the value of PWNSs, $Q_{[\gamma,\delta]}$ and $L_{[\theta,\phi]}$.

Step 2: Compute the $\wedge$-product matrix, D_G.

Step 3: Establish the weight (w_j) of the parameters.

Step 4: Create the weighted aggregated decision matrix $D_G^w = \langle d_{ij}^{w_j} \rangle_{n\times r}$

Step 5: Recognize the relative positive ideal solution (R_G^{w+}) and relative negative ideal solution (R_G^{w-}).

Step 6: Determine the normalized Euclidean distance from relative positive ideal solution (R_G^{w+}) and relative negative ideal solution of the alternatives.

Step 7: Find the relative closeness co-efficient ρ_i^*.

Step 8: Based on the relative closeness rank the preference order of alternatives.

A NUMERICAL EXAMPLE

Assume that U = $\{C_1, C_2, C_3\}$ is a set of three colleges and E = $\{e_1, e_2, e_3\}$ = {Academics, Sports, Fine Arts} is a set of parameters which is best college among the three, α-represents priority given to the parameters by the college and β-represents the weightage given to the parameters by the experts.

Suppose that a committee wants to award a particular college.

Step 1: The PWNSs $Q_{[\gamma,\delta]}$ and $L_{[\theta,\phi]}$, constructed by the experts are:

$$Q_{[\gamma,\delta]} = \begin{Bmatrix} Q_{[\gamma,\delta]}(e_1) = \left\{\left(\frac{u_1}{(\mathbf{0.6,0.5,0.6})}, 0.7\right), \left(\frac{u_2}{(\mathbf{0.7,0.3,0.5})}, 0.3\right), \left(\frac{u_3}{(\mathbf{0.8,0.7,0.6})}, 0.5\right), 0.9\right\} \\ Q_{[\gamma,\delta]}(e_2) = \left\{\left(\frac{u_1}{(\mathbf{0.4,0.3,0.6})}, 0.5\right), \left(\frac{u_2}{(\mathbf{0.8,0.9,0.4})}, 0.6\right), \left(\frac{u_3}{(\mathbf{0.3,0.5,0.5})}, 0.7\right), 0.8\right\} \\ Q_{[\gamma,\delta]}(e_3) = \left\{\left(\frac{u_1}{(\mathbf{0.8,0.7,0.6})}, 0.5\right), \left(\frac{u_2}{(\mathbf{0.5,0.6,0.3})}, 0.4\right), \left(\frac{u_3}{(\mathbf{0.6,0.4,0.6})}, 0.3\right), 0.7\right\} \end{Bmatrix}$$

$$L_{[\theta,\phi]} = \left\{ \begin{array}{l} L_{[\theta,\phi]}(e_1) = \left\{ \left(\frac{u_1}{(\mathbf{0.4},\mathbf{0.5},\mathbf{0.6})}, 0.7 \right), \left(\frac{u_2}{(\mathbf{0.8},\mathbf{0.4},\mathbf{0.5})}, 0.3 \right), \left(\frac{u_3}{(\mathbf{0.5},\mathbf{0.6},\mathbf{0.3})}, 0.5 \right), 0.7 \right\} \\ L_{[\theta,\phi]}(e_2) = \left\{ \left(\frac{u_1}{(\mathbf{0.5},\mathbf{0.7},\mathbf{0.6})}, 0.5 \right), \left(\frac{u_2}{(\mathbf{0.3},\mathbf{0.6},\mathbf{0.4})}, 0.8 \right), \left(\frac{u_3}{(\mathbf{0.5},\mathbf{0.7},\mathbf{0.3})}, 0.9 \right), 0.8 \right\} \\ L_{[\theta,\phi]}(e_3) = \left\{ \left(\frac{u_1}{(\mathbf{0.3},\mathbf{0.2},\mathbf{0.7})}, 0.8 \right), \left(\frac{u_2}{(\mathbf{0.9},\mathbf{0.5},\mathbf{0.6})}, 0.4 \right), \left(\frac{u_3}{(\mathbf{0.7},\mathbf{0.5},\mathbf{0.4})}, 0.5 \right), 0.6 \right\} \end{array} \right\}$$

Step 2: Let us consider PWNS ∧-product $P_{[\alpha,\beta]} = Q_{[\gamma,\delta]} \wedge L_{[\theta,\phi]}$ defined by ∧**:** E × E ⟶ N(U) × F(U) × F(U) given by:

$P_{[\alpha,\beta]} \wedge Q_{[\gamma,\delta]} = D_G =$

$$\begin{array}{c|cccc} \wedge & u_1, \alpha & u_2, \alpha & u_3, \alpha & \beta \\ \hline d_1 & (\langle 0.4,0.5,0.6\rangle, 0.7) & (\langle 0.7,0.3,0.5\rangle, 0.3) & (\langle 0.5,0.6,0.6\rangle, 0.5) & (0.7) \\ d_2 & (\langle 0.5,0.5,0.6\rangle, 0.5) & (\langle 0.3,0.3,0.5\rangle, 0.3) & (\langle 0.5,0.7,0.6\rangle, 0.5) & (0.8) \\ d_3 & (\langle 0.3,0.2,0.7\rangle, 0.6) & (\langle 0.7,0.3,0.6\rangle, 0.2) & (\langle 0.7,0.5,0.6\rangle, 0.6) & (0.6) \\ d_4 & (\langle 0.4,0.3,0.6\rangle, 0.5) & (\langle 0.8,0.4,0.5\rangle, 0.3) & (\langle 0.3,0.5,0.5\rangle, 0.5) & (0.7) \\ d_5 & (\langle 0.4,0.3,0.6\rangle, 0.5) & (\langle 0.3,0.6,0.4\rangle, 0.6) & (\langle 0.3,0.5,0.5\rangle, 0.7) & (0.8) \\ d_6 & (\langle 0.3,0.2,0.7\rangle, 0.5) & (\langle 0.8,0.5,0.6\rangle, 0.5) & (\langle 0.3,0.5,0.5\rangle, 0.5) & (0.6) \\ d_7 & (\langle 0.4,0.3,0.6\rangle, 0.5) & (\langle 0.5,0.4,0.5\rangle, 0.3) & (\langle 0.5,0.4,0.6\rangle, 0.3) & (0.7) \\ d_8 & (\langle 0.5,0.3,0.6\rangle, 0.5) & (\langle 0.3,0.6,0.4\rangle, 0.4) & (\langle 0.5,0.4,0.6\rangle, 0.3) & (0.7) \\ d_9 & (\langle 0.3,0.2,0.6\rangle, 0.5) & (\langle 0.5,0.5,0.6\rangle, 0.4) & (\langle 0.6,0.4,0.6\rangle, 0.3) & (0.6) \end{array}$$

Step 3: Calculation of Entropy value of the parametric weights H_j (j = 1, 2, 3, 4,⋯,9) with respect to the j-th choice can be obtained by:

$H_1 = 0.9407$, $H_2 = 0.9157$, $H_3 = 0.9072$, $H_4 = 0.9351$, $H_5 = 0.9242$, $H_6 = 0.94$,
$H_7 = 0.9239$, $H_8 = 0.9202$, $H_9 = 0.9316$.

The normalized entropy weights are determined as :

$w_1 = 0.0896$, $w_2 = 0.1274$, $w_3 = 0.1403$, $w_4 = 0.0981$, $w_5 = 0.1145$, $w_6 = 0.0907$,
$w_7 = 0.1150$, $w_8 = 0.1206$, $w_9 = 0.1034$,
where $\sum_{j-1}^{9} w_j = 1$.

Step 4: Formulating the weighted decision

D_G^w	$\mathbf{u_1}$	$\mathbf{u_2}$	$\mathbf{u_3}$
$\mathbf{d_{11}} \times \mathbf{w_1}$	(0.0358, 0.0448, 0.0537)	(0.0627, 0.0268, 0.0448)	(0.0448, 0.0537, 0.0537)
$\mathbf{d_{12}} \times \mathbf{w_2}$	(0.0637, 0.0637, 0.0764)	(0.0382, 0.0382, 0.0637)	(0.0637, 0.0892, 0.0764)
$\mathbf{d_{13}} \times \mathbf{w_3}$	(0.0421, 0.0280, 0.0982)	(0.0982, 0.0421, 0.0842)	(0.0982, 0.0701, 0.0842)
$\mathbf{d_{21}} \times \mathbf{w_4}$	(0.0392, 0.0294, 0.0588)	(0.0784, 0.0392, 0.0490)	(0.0294, 0.0490, 0.0490)
$\mathbf{d_{22}} \times \mathbf{w_5}$	(0.0458, 0.0343, 0.0687)	(0.0343, 0.0687, 0.0458)	(0.0343, 0.0572, 0.0572)
$\mathbf{d_{23}} \times \mathbf{w_6}$	(0.0272, 0.0181, 0.0635)	(0.0725, 0.0453, 0.0544)	(0.0272, 0.0453, 0.0453)
$\mathbf{d_{31}} \times \mathbf{w_7}$	(0.0460, 0.0345, 0.0690)	(0.0575, 0.0460, 0.0575)	(0.0575, 0.0460, 0.0690)
$\mathbf{d_{32}} \times \mathbf{w_8}$	(0.0603, 0.0362, 0.0724)	(0.0362, 0.724, 0.0482)	(0.0603, 0.0482, 0.0724)
$\mathbf{d_{33}} \times \mathbf{w_9}$	(0.0310, 0.0206, 0.0620)	(0.0517, 0.0517, 0.0620)	(0.0620, 0.0413, 0.0620)

weighted decision matrix

Step 5: Calculation of RPIS and RNIS

From the weighted decision matrix the RPIS (R_G^+) and RNIS (R_G^-)is attained as:

R_G^+ $\langle(0.0637, 0.0181, 0.0537);\ (0.0982, 0.0268, 0.0448);$
$(0.0982, 0.0413, 0.0453)\rangle$

R_G^- $\langle(0.0272, 0.0637, 0.0982);\ (0.0343, 0.0724, 0.0842);$

$(0.0272, 0.0892, 0.0842)\rangle.$

Step 6: Calculate the distance measure of the alternative from the RPIS and RNIS,

The distance measures of the alternative from the RPIS are established as:

$$D_{euc}^{1+} = 0.0424,\ D_{euc}^{2+} = 0.0535,\ D_{euc}^{3+} = 0.0578$$

Likewise, the distance measures of the alternative from the RNIS are established as:

$$D_{euc}^{1-} = 0.0485,\ D_{euc}^{2-} = 0.0520,\ D_{euc}^{3-} = 0.0550$$

Step 7: Computation of relative closeness co-efficient

Now compute the relative closeness co-efficient ρ_i^* , i = 1, 2, 3

ρ_1^* 0.5332 (0.4667), ρ_2^* 0.4926 (0.5073), ρ_3^* 0.4876 (0.5123)

Step 8: Ranking the alternatives

The ranking order of alternatives with respect to relative closeness coefficient is given as follows:

$C_3 > C_2 > C_1$

Therefore, C_3 is the best choice.

CONCLUSIONS

In this paper, a new hybrid priority weighted neutrosophic sets are presented and some of its basic properties are discussed. We define various operations on PWNSs and prove some results on them. We have proposed a TOPSIS method for solving MADM problem with priority weighted neutrosophic soft information. In the decision making context, the rating of performance values of the alternatives with respect to the parameters are presented in terms of PWNSs. We employ AND operator of PWNSs to combine opinions of the DMs based on the choice parameters of the evaluator. We construct weighted decision matrix after obtaining the weights of the choice parameters by using information entropy method. Then, we define RPIS and RNIS from the weighted decision matrix and Euclidean distance measure is used to compute distance of each alternative from RPISs as well as RNISs. Finally, relative closeness co-efficient of each alternative is calculated in order to select the best alternative.

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