$\psi\alpha$-Irresolute Functions in Topological Spaces

KARTHIKA A P

(17PMA010)

Thesis Submitted to
Avinashilingam Institute for Home Science and Higher Education for Women
Coimbatore - 641043

In Partial Fulfilment of the Requirements for the Degree of
Master of Science in Mathematics

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Signature of the Head of the Department

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CONTENT

INTRODUCTION

Topology is the mathematical study of the properties that are preserved through deformations, twisting and stretching of objects. General topology is a branch of mathematics deals with most fundamental structures of geometric figures or point sets. The word topology means “The study of mappings”. In mathematics it is a large area of study with many more specific sub-fields of topology which include algebraic topology, geometric topology, differential topology or manifold theory and knot theory. This can be studied by considering a collection of subsets, called open sets that satisfy certain properties, turning the given set into what is known as a topological spaces.

The notion of open sets is the powerful tool for defining a topological space. Stone (1937) introduced the regular open sets in topological spaces, as a stronger form of open sets. The notion of semi-open sets was first introduced and investigated by Levine (1963). Njastad (1965) introduced the concept of α-open sets in topological spaces. Further Levine (1970) introduced a new type of closed sets called generalized closed sets in topological spaces in order to extend some important properties of closed sets to a larger family of sets. Veera Kumar (2000) introduced ψ-closed sets in topological spaces. Recently Shakila and Balamani (2017) introduced ψα-closed sets in topological spaces.

The notion of continuous function is one of the most important concepts in mathematics. Levine (1970) introduced the concepts of continuous functions in topological spaces. Mashhour et al. (1983) introduced and studied α-continuous functions in topological spaces. The generalized continuous functions were introduced and studied by Balachandran et al. (1991). Crossley and Hildebrand (1972) introduced the concept of irresolute functions in topological spaces.

Concepts discussed in this thesis are

1) Totally ψα-continuous functions in topological spaces

2) Contra ψα-continuous functions in topological spaces
3) ψα-irresolute functions in topological spaces

Chapter 1 deals with preliminary definitions that are needed for the present study.

In chapter 2, the concepts of totally ψα-continuous functions and ψα-totally continuous functions in topological spaces are defined and derived their interrelations.

In chapter 3, two new types of continuous functions called contra ψα-continuous functions and almost contra ψα-continuous functions in topological spaces are introduced and studied their properties.

In the final chapter, a stronger form of ψα-continuous functions called ψα-irresolute functions are defined in topological spaces. Also the interrelations between ψα-irresolute functions with already existing various irresolute functions are derived.

REVIEW OF LITERATURE

Topology is one of the widely studied areas of mathematics emerged through the works of the great mathematician Henri Poincare in the 19th century. Topology is the branch of mathematics through which we elucidate and investigate the ideas of continuity, within the framework of mathematics.

Initially the topological spaces were characterized by open sets. Stone (1937) introduced regular openness which is stronger than openness. Levine (1963) introduced the concept of semi openness which is weaker than the notion of openness. Njastad (1965) introduced the concept of α-open sets which are weaker than open sets and proved that the collection of all α-open sets forms a finer topology. Levine (1970) introduced the notion of generalized closed (briefly g-closed) sets in topological spaces.

Bhattacharya and Lahiri (1987) introduced semi generalized closed sets with the help of semi open sets. Veerakumar (2000) investigated a new class of closed sets called ψ-closed sets in topological spaces. Ramya and Parvathi (2013) introduced ψg-closed sets in topological spaces and studied their properties. Balamani and Parvathi (2016) introduced a stronger form of ψg-closed sets called $\psi^*\alpha$-closed sets in topological spaces. Shakila and Balamani (2017) introduced $\psi\alpha$-closed sets in topological spaces and proved that $\psi\alpha$-closed sets form a topology.

Continuous functions are an important notion in the study of mathematical sciences. Many generalizations of continuous functions have been introduced over years and many interesting results have been obtained. Levine (1970) introduced continuous functions in topological spaces.

Arya and Gupta (1974) introduced completely continuous functions in topological spaces. Balachandran et al. (1991) introduced and studied g-continuous functions in topological spaces. Mashhour et.al, (1983) defined and studied α-continuous functions in topological spaces. Veera Kumar (2000) introduced ψ-continuous functions in topological spaces. Shakila and Balamani (2017) introduced the concept of ψα-continuous functions in topological spaces.

Jain (1980) introduced the totally continuous functions in topological spaces. Dontchev (1996) introduced contra continuous functions in topological spaces. Jafari and Noiri (2001) introduced the contra α-continuous functions in topological spaces.

Singal M.K and Singal A.R (1968) introduced and analyzed the concept of almost continuous functions in topological spaces. Ekici (2004) introduced the concept of almost contra continuous functions in topological spaces.

Crossley and Hildebrand (1972) investigated irresolute functions in topological spaces. Maheshwari and Thakur (1980) introduced α-irresolute functions in topological spaces. Ramya and Parvathi (2013) introduced the ψg-irresolute functions in topological spaces. Balamani and Parvathi (2017) introduced the notation of $\psi^*\alpha$-irresolute functions in topological spaces.

CHAPTER 1

PRELIMINARIES

Definition 1.1[19]

A subset A of a topological space (X,τ) is called **regular open** if A = int(cl(A)) and **regular closed** A = cl(int (A)).

Definition 1.2[12]

A subset A of a topological space (X,τ) is called **semi-open** if A ⊆ cl(int(A)) and **semi-closed** if int (cl(A)) ⊆ A.

Definition 1.3[15]

A subset A of a topological space (X,τ) is called **α-open** if A ⊆ int(cl(int(A))) and **α-closed** if cl(int(cl(A))) ⊆ A.

Definition 1.4[11]

A subset A of a topological space (X,τ) is called **generalized closed** if cl(A) ⊆ U whenever A⊆ U and U is open in (X,τ).

Definition 1.5[5]

A subset A of a topological space (X,τ) is called **semi generalized closed** if scl(A) ⊆ U whenever A⊆ U and U is semi-open in (X,τ).

Definition 1.6[20]

A subset A of a topological space (X,τ) is called **ψ-closed** if scl(A) ⊆ U whenever A⊆ U and U is sg-open in (X,τ).

Definition 1.7[16]

A subset A of a topological space (X,τ) is called **ψg-closed** if ψcl(A) ⊆ U whenever A⊆ U and U is open in (X,τ)

Definition 1.8[3]

A subset A of a topological space (X,τ) is called **$\psi^*\alpha$-closed** if αcl(A) ⊆ U whenever A ⊆ U and U is ψg-open in (X,τ)

Definition 1.9[17]

A subset A of a topological space (X,τ) is called **ψα-closed** if ψcl(A) ⊆ U whenever A ⊆ U and U is α-open in (X,τ).

Definition 1.10[17]

A subset A of a topological space (X,τ) is called **ψα-clopen** if it is both ψα-open and ψα-closed in (X,τ).

Results 1.11

- Every closed (open) subset in (X,τ) is ψα-closed (ψα-open).
- Every clopen subset in (X,τ) is ψα- clopen.
- Every regular open (regular closed) subset in (X,τ) is open (closed).
- Every α-open subset in (X,τ) is ψα-open.

Definition 1.12[11]

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **continuous** if $f^{-1}(V)$ is closed in (X,τ) for every closed set V of (Y,σ).

Definition 1.13[14]

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **α-continuous** if $f^{-1}(V)$ is α-closed in (X,τ) for every closed set V of (Y,σ).

Definition 1.14[2]

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **g-continuous** if $f^{-1}(V)$ is g-closed in (X,τ) for every closed set V of (Y,σ).

Definition 1.15[20]

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **ψ-continuous** if $f^{-1}(V)$ is ψ-closed in (X,τ) for every closed set V of (Y,σ).

Definition 1.16[17]

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **ψα- continuous** if $f^{-1}(V)$ is ψα-closed in (X,τ) for every closed set V of (Y,σ).

Definition 1.17[18]

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **almost continuous** if $f^{-1}(V)$ is closed in (X,τ) for every regular closed set V of (Y,σ).

Definition 1.18[1]

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **completely continuous** if $f^{-1}(V)$ is regular open in (X,τ) for every open set V of (Y,σ).

Definition 1.19[10]

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **totally continuous** if $f^{-1}(V)$ is clopen in (X,τ) for every open set V of (Y,σ).

Definition 1.20[7]

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **contra continuous** if $f^{-1}(V)$ is closed in (X,τ) for every open set V of (Y,σ).

Definition 1.21[9]

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **contra α-continuous** if $f^{-1}(V)$ is α-closed in (X,τ) for every open set V of (Y,σ).

Definition 1.22[8]

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **almost contra continuous** if $f^{-1}(V)$ is closed in (X,τ) for every regular open set V of (Y,σ).

Definition 1.23[6]

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **irresolute** if $f^{-1}(V)$ is semi closed in (X,τ) for every semi closed set V of (Y,σ).

Definition 1.24[13]

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **α-irresolute** if $f^{-1}(V)$ is α-closed in (X,τ) for every α-closed set V of (Y,σ).

Definition 1.25[16]

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **ψg-irresolute** if $f^{-1}(V)$ is ψg-closed in (X,τ) for every ψg-closed set V of (Y,σ).

Definition 1.26[4]

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **ψ^{*}α-irresolute** if $f^{-1}(V)$ is $\psi^{*}\alpha$-closed in (X,τ) for every $\psi^{*}\alpha$-closed set V of (Y,σ).

CHAPTER 2

ψα-Totally Continuous Functions in Topological Spaces

2.1 Introduction

Many different forms of continuous functions have been introduced over the years. Levine [1970] introduced the idea of continuous functions in topological spaces. Jain [1980] introduced totally continuous functions in topological spaces. Mashhour [1983] introduced and studied α-continuous functions in topological spaces. Balachandran et al. [1991] introduced generalized continuous functions in topological spaces. Veera Kumar [2000] introduced and studied ψ-continuous functions in topological spaces. Shakila and Balamani [2017] introduced ψα-continuous functions in topological spaces.

In this chapter we introduce totally ψα-continuous functions and ψα-totally continuous functions in topological spaces.

2.2 Totally ψα-continuous functions

In this section we introduce a new class of continuous functions called totally ψα-continuous functions in topological spaces and study some of their properties.

Definition 2.2.1

Let (X,τ) and (Y,σ) be two topological spaces. A function $f: (X,\tau) \rightarrow (Y,\sigma)$ is said to be **totally ψα-continuous** if $f^{-1}(V)$ is ψα-clopen subset of (X,τ) for every open set V of (Y,σ).

Example 2.2.2

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{b,c\},X\}$ and $\sigma = \{\phi,\{a\},Y\}$. Let $f :(X,\tau) \rightarrow (Y,\sigma)$ be a function defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. Then f is totally ψα-continuous.

Theorem 2.2.3

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is totally $\psi\alpha$-continuous if and only if the inverse image of every closed subset of (Y,σ) is a $\psi\alpha$-clopen subset of (X,τ).

Proof:

(Necessity): Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be a totally $\psi\alpha$-continuous function**.** Let V be any closed set in (Y,σ). Then Y-V is open in (Y, σ). Since f is totally $\psi\alpha$-continuous**,** f^{-1} (Y- V) = X - f^{-1}(V) is $\psi\alpha$-clopen in (X,τ) which implies that f^{-1}(V) is $\psi\alpha$-clopen in (X,τ).

(Sufficiency): Assume that U is any open set in (Y, σ).Then Y-U is closed in (Y, σ). By assumption, f^{-1}(Y- U) = X- f^{-1}(U) is $\psi\alpha$-clopen in (X,τ) which implies that f^{-1}(U) is $\psi\alpha$-clopen in (X,τ). Hence f is a totally $\psi\alpha$-continuous function.

Proposition 2.2.4

Every totally continuous function is a totally $\psi\alpha$-continuous function but not conversely.

Proof:

Let V be any open set in (Y,σ). Since f is totally continuous, f^{-1}(V) is clopen in (X,τ). By result 1.11, f^{-1}(V) is $\psi\alpha$-clopen in (X,τ). Hence f is a totally $\psi\alpha$-continuous function.

Example 2.2.5

Let X =Y ={a,b,c}, $\tau = \{\phi, \{a\},\{b,c\},X\}$ and $\sigma = \{\phi, \{a\},Y\}$. Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be a function defined by f(a) = c, f(b) = a ,f(c) =b. Then f is a totally $\psi\alpha$ - continuous function but not a totally continuous function, since for the open set {a} in (Y,σ), f^{-1}({a})= {b} is $\psi\alpha$-clopen in (X,τ) but not clopen in (X,τ).

Proposition 2.2.6

Every totally ψα-continuous function is a ψα-continuous function but not conversely.

Proof:

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be a totally ψα-continuous function**.** Let V be any open set in (Y,σ). Since f is totally ψα-continuous, $f^{-1}(V)$ is ψα-clopen in (X,τ) which implies that $f^{-1}(V)$ is ψα-open in (X,τ) **.** Hence f is a ψα-continuous function.

Example 2.2.7

Let $X = Y = \{a,b,c\}, \tau = \{\phi,\{a\},\{a,b\},X\}$ and $\sigma = \{\phi, \{a\},Y\}$. Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be the identity function. Then f is ψα-continuous but not totally ψα- continuous, since for the open set {a} in (Y,σ), $f^{-1}(\{a\}) = \{a\}$ is ψα-open but not ψα-closed in (X,τ).

Proposition 2.2.8

Every continuous function is independent from totally ψα-continuous function.

Example 2.2.9

Let $X= Y = \{a,b,c\}, \tau = \{\phi,\{a\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a,b\},Y\}$.Let f:$(X,\tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is continuous but not totally ψα-continuous, since for the open set {a,b} in (Y,σ), $f^{-1}(\{a,b\}) = \{a,b\}$ is open but not ψα-clopen in (X,τ).

Example 2.2.10

Let $X = Y = \{a,b,c\}, \tau = \{\phi,\{a\},\{b,c\},X\}$ and $\sigma = \{\phi, \{a,b\},Y\}$.Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be defined by the identity function.Then f is totally ψα-continuous but not continuous, since for the open set {a,b} in (Y,σ), $f^{-1}(\{a,b\}) = \{a,b\}$ is ψα-clopen but not open in (X,τ).

Proposition 2.2.11

If f: (X,τ) →(Y,σ) is a totally ψα-continuous function and X is ψα-connected, then Y is an indiscrete space.

Proof:

Suppose that Y is not an indiscrete space. Let V be a non-empty open subset of Y. Since f is totally ψα-continuous, $f^{-1}(V)$ is non-empty ψα-clopen subset of X. Then $X = f^{-1}(V) \cup (f^{-1}(V))^C$. Thus X is union of two non-empty disjoint ψα-open sets which is a contradiction to the fact that X is ψα-connected. Therefore Y must be indiscrete space.

Theorem 2.2.12

Let f: (X,τ) →(Y,σ) be a continuous function from discrete space (X,τ) into a topological space (Y,σ) if and only if f is a totally ψα-continuous function.

Proof:

(Necessity): Assume that f is a totally ψα-continuous function. Let V be any open set in (Y,σ). Since f is a totally ψα-continuous function, f^{-1} (V) is ψα-clopen in (X,τ). Since (X,τ) is discrete space, every subset of (X,τ) is open and closed subset in (X,τ) which implies that f^{-1} (V) is open in (X,τ). Therefore f is a continuous function.

(Sufficiency): Assume that f is a continuous function. Let V be any open set in (Y,σ). Then f^{-1} (V) is open in (X,τ). Since (X,τ) is discrete space, every subset of (X,τ) is clopen in (X,τ) which implies that f^{-1} (V) is ψα-clopen in (X,τ). Therefore f is totally ψα-continuous.

Theorem 2.2.13

Let f: (X,τ) →(Y,σ) be a ψα-continuous function from a discrete space (X,τ) into a topological space (Y,σ) if and only if f is a totally ψα-continuous function.

Proof:

(Necessity): Assume that f is a totally ψα-continuous function. Let V be any open set in (Y,σ).Then f^{-1} (V) is ψα-clopen in (X,τ). Since (X,τ) is discrete space, every subset of (X,τ) is open and closed subset in (X,τ) which implies that f^{-1} (V) is clopen in (X,τ). By result 1.11, f^{-1} (V) is ψα-open in (X,τ).Therefore f is ψα-continuous.

(Sufficiency): Assume that f is ψα-continuous. Let V be any open set in (Y,σ).Then f^{-1} (V) is ψα -open in (X,τ). Since (X,τ) is discrete space, f^{-1} (V) is clopen in (X,τ). By result 1.11, f^{-1} (V) is ψα-clopen in (X,τ).Therefore f is totally ψα-continuous.

Proposition 2.2.14

The composition of two totally ψα-continuous functions need not be a totally ψα-continuous function as seen from the following example.

Example 2.2.15

Let X = Y = Z = {a,b,c}, τ = {ϕ,{a},{b},{a,b},X} , σ = {ϕ,{a},{b,c},Y} and η = {ϕ, {a,b},Z}. Let f:(X,τ)→(Y,σ) be the identity function and g:(Y,σ) →(Z, η) be the identity function. Then f and g are totally ψα-continuous but their composition g o f :(X,τ)→(Z,η) is not totally ψα-continuous, since {a,b} is open in (Z,η), whereas $(g \circ f)^{-1}$({a,b}) ={a,b} is not ψα - clopen in (X,τ).

Proposition 2.2.16

Let f:(X,τ) →(Y,σ) be a totally ψα-continuous function and g:(Y,σ) →(Z, η) be a continuous function. Then g o f : (X,τ) →(Z, η) is a totally ψα-continuous function.

Proof:

Let V be any open set in (Z,η). Since g is a continuous function, $g^{-1}(V)$ is open in (Y,σ). Since f is totally $\psi\alpha$-continuous $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$-clopen in (X,τ). Hence g o f is a totally $\psi\alpha$-continuous function.

Proposition 2.2.17

Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a totally $\psi\alpha$-continuous function and $g:(Y,\sigma)\rightarrow(Z, \eta)$ be a totally continuous function. Then $g \circ f :(X,\tau) \rightarrow(Z, \eta)$ is a totally $\psi\alpha$-continuous function.

Proof:

Let V be any open set in (Z,η). Since g is totally continuous, $g^{-1}(V)$ is clopen in (Y,σ) which implies that $g^{-1}(V)$ is open in (Y,σ) . Since f is totally $\psi\alpha$-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$-clopen in (X,τ). Hence g o f is a totally $\psi\alpha$-continuous function.

Proposition 2.2.18

Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a totally continuous function and $g:(Y,\sigma)\rightarrow(Z, \eta)$ be a totally continuous function. Then $g \circ f :(X,\tau) \rightarrow(Z, \eta)$ is a totally $\psi\alpha$-continuous function.

Proof:

Let V be any open set in (Z,η). Since g is totally continuous, $g^{-1}(V)$ is clopen in (Y,σ) which implies that $g^{-1}(V)$ is open in (Y,σ) . Since f is totally continuous, $f^{-1}(g^{-1}(V))$ is clopen in (X,τ), By result 1.11, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$-clopen in (X,τ). Hence g o f is totally $\psi\alpha$-continuous.

2.3 ψα-totally continuous functions

In this section, ψα-totally continuous functions in topological spaces are introduced and their properties are derived. It is also proved that composition of two ψα-totally continuous functions is a ψα-totally continuous function.

Definition 2.3.1

A function $f: (X,\tau) \rightarrow (Y,\sigma)$ is called **ψα-totally continuous** if $f^{-1}(V)$ is clopen subset of (X,τ) for every ψα -open set V of (Y,σ) .

Theorem 2.3.2

A function $f: (X,\tau) \rightarrow (Y,\sigma)$ is ψα-totally continuous if and only if the inverse image of every ψα-closed subset of (Y,σ) is a clopen subset of (X,τ).

Proof:

(Necessity): Let $f: (X,\tau) \rightarrow (Y,\sigma)$ be a ψα-totally continuous function. Let V be any ψα-closed set in (Y,σ). Then Y-V is ψα-open in (Y, σ). Since f is a ψα-totally continuous function, $f^{-1}(Y- V) = X - f^{-1}(V)$ is clopen in (X,τ) which implies that $f^{-1}(V)$ is clopen in (X,τ).

(Sufficiency): Assume that U is any ψα-open set in (Y, σ). Then Y-U is ψα-closed in (Y, σ). By assumption, $f^{-1}(Y - U) = X - f^{-1}(U)$ is clopen in (X,τ) which implies that $f^{-1}(U)$ is clopen in (X,τ). Hence f is ψα -totally continuous.

Proposition 2.3.3

If $f: (X,\tau) \rightarrow (Y,\sigma)$ is a ψα-totally continuous function, then f is a totally continuous function.

Proof:

Let V be any open set in (Y,σ). By result 1.11, V is a ψα-open set in (Y, σ). Since f is a ψα-totally continuous function, $f^{-1}(V)$ is clopen in (X,τ). Therefore f is totally continuous.

The converse of proposition 2.3.3 need not be true as seen from the following example

Example 2.3.4

Let X = Y ={a,b,c}, τ = {ϕ,{a},{b},{a,b},{a,c},X}and σ = {ϕ,{a},Y}. Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be a function defined by f(a) = b, f(b) = a, f(c) = c. Then f is totally continuous, but f is not ψα-totally continuous, since for the ψα-open set {b} in (Y,σ), $f^{-1}(\{b\})$ ={a} is not clopen subset in (X,τ).

Proposition 2.3.5

If f: (X,τ) →(Y,σ) is a ψα-totally continuous function, then f is a totally ψα-continuous function.

Proof:

Let V be any open set in (Y,σ). By result 1.11, V is a ψα-open set in (Y, σ). Since f is ψα-totally continuous, $f^{-1}(V)$ is clopen in (X,τ) which implies that $f^{-1}(V)$ is ψα-clopen in (X,τ). Therefore f is totally ψα-continuous.

The converse of proposition 2.3.5 need not be true as seen from the following example.

Example 2.3.6

Let X = Y ={a,b,c}, τ = {ϕ,{a},{b},{a,b},{a,c},X}and σ = {ϕ,{a},Y}. Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be defined by f(a) = b, f(b) = a, f(c) = c. Then f is totally ψα-continuous, but f is not ψα-totally continuous, since for the ψα-open set {a,b}in (Y,σ), $f^{-1}(\{a,b\})$ ={a,b} is not clopen in (X,τ).

Proposition 2.3.7

Every $\psi\alpha$ -totally continuous function is continuous but not conversely.

Proof:

Let V be any open set in (Y,σ). By result 1.11, V is a $\psi\alpha$-open set in (Y, σ). Since f is $\psi\alpha$-totally continuous, $f^{-1}(V)$ is clopen in (X,τ) which implies that $f^{-1}(V)$ is open in (X,τ). Therefore f is continuous.

Example 2.3.8

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{a,b\},X\}$and $\sigma = \{\phi,\{a,b\},Y\}$. Let $f: (X,\tau)\to(Y,\sigma)$ be the identity function. Then f is continuous, but not $\psi\alpha$-totally continuous, since for the $\psi\alpha$-open set $\{b\}$ in (Y,σ), $f^{-1}(\{b\}) = \{b\}$ is not clopen in (X,τ).

Proposition 2.3.9

Every $\psi\alpha$-totally continuous function is $\psi\alpha$-continuous but not conversely.

Proof:

Let V be any open set in (Y,σ). By result 1.11, V is $\psi\alpha$-open in (Y, σ). Since f is $\psi\alpha$-totally continuous, $f^{-1}(V)$ is clopen in (X,τ) which implies that $f^{-1}(V)$ is open in (X,τ). By result 1.11, $f^{-1}(V)$ is $\psi\alpha$-open in (X,τ). Therefore f is $\psi\alpha$-continuous.

Example 2.3.10

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{a,b\},X\}$and $\sigma = \{\phi,\{a\},Y\}$. Let $f: (X,\tau)\to(Y,\sigma)$ be the identity function. Then f is $\psi\alpha$-continuous but not $\psi\alpha$-totally continuous, since for the $\psi\alpha$-open set $\{a,c\}$ in (Y,σ), $f^{-1}(\{a,c\}) = \{a,c\}$ is not clopen in (X,τ).

Proposition 2.3.11

Let f: (X,τ) →(Y,σ) be any function from discrete space (X,τ) into a topological space (Y,σ). If f is a ψα-totally continuous function then f is a continuous function.

Proof:

Let V be any open set in (Y,σ).By result 1.11, V is ψα-open in (Y, σ). Since f is ψα-totally continuous, f^{-1} (V) is clopen in (X,τ) which implies that f^{-1} (V) is open in (X,τ). Therefore f is continuous.

Proposition 2.3.12

Let f: (X,τ) →(Y,σ) be any function from discrete space (X,τ) into a topological space (Y,σ). If f is a ψα-totally continuous function, then f is a ψα-continuous function.

Proof:

Let V be any open set in (Y,σ).By result 1.11, V is a ψα-open set in (Y, σ). Since f is ψα- totally continuous, f^{-1} (V) is clopen in (X,τ) which implies that f^{-1} (V) is open in (X,τ). By result 1.11, f^{-1} (V) is ψα-open in (X,τ). Therefore f is ψα-continuous.

Proposition 2.3.13

Let f:(X,τ) →(Y,σ) be a ψα-totally continuous function and g: (Y,σ)→(Z, η) be a totally ψα- continuous function. Then g o f :(X,τ) →(Z, η) is a totally ψα -continuous function.

Proof:

Let V be any open set in (Z, η). Since g is totally ψα-continuous, g^{-1} (V) is ψα - clopen in (Y,σ) which implies that g^{-1} (V) is ψα-open in (Y, σ). Since f is a ψα-totally continuous function, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). By result 1.11, $(g \circ f)^{-1}(V)$ is ψα-clopen in (X,τ). Therefore g o f is totally ψα-continuous.

Proposition 2.3.14

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be a $\psi\alpha$-totally continuous function and g: $(Y,\sigma)\rightarrow(Z, \eta)$ be a totally continuous function. Then g o f :$(X,\tau) \rightarrow(Z, \eta)$ is a totally $\psi\alpha$-continuous function.

Proof:

Let V be any open set in (Z, η). Since g is totally continuous, g^{-1} (V) is clopen in (Y,σ).Which implies that $g^{-1}(V)$ is open in (Y,σ) . By result 1.11, $g^{-1}(V)$ is $\psi\alpha$-open in (Y,σ). Since f is $\psi\alpha$-totally continuous, $f^{-1}(g^{-1}(V))$ is clopen in (X,τ). By result 1.11, $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$-clopen in (X,τ). Therefore g o f is totally $\psi\alpha$-continuous.

Proposition 2.3.15

Let f: $(X,\tau) \rightarrow(Y,\sigma)$ be a $\psi\alpha$-totally continuous function and g: $(Y,\sigma)\rightarrow(Z, \eta)$ be a continuous function. Then g o f :$(X,\tau) \rightarrow(Z, \eta)$ is a totally $\psi\alpha$-continuous function.

Proof:

Let V be any open set in (Z, η). Since g is continuous, g^{-1} (V) is open in (Y,σ). By result 1.11, $g^{-1}(V)$ is $\psi\alpha$-open in (Y,σ).Since f is $\psi\alpha$-totally continuous, $f^{-1}(g^{-1}(V))$ is clopen in (X,τ). By result 1.11, $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$-clopen in (X,τ). Therefore g o f is totally $\psi\alpha$-continuous.

Proposition 2.3.16

Let f: $(X,\tau) \rightarrow(Y,\sigma)$ be a $\psi\alpha$-totally continuous function and g: $(Y,\sigma)\rightarrow(Z, \eta)$ be a $\psi\alpha$-continuous function. Then g o f: $(X,\tau) \rightarrow(Z, \eta)$ is a totally $\psi\alpha$-continuous function.

Proof:

Let V be any open set in (Z, η). Since g is $\psi\alpha$-continuous, g^{-1} (V) is $\psi\alpha$-open in (Y,σ).Since f is $\psi\alpha$- totally continuous, By result 1.11 $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$-clopen in (X,τ). Therefore gof is totally $\psi\alpha$-continuous.

Proposition 2.3.17

Let f: (X,τ) →(Y,σ) be a totally continuous function and g: (Y,σ)→(Z, η) be a ψα-totally continuous function. Then g o f: (X,τ) →(Z, η) is a ψα-totally continuous function.

Proof:

Let V be any ψα-open set in (Z, η). Since g is ψα-totally continuous, g^{-1} (V) is clopen in (Y,σ), which implies that g^{-1} (V) is open in (Y,σ) . Since f is a totally continuous function, $(g \text{ o } f)^{-1}$(V) is clopen in (X,τ). Therefore g o f is ψα-totally continuous.

Proposition 2.3.18

Let f: (X,τ) →(Y,σ) be a continuous function and g: (Y,σ)→(Z, η) be a ψα-totally continuous function. Then g o f : (X,τ) →(Z, η) is a continuous function.

Proof:

Let V be any open set in (Z, η). By result 1.11, V is ψα-open in (Z, η). Since g is ψα-totally continuous, g^{-1} (V) is clopen in (Y,σ) which implies that g^{-1} (V) is open in (Y,σ). Since f is continuous, $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is open in (X,τ). Therefore g o f is continuous.

Proposition 2.3.19

Let f: (X,τ) →(Y,σ) be a ψα-continuous function and g: (Y,σ)→(Z, η) be a ψα-totally continuous function. Then g o f: (X,τ) →(Z, η) is a ψα-continuous function.

Proof:

Let V be any open set in (Z, η). By result 1.11, V is ψα -open in (Z, η). Since g is ψα -totally continuous, g^{-1} (V) is clopen in (Y,σ) which implies that g^{-1} (V) is open in (Y,σ). Since f is ψα-continuous, $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψα-open in (X,τ). Therefore g o f is ψα-continuous.

Proposition 2.3.20

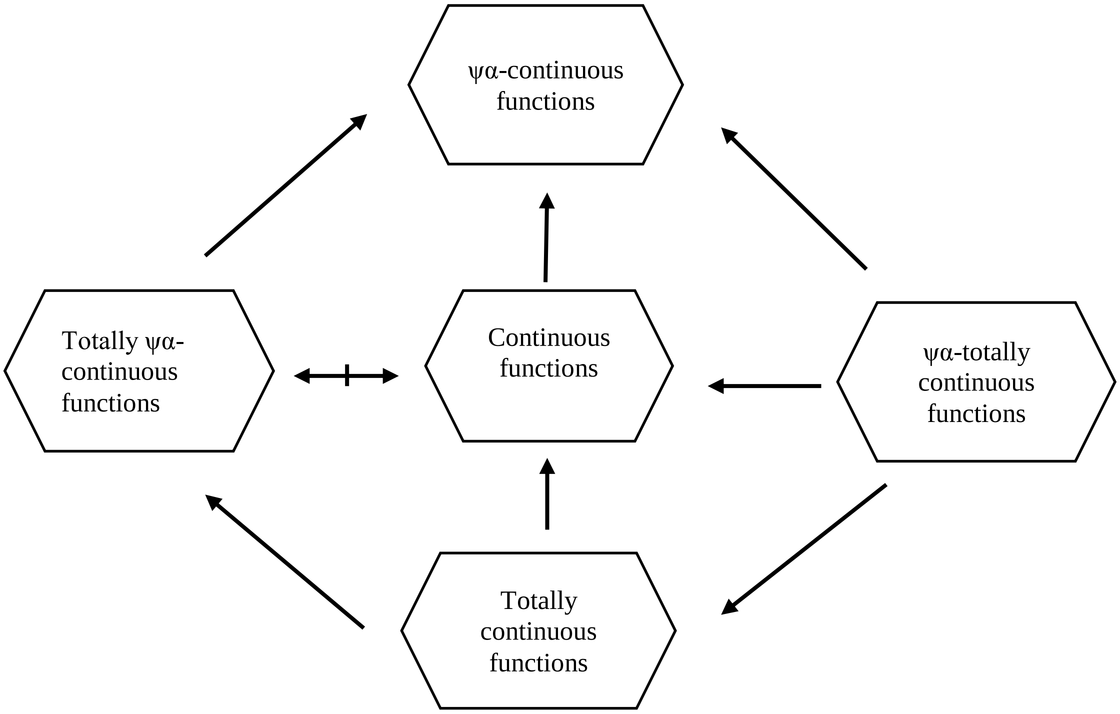
Let $f:(X,\tau)\rightarrow(Y,\sigma)$ and $g:(Y,\sigma)\rightarrow(Z,\eta)$ be two $\psi\alpha$-totally continuous functions. Then $g \circ f: (X,\tau) \rightarrow(Z,\eta)$is also $\psi\alpha$-totally continuous function.

Proof:

Let V be any $\psi\alpha$-open set in (Z, η). Since g is $\psi\alpha$-totally continuous, g^{-1} (V) is clopen in (Y,σ) By result 1.11, g^{-1} (V) is $\psi\alpha$-clopen in (Y,σ) which implies that g^{-1} (V) is $\psi\alpha$-open in (Y,σ). Since f is $\psi\alpha$-totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). Therefore g o f is $\psi\alpha$-totally continuous.

Remark 2.3.21

The newly defined continuous functions are compared with various types of continuous functions and the results are given in the following diagram.



CHAPTER 3

Contra ψα-Continuous Functions in Topological Spaces

3.1 Introduction

Singal M. K and Singal A.R [1968] introduced almost continuous functions in topological spaces. Dontchev [1996] introduced the notion of contra continuous functions in topological spaces. Jafari and Noiri [2001] introduced and studied the new form of functions called contra α-continuous functions in topological spaces. Ekici [2004] introduced the concept of almost contra continuous functions in topological spaces.

In this chapter we introduce contra ψα-continuous functions in topological spaces.

3.2 Contra ψα-continuous functions

In this section, contra ψα-continuous functions in topological spaces are introduced and some of their properties are analyzed.

Definition 3.2.1

A function $f: (X,\tau) \to (Y,\sigma)$ is called **contra ψα-continuous** if $f^{-1}(V)$ is ψα-open in (X,τ) for every closed set V of (Y,σ).

Example 3.2.2

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},X\}$ and $\sigma = \{\phi,\{a,b\},Y\}$. Let $f: (X,\tau)\to(Y,\sigma)$ be a function defined by f(a) = c, f(b) = b, f(c) = a. Then f is contra ψα-continuous.

Theorem 3.2.3

A function f: $(X,\tau) \to (Y,\sigma)$ is contra ψα-continuous if and only if $f^{-1}(V)$ is ψα -closed in (X,τ) for every open set V of (Y,σ).

Proof:

(Necessity): Let f: $(X,\tau) \to (Y,\sigma)$ be contra ψα-continuous and V be any open set in (Y,σ). Then Y-V is closed in (Y,σ). Since f is contra ψα-continuous, $f^{-1}(Y - V) = X - f^{-1}(V)$ is ψα-open in (X,τ) which implies that $f^{-1}(V)$ is ψα-closed in (X,τ).

(Sufficiency): Let U be any closed set in (Y,σ). Then Y-U is open in (Y,σ). By assumption, $f^{-1}(Y-U) = X - f^{-1}(U)$ is ψα-closed in (X,τ) which implies that $f^{-1}(U)$ is ψα-open in (X,τ). Hence f is contra ψα-continuous.

Proposition 3.2.4

Every contra continuous function is a contra ψα-continuous function but not conversely.

Proof:

Let V be any open set in (Y,σ). Since f is contra continuous, $f^{-1}(V)$ is closed in (X,τ). By result 1.11, $f^{-1}(V)$ is ψα-closed in (X,τ). Hence f is contra ψα-continuous.

Example 3.2.5

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a\},Y\}$. Let f: $(X,\tau) \to (Y,\sigma)$ be a function defined by f(a) = b, f(b) = a, f(c) = c. Then f is contra ψα-continuous but not contra continuous, since for the open set {a} in (Y,σ), $f^{-1}(\{a\}) = \{b\}$ is ψα-closed but not closed in (X,τ).

Proposition 3.2.6

Every contra α-continuous function is a contra ψα-continuous function but not conversely.

Proof:

Let V be any closed set in (Y,σ). Since f is contra α-continuous, $f^{-1}(V)$ is α-open in (X,τ). By result 1.11, $f^{-1}(V)$ is ψα-open in (X,τ). Hence f is contra ψα-continuous.

Example 3.2.7

Let X = Y = {a,b,c}, τ = {ϕ,{a}, {b}, {a,b},X}and σ = {ϕ,{a},Y}. Let f: (X,τ)→(Y,σ) be a function defined by f(a) = b, f(b) = a ,f(c) = c. Then f is contra ψα-continuous but not contra α-continuous, since for the closed set {b,c} in (Y,σ), f^{-1}({b,c}) = {a,c} is ψα-open but not α-open in (X,τ).

Proposition 3.2.8

Every totally continuous function is a contra ψα-continuous function but not conversely.

Proof:

Let V be any open set in (Y,σ). Since f is totally continuous, $f^{-1}(V)$ is clopen in (X,τ). By result 1.11, $f^{-1}(V)$ is ψα-clopen in (X,τ) which implies that $f^{-1}(V)$ is ψα-closed in (X,τ). Hence f is contra ψα-continuous.

Example 3.2.9

Let X = Y = {a,b,c}, τ = {ϕ,{a},{a,b},X}and σ = {ϕ,{a},Y}. Let f: (X,τ)→(Y,σ) be a function defined f(a) = b, f(b) = a, f(c) = c. Then f is contra ψα-continuous but not totally continuous, since for the open set {a} in (Y,σ), $f^{-1}(\{a\}) = \{b\}$ is ψα- closed but not clopen in (X,τ).

Proposition 3.2.10

Every totally ψα-continuous function is a contra ψα-continuous function but not conversely.

Proof:

Let V be any closed set in (Y,σ). Since f is totally ψα-continuous, $f^{-1}(V)$ is ψα-clopen in (X,τ) which implies that $f^{-1}(V)$ is ψα-open in (X,τ). Hence f is contra ψα-continuous.

Example 3.2.11

Let X = Y = {a,b,c}, $\tau = \{\phi,\{a\},X\}$and σ = {ϕ,{a,b},Y}. Let f: (X,τ)→(Y,σ) be a function defined by f(a) = c, f(b) = b ,f(c) = a. Then f is contra ψα-continuous but not totally ψα-continuous, since for the closed set {c} in (Y,σ), $f^{-1}(\{c\}) = \{a\}$ is ψα-open but not ψα-closed in (X,τ).

Proposition 3.2.12

Every ψα- totally continuous function is a contra ψα-continuous function but not conversely.

Proof:

Let V be any open set in (Y,σ). By result 1.11, V is ψα-open in(Y,σ). Since f is ψα-totally continuous, $f^{-1}(V)$ is clopen in (X,τ). By result 1.11, $f^{-1}(V)$ is ψα-clopen in (X,τ) which implies that $f^{-1}(V)$ is ψα-closed in (X,τ). Hence f is contra ψα-continuous.

Example 3.2.13

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{b\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a,b\},Y\}$. Let $f: (X,\tau)\to(Y,\sigma)$ be a function defined $f(a) = a$, $f(b) = c$, $f(c) = b$. Then f is contra ψα-continuous but not ψα-totally continuous, since for the ψα-open set $\{a,b\}$ in (Y,σ), $f^{-1}(\{a,b\}) = \{a,c\}$is not clopen in (X,τ).

Remark 3.2.14

Contra ψα-continuous function is independent from ψα-continuous function as seen from the following examples.

Example 3.2.15

Let $X =Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},X\}$ and $\sigma = \{\phi,\{a,b\},Y\}$. Let $f: (X,\tau)\to(Y,\sigma)$ be a function defined by $f(a) = c$,$f(b) = b$, $f(c) = a$ .Then f is contra ψα-continuous but not ψα-continuous, since for the closed set $\{c\}$ in (Y,σ), $f^{-1}(\{c\}) = \{a\}$ is ψα-open but not ψα-closed in (X,τ).

Example 3.2.16

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a,b\},Y\}$. Let $f: (X,\tau)\to(Y,\sigma)$ be the identity function. Then f is ψα-continuous but not contra ψα-continuous, since for the closed set $\{c\}$ in (Y,σ), $f^{-1}(\{c\}) = \{c\}$ is ψα-closed but not ψα-open in (X,τ).

Proposition 3.2.17

If $f:(X,\tau)\to(Y,\sigma)$ is a contra ψα-continuous function and $g:(Y,\sigma)\to(Z, \eta)$ is a continuous function, then $g \circ f :(X,\tau) \to(Z,\eta)$ is a contra ψα-continuous function.

Proof:

Let V be any closed set in (Z, η). Since g is continuous, $g^{-1}(V)$ is closed in (Y,σ). Since f is contra $\psi\alpha$-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$-open in (X,τ). Hence g o f is a contra $\psi\alpha$-continuous function.

Proposition 3.2.18

If f: $(X,\tau)\to(Y,\sigma)$ is a totally $\psi\alpha$-continuous function and g: $(Y,\sigma)\to(Z, \eta)$ is a continuous function, then g o f :$(X,\tau) \to(Z,\eta)$ is a contra $\psi\alpha$-continuous function.

Proof:

Let V be any closed set in (Z, η). Since g is continuous, $g^{-1}(V)$ is closed in (Y,σ). Since f is totally $\psi\alpha$-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$-clopen in (X,τ) which implies that $(g \circ f)^{-1}(V)$ is $\psi\alpha$-open in (X,τ). Hence g o f is a contra $\psi\alpha$-continuous function.

Proposition 3.2.19

If f:$(X,\tau)\to(Y,\sigma)$ is a totally continuous function and g:$(Y,\sigma)\to(Z, \eta)$ is a continuous function, then g o f :$(X,\tau) \to(Z,\eta)$ is a contra $\psi\alpha$-continuous function.

Proof:

Let V be any closed set in (Z, η). Since g is continuous, $g^{-1}(V)$ is closed in (Y,σ). Since f is totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). By result 1.11, $(g \circ f)^{-1}(V)$ is $\psi\alpha$-clopen in (X,τ) which implies that $(g \circ f)^{-1}(V)$ is $\psi\alpha$-open in (X,τ) . Hence g o f is contra $\psi\alpha$-continuous.

Proposition 3.2.20

If f:$(X,\tau)\to(Y,\sigma)$ is a $\psi\alpha$-totally continuous function and g:$(Y,\sigma)\to(Z, \eta)$ is a continuous function, then g o f :$(X,\tau) \to(Z,\eta)$ is a contra $\psi\alpha$-continuous function.

Proof:

Let V be any closed set in (Z, η). Since g is continuous, $g^{-1}(V)$ is closed in (Y,σ). By result 1.11, $g^{-1}(V)$ is $\psi\alpha$-closed in (Y,σ). Since f is $\psi\alpha$-totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ) By result 1.11, $(g \circ f)^{-1}(V)$ is $\psi\alpha$-clopen in (X,τ) which implies that $(g \circ f)^{-1}(V)$ is $\psi\alpha$-open in (X,τ). Hence g o f is a contra $\psi\alpha$-continuous function.

Proposition 3.2.21

If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a contra $\psi\alpha$-continuous function and $g:(Y,\sigma)\rightarrow(Z, \eta)$ is a totally continuous function, then $g \circ f :(X,\tau) \rightarrow(Z,\eta)$ is a contra $\psi\alpha$-continuous function.

Proof:

Let V be any closed set in (Z, η). Since g is totally continuous, $g^{-1}(V)$ is clopen in (Y,σ) which implies that $g^{-1}(V)$ is closed in (Y,σ). Since f is contra $\psi\alpha$-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$-open in (X,τ). Hence g o f is a contra $\psi\alpha$-continuous function.

Proposition 3.2.22

If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a contra $\psi\alpha$-continuous function and $g:(Y,\sigma)\rightarrow(Z, \eta)$ is a $\psi\alpha$- totally continuous function, then $g \circ f :(X,\tau) \rightarrow(Z,\eta)$ is a contra $\psi\alpha$-continuous function.

Proof:

Let V be any closed set in (Z, η). By result 1.11V is $\psi\alpha$-closed in (Z, η). Since g is $\psi\alpha$-totally continuous, $g^{-1}(V)$ is clopen in (Y,σ) which implies that $g^{-1}(V)$ is closed in (Y,σ). Since f is contra $\psi\alpha$-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$-open in (X,τ). Hence g o f is a contra $\psi\alpha$-continuous function.

Proposition 3.2.23

If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a totally $\psi\alpha$-continuous function and $g:(Y,\sigma)\rightarrow(Z, \eta)$ is a completely continuous function, then $g \text{ o } f :(X,\tau) \rightarrow(Z,\eta)$ is a contra $\psi\alpha$-continuous function.

Proof:

Let V be any open set in (Z, η). Since g is completely continuous, $g^{-1}(V)$ is regular open in (Y,σ). By result 1.11, $g^{-1}(V)$ is open in (Y,σ). Since f is totally $\psi\alpha$-continuous, $(g \text{ o } f)^{-1}(V) =f^{-1}(g^{-1}(V))$ is $\psi\alpha$-clopen in (X,τ) which implies that $(g \text{ o } f)^{-1}(V)$ is $\psi\alpha$-closed in (X,τ). Hence g o f is a contra $\psi\alpha$-continuous function.

Proposition 3.2.24

If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a contra continuous function and $g:(Y,\sigma)\rightarrow(Z, \eta)$ is a continuous function, then $g \text{ o } f : (X,\tau) \rightarrow(Z,\eta)$ is a contra $\psi\alpha$-continuous function.

Proof:

Let V be any open set in (Z, η). Since g is continuous, $g^{-1}(V)$ is open in (Y,σ). Since f is contra continuous, $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is closed in (X,τ). By result 1.11,$(g \text{ o } f)^{-1}(V)$ is $\psi\alpha$-closed in (X,τ). Hence g o f is a contra $\psi\alpha$-continuous function.

Remark 3.2.25

The composition of two contra $\psi\alpha$-continuous functions need not be a contra $\psi\alpha$-continuous function as seen from the following example.

Example 3.2.26

Let $X=Y = Z = \{a,b,c\}$, $\tau = \{\phi, \{a\}, \{a,b\}, X\}$ and $\sigma = \{\phi,\{a,b\}, Y\}$,$\eta =\{\phi, \{a\}, Z\}$. Let $f: (X,\tau)\rightarrow(Y,\sigma)$ be a function defined by $f(a) = c$, $f(b) = b$,$f(c) = a$ and $g: (Y,\sigma)\rightarrow(Z, \eta)$

be a function defined by g(a) = c, g(b) = b , g(c) = a. Then the functions f and g are contra $\psi\alpha$-continuous, but their composition g o f : $(X,\tau) \rightarrow (Z,\eta)$ is not contra $\psi\alpha$-continuous, since for the closed set $\{b,c\}$ in (Z,η), $(g \text{ o } f)^{-1}(\{b,c\}) = \{b,c\}$ is not $\psi\alpha$-open in (X,τ).

3.3 Almost contra ψα-continuous Functions

In this section, almost contra $\psi\alpha$-continuous functions are introduced and studied the relationship between newly defined functions with already existing functions in topological spaces.

Definition 3.3.1

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **almost contra ψα-continuous** if $f^{-1}(V)$ is $\psi\alpha$-closed in (X,τ) for every regular open set V of (Y,σ).

Example 3.3.2

Let $X=Y=\{a,b,c\}$, $\tau=\{\phi,\{a\},\{a,b\},X\}$ and $\sigma=\{\phi,\{a\},\{b\},\{a,b\},Y\}$. Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be a function defined by f(a) = c, f(b) = b, f(c) = a. Then f is a almost contra $\psi\alpha$-continuous function.

Theorem 3.3.3

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is almost contra $\psi\alpha$-continuous if and only if the inverse image of every regular open subset of (Y,σ) is $\psi\alpha$-closed in (X,τ) .

Proof:

(Necessity): Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be almost contra $\psi\alpha$-continuous. Let V be any regular open set in (Y,σ). Then Y-V is regular closed in (Y,σ). Since f is almost contra $\psi\alpha$-continuous, $f^{-1}(Y - V) = X - f^{-1}(V)$ is $\psi\alpha$-open in (X,τ) which implies that $f^{-1}(V)$ is $\psi\alpha$-closed in (X,τ).

(Sufficiency): Let U be any regular closed set in (Y,σ). Then Y-U is regular open in (Y,σ). By assumption, f^{-1}(Y -U) = X - f^{-1}(U) is $\psi\alpha$-closed in (X,τ) which implies that f^{-1}(U) is $\psi\alpha$-open in (X,τ). Hence f is almost contra $\psi\alpha$-continuous.

Proposition 3.3.4

Every contra $\psi\alpha$-continuous function is a almost contra $\psi\alpha$- continuous function but not conversely.

Proof:

Let V be any regular open set in (Y,σ). By result 1.11, V is open in (Y,σ). Since f is contra $\psi\alpha$-continuous, f^{-1}(V) is $\psi\alpha$-closed in (X,τ). Hence f is almost contra $\psi\alpha$-continuous.

Example 3.3.5

Let X = Y = {a,b,c}, τ = {ϕ,{a},{b},{a,b},X}and σ = {ϕ,{a},{b},{a,b},{a,c},Y}. Let f: (X,τ)→(Y,σ) be the identity function . Then f is almost contra $\psi\alpha$-continuous but not contra $\psi\alpha$-continuous, since for the open set {a,b} in (Y,σ), f^{-1}({a,b}) = {a,b} is not $\psi\alpha$-closed in (X,τ).

Proposition 3.3.6

Every contra continuous function is a almost contra $\psi\alpha$-continuous function but not conversely.

Proof:

Let V be any regular open set in (Y,σ). By result 1.11, V is open in (Y,σ). Since f is contra continuous, f^{-1}(V) is closed in (X,τ). By result 1.11, f^{-1}(V) is $\psi\alpha$-closed in(X,τ). Hence f is almost contra $\psi\alpha$-continuous.

Example 3.3.7

Let $X=Y=\{a,b,c\}$, $\tau=\{\phi,\{a\},\{a,b\},X\}$ and $\sigma=\{\phi,\{a\},\{b\},\{a,b\},Y\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a function defined by $f(a) = c$, $f(b) = b$, $f(c) = a$. Then f is almost contra ψα-continuous but not contra continuous, since for the open set $\{b\}$ in (Y,σ), $f^{-1}(\{b\}) = \{b\}$ is not closed in (X,τ).

Proposition 3.3.8

Every totally ψα-continuous function is a almost contra ψα-continuous function but not conversely.

Proof:

Let V be any regular open set in (Y,σ). By result 1.11, V is open in (Y,σ). Since f is totally ψα- continuous, $f^{-1}(V)$ is ψα- clopen in (X,τ) which implies that $f^{-1}(V)$ is ψα- closed in (X,τ). Hence f is almost contra ψα-continuous.

Example 3.3.9

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{b\},\{a,b\},\{a,c\},X\}$ and $\sigma = \{\phi,\{a\},\{b\},\{a,b\},Y\}$. Let $f: (X,\tau)\rightarrow(Y,\sigma)$ be a function defined by $f(a) = c$, $f(b) = b$, $f(c) = a$. Then f is almost contra ψα-continuous but not totally ψα-continuous, since for the open set $\{a\}$ in(Y,σ), $f^{-1}(\{a\}) = \{c\}$ is not ψα-clopen in (X,τ).

Proposition 3.3.10

Every ψα-totally continuous function is a almost contra ψα-continuous function but not conversely.

Proof:

Let V be any regular open set in (Y,σ). By result 1.11, V is open in (Y,σ) which implies that V is ψα-open in (Y,σ). Since f is ψα-totally continuous, $f^{-1}(V)$ is clopen in (X,τ). By result 1.11, $f^{-1}(V)$ is ψα- clopen in (X,τ) which implies that $f^{-1}(V)$ is ψα-closed in (X,τ). Hence f is almost contra ψα-continuous.

Example 3.3.11

Let X =Y ={a,b,c},τ ={ϕ,{a},{a,b},X} and σ ={ϕ,{a},{b},{a,b},Y}. Let f: (X,τ)→(Y,σ) be a function defined by f(a) = c, f(b) = b, f(c) = a. Then f is almost contra ψα- continuous but not ψα- totally continuous, since for the ψα-open set {a,c} in (Y,σ), f^{-1}({a,c}) = {a,c} is not clopen in (X,τ).

Proposition 3.3.12

Every totally continuous function is a almost contra ψα-continuous function but not conversely.

Proof:

Let V be any regular open set in (Y,σ). By result 1.11, V is open in (Y,σ). Since f is totally continuous, $f^{-1}(V)$ is clopen in (X,τ). By result 1.11, $f^{-1}(V)$ is ψα- clopen in (X,τ) which implies that $f^{-1}(V)$ is ψα-closed in (X,τ). Hence f is almost contra ψα-continuous.

Example 3.3.13

Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},X} and σ = {ϕ,{a},{b,c},Y}. Let f: (X,τ)→(Y,σ) be the identity function. Then f is almost contra ψα- continuous but not totally continuous, since for the open set {a} in (Y,σ), f^{-1}({a}) = {a} is not clopen in (X,τ).

Proposition 3.3.14

If f: $(X,\tau)\to(Y,\sigma)$ is a almost contra $\psi\alpha$-continuous function and $g:(Y,\sigma)\to(Z,\eta)$ is a completely continuous function, then g o f $:(X,\tau)\to(Z,\eta)$ is a almost contra $\psi\alpha$-continuous function.

Proof:

Let V be any regular open set in (Z,η). By result 1.11, V is open in (Z,η). Since g is completely continuous function, $g^{-1}(V)$ is regular open in (Y,σ). Since f is almost contra $\psi\alpha$- continuous, $(g\ o\ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$-closed in (X,τ). Hence g o f is a almost contra $\psi\alpha$- continuous function.

Proposition 3.3.15

If $f:(X,\tau)\to(Y,\sigma)$ is a contra $\psi\alpha$-continuous function and $g:(Y,\sigma)\to(Z,\eta)$ is a almost continuous function, then g o f $:(X,\tau)\to(Z,\eta)$ is a almost contra $\psi\alpha$-continuous function.

Proof:

Let V be any regular open set in (Z,η). Since g is almost continuous, $g^{-1}(V)$ is open in (Y,σ). Since f is contra $\psi\alpha$ continuous, $(g\ o\ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$-closed in (X,τ). Hence g o f is a almost contra $\psi\alpha$-continuous function.

Proposition 3.3.16

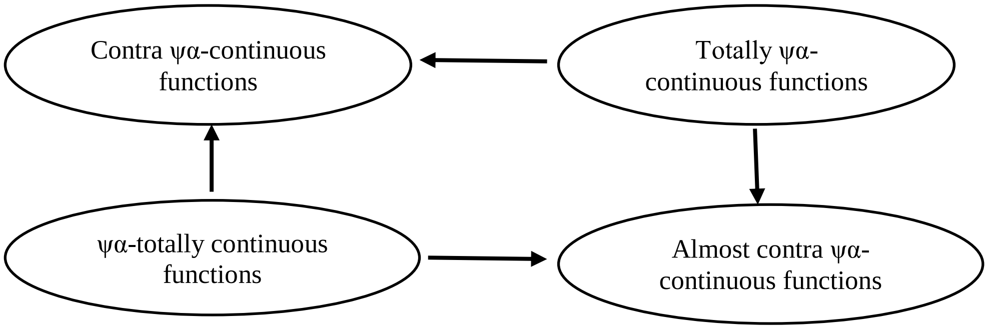
If $f:(X,\tau)\to(Y,\sigma)$ is a almost contra $\psi\alpha$-continuous function and $g:(Y,\sigma)\to(Z,\eta)$ is a completely continuous function, then g o f $:(X,\tau)\to(Z,\eta)$ is a contra $\psi\alpha$-continuous function.

Proof:

Let V be any open set in (Z, η). Since g is completely continuous, $g^{-1}(V)$ is regular open in (Y,σ). Since f is almost contra ψα-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψα-closed in (X,τ). Hence g o f is a contra ψα-continuous function.

Remark 3.3.17

The above results are depicted in the following diagram.



Chapter 4

ψα-Irresolute Functions in Topological Spaces

4.1 Introduction

Crossley and Hildebrand [1972] introduced the concept of irresolute functions in topological spaces. Maheshwari and Thakur [1980] introduced α-irresolute functions in topological spaces. Ramya and Parvathi [2013] introduced the notion of ψg-irresolute functions in topological spaces.

In this chapter we introduce ψα-irresolute functions in topological spaces.

4.2 ψα-irresolute functions

In this section, the stronger form ψα-continuous functions, namely ψα-irresolute functions is introduced and their properties are analysed. It is shown that composition of two ψα-irresolute functions is also a ψα-irresolute function.

Definition 4.2.1

A function $f: (X,\tau) \to (Y,\sigma)$ is called **ψα-irresolute** if $f^{-1}(V)$ is ψα-closed in (X,τ) for every ψα-closed V of (Y,σ).

Example 4.2.2

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a\},Y\}$. Let $f: (X,\tau)\to(Y,\sigma)$ be the identity function. Then f is a ψα-irresolute function.

Theorem 4.2.3

A function f: (X,τ) →(Y,σ) is called ψα-irresolute if and only if f^{-1}(V) is ψα-open in (X,τ) for every ψα-open set V of (Y,σ).

Proof:

(Necessity): Let f: (X,τ)→(Y,σ) be a ψα-irresolute function. Let V be any ψα-open set in (Y,σ). Then Y-V is ψα-closed in (Y,σ). Since f is ψα-irresolute, f^{-1}(Y-V) = X-f^{-1}(V) is ψα-closed in (X,τ) which implies that f^{-1}(V) is ψα-open in (X,τ). Hence f^{-1}(V) is ψα-open in (X,τ).

(Sufficiency): Assume that f^{-1}(V) is ψα-open in (X,τ) for each ψα-open set V in (Y,σ). Let U be any ψα-closed set in (Y,σ). Then Y-U is ψα-open in (Y,σ). By assumption, f^{-1}(Y -U) = X-f^{-1}(U) is ψα-open in (X,τ) which implies that f^{-1}(U) is ψα-closed in (X,τ). Hence f is ψα-irresolute.

Proposition 4.2.4

If a function f : (X,τ)→(Y,σ) is ψα-irresolute, then for every subset A of (X,τ) such that f(A) is ψα-closed in (Y,σ), f(ψαcl(A))⊆ ψαcl(f(A)).

Proof:

Let A be a subset of (X,τ) such that f(A) is ψα-closed in (Y,σ). Then ψαcl(f(A)) is ψα-closed in (Y,σ). Since f is ψα-irresolute, f^{-1}(ψαcl(f(A))) is ψα-closed in (X,τ). Now A ⊆ f^{-1}(f(A)) ⊆ f^{-1}((ψαcl(f(A)))). Therefore ψαcl(A) ⊆ f^{-1}((ψαcl(f(A)))) and hence f(ψαcl(A)) ⊆ f(f^{-1}((ψαcl(f(A)))) ⊆ ψαcl(f(A)).

Proposition 4.2.5

If a function f: (X,τ)→(Y,σ) is ψα-irresolute, then for every ψα-closed set B⊆ Y, ψαcl(f^{-1} (B)) ⊆ f^{-1}(ψαcl(B).

Proof:

Let B be a ψα-closed set in (Y,σ). Then ψαcl(B) is ψα-closed in (Y,σ). Since f is ψα-irresolute, f^{-1}(ψαcl(B)) is ψα-closed in (X,τ). Since B ⊆ ψαcl(B), f^{-1}(B) ⊆ f^{-1}(ψαcl(B)). By definition of ψα-closure, ψαcl(f^{-1}(B)) ⊆ f^{-1}(ψαcl(B)).

Proposition 4.2.6

If f: (X,τ)→(Y,σ) is a ψα-irresolute function, then it is a ψα-continuous function but not conversely.

Proof:

Let V be any closed set in (Y,σ). By result 1.11, V is ψα-closed in (Y,σ). Since f is ψα-irresolute, f^{-1}(V) is ψα-closed in (X,τ). Hence f is a ψα-continuous function.

Example 4.2.7

Let X = Y = {a,b,c}, τ ={ϕ,{a},{b},{a,b},X} and σ={ϕ,{a},{b,c},Y}. Let f: (X,τ)→(Y,σ) be a function defined by f(a) =b, f(b) = a, f(c) = c. Then f is ψα-continuous but not ψα-irresolute, since for the ψα-closed set {a,b} in (Y,σ), f^{-1}({a,b}) = {a,b} is not ψα-closed in (X,τ).

Remark 4.2.8

Irresolute function is independent from ψα-irresolute function as seen from the following examples.

Example 4.2.9

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a,b\},Y\}$. Let $f: (X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f is irresolute but not $\psi\alpha$-irresolute, since for the $\psi\alpha$-closed set $\{a,c\}$ in (Y,σ), $f^{-1}(\{a,c\}) = \{a,c\}$ is not $\psi\alpha$-closed in (X,τ).

Example 4.2.10

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{b,c\},X\}$ and $\sigma = \{\phi,\{a\},\{b\},\{a,b\},Y\}$. Let $f: (X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f is $\psi\alpha$-irresolute but not irresolute, since for the semi-closed set $\{b\}$ in (Y,σ), $f^{-1}(\{b\}) = \{b\}$ is not semi-closed in (X,τ).

Remark 4.2.11

α-irresolute function is independent from $\psi\alpha$-irresolute function as seen from the following examples.

Example 4.2.12

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a,b\},Y\}$. Let $f: (X,\tau)\rightarrow(Y,\sigma)$ be a function defined by $f(a)=a$, $f(b)=c$, $f(c)=b$. Then f is α-irresolute but not $\psi\alpha$-irresolute, since for the $\psi\alpha$-closed set $\{a,c\}$ in (Y,σ), $f^{-1}(\{a,c\}) = \{a,b\}$ is not $\psi\alpha$-closed in (X,τ).

Example 4.2.13

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{b,c\},X\}$ and $\sigma = \{\phi,\{a\},Y\}$. Let $f: (X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f is $\psi\alpha$-irresolute but not α-irresolute, since for the α-closed sets $\{b\}$ and $\{c\}$ in (Y,σ), $f^{-1}(\{b\}) = \{b\}$and $f^{-1}(\{c\}) = \{c\}$ is not α-closed in (X,τ).

Remark 4.2.14

ψg-irresolute function is independent from $\psi\alpha$-irresolute function as seen from the following examples.

Example 4.2.15

Let $X = Y = \{a,b,c\}, \tau = \{\phi,\{a\},X\}$ and $\sigma = \{\phi,\{a,b\},Y\}$. Let $f: (X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f is ψg-irresolute but not $\psi\alpha$-irresolute, since for the $\psi\alpha$-closed set $\{a,c\}$ in (Y,σ), $f^{-1}(\{a,c\}) = \{a,c\}$ is not $\psi\alpha$-closed in (X,τ).

Example 4.2.16

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a\},Y\}$. Let $f: (X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f is $\psi\alpha$-irresolute but not ψg-irresolute, since for the ψg-closed set $\{a,b\}$ in (Y,σ), $f^{-1}(\{a,b\}) = \{a,b\}$ is not ψg-closed in (X,τ).

Remark 4.2.17

$\psi^{*}\alpha$-irresolute function is independent from $\psi\alpha$-irresolute function as seen from the following examples.

Example 4.2.18

Let $X = Y = \{a,b,c\}, \tau =\{\phi,\{a,b\},X\}$ and $\sigma = \{\phi,\{a\},\{b\},\{a,b\},Y\}$. Let $f: (X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f is $\psi^{*}\alpha$-irresolute but not $\psi\alpha$-irresolute, since for the $\psi\alpha$-closed set $\{a\}$ in (Y,σ), $f^{-1}(\{a\}) = \{a\}$ is not $\psi\alpha$-closed in (X,τ).

Example 4.2.19

Let X = Y = {a,b,c},τ = { ϕ ,{a},{b,c},X }and σ = {ϕ,{a,b},Y}. Let f: (X,τ)→(Y,σ) be the identity function. Then f is ψα-irresolute but not ψ^*α-irresolute, since for the ψ^*α-closed set {a,c} in (Y,σ), f^{-1}({a,c}) = {a,c} is not ψ^*α-closed in (X,τ).

Remark 4.2.20

Totally ψα-continuous function is independent from ψα-irresolute function as seen from the following examples.

Example 4.2.21

Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},X}and σ = {ϕ,{a},{b,c},Y}. Let f: (X,τ)→(Y,σ) be the identity function. Then f is totally ψα-continuous but not ψα- irresolute, since for the ψα-closed set {a,b} in (Y,σ), f^{-1}({a,b}) = {a,b} is not ψα-closed in (X,τ).

Example 4.2.22

Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},X}and σ = {ϕ,{a},{b},{a,b},{a,c},Y}. Let f: (X,τ)→(Y,σ) be the identity function. Then f is ψα-irresolute but not totally ψα-continuous, since for the open set {a,b} in (Y,σ), f^{-1}({a,b}) = {a,b} is not ψα-clopen in (X,τ).

Proposition 4.2.23

Every ψα-totally continuous function is a ψα-irresolute function but not conversely.

Proof:

Let V be any ψα-closed set in (Y,σ). Since f is ψα-totally continuous, $f^{-1}(V)$ is clopen in (X,τ). By result 1.11, $f^{-1}(V)$ is ψα-clopen in (X,τ) which implies that $f^{-1}(V)$ is ψα-closed in (X,τ). Hence f is ψα-irresolute.

Example 4.2.24

Let X = Y = {a,b,c},τ = {ϕ,{a},{a,b},X}and σ = {ϕ,{a},Y}. Let f: (X,τ)→(Y,σ) be the identity function. Then f is ψα-irresolute but not ψα-totally continuous, since for the ψα-closed set{c} in (Y,σ), f^{-1}({c}) = {c} is not clopen in (X,τ).

Remark 4.2.25

The following examples show that contra ψα-continuous function and ψα-irresolute function are independent.

Example 4.2.26

Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},X} and σ = {ϕ,{a},{b,c},Y}. Let f: (X,τ)→(Y,σ) be a function defined by f(a) =b, f(b) = a, f(c) = c. Then f is contra ψα-continuous but not ψα -irresolute, since for the ψα-closed set {a,b} in (Y,σ), f^{-1}({a,b}) = {a,b} is not ψα-closed in (X,τ).

Example 4.2.27

Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},X }and σ = {ϕ,{a},{a,b},{a,c},Y}. Let f: (X,τ)→(Y,σ) be the identity function. Then f is ψα-irresolute but not contra ψα-continuous, since for the open set {a,b} in (Y,σ), f^{-1}({a,b}) = {a,b} is not ψα-closed in (X,τ).

Proposition 4.2.28

If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a ψα-irresolute function and $g:(Y,\sigma)\rightarrow(Z, \eta)$ is a ψα-irresolute function, then $g \circ f :(X,\tau) \rightarrow(Z,\eta)$ is a ψα-irresolute function.

Proof:

Let V be any ψα-closed set in (Z, η). Since g is ψα-irresolute, $g^{-1}(V)$ is ψα-closed in (Y,σ). Since f is ψα-irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψα-closed in (X,τ). Hence g o f is a ψα-irresolute function.

Proposition 4.2.29

If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a ψα-irresolute function and $g:(Y,\sigma)\rightarrow(Z, \eta)$ is a ψα-continuous function, then $g \circ f :(X,\tau) \rightarrow(Z,\eta)$ is a ψα-continuous function.

Proof:

Let V be any closed set in (Z, η). Since g is ψα-continuous, $g^{-1}(V)$ is ψα-closed in (Y,σ). Since f is ψα-irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψα-closed in (X,τ). Hence g o f is a ψα-continuous function.

Proposition 4.2.30

If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a ψα-irresolute function and $g:(Y,\sigma)\rightarrow(Z, \eta)$ is a contra ψα-continuous function, then $g \circ f :(X,\tau) \rightarrow(Z,\eta)$ is a contra ψα-continuous function.

Proof:

Let V be any open set in (Z, η). Since g is contra ψα-continuous, $g^{-1}(V)$ is ψα-closed in (Y,σ). Since f is ψα-irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψα-closed in (X,τ). Hence g o f is a contra ψα-continuous function.

Proposition 4.2.31

If f:(X,τ)→(Y,σ) is a ψα -irresolute function and g:(Y,σ)→(Z, η) is a continuous function, then g o f :(X,τ) →(Z,η) is a ψα-continuous function.

Proof:

Let V be any closed set in (Z, η). Since g is continuous, $g^{-1}(V)$ is closed in (Y,σ). By result 1.11, $g^{-1}(V)$ is ψα-closed in (Y,σ). Since f is ψα-irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψα-closed in (X,τ). Hence g o f is a ψα- continuous function.

Proposition 4.2.32

If f:(X,τ)→(Y,σ) is a ψα-irresolute function and g:(Y,σ)→(Z, η) is a totally ψα- continuous function, then g o f :(X,τ) →(Z,η) is a ψα-continuous function.

Proof:

Let V be any closed set in (Z, η). Since g is totally ψα-continuous, $g^{-1}(V)$ is ψα-clopen in (Y,σ) which implies that $g^{-1}(V)$ is ψα-closed in (Y,σ). Since f is ψα-irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψα-closed in (X,τ). Hence g o f is a ψα - continuous function.

Proposition 4.2.33

If f:(X,τ)→(Y,σ) is a ψα-irresolute function and g:(Y,σ)→(Z, η) is a totally continuous function, then g o f :(X,τ) →(Z,η) is a ψα-continuous function.

Proof:

Let V be any closed set in (Z, η).Since g is totally continuous, $g^{-1}(V)$ is clopen in (Y,σ). By result 1.11, $g^{-1}(V)$ is ψα-clopen in (Y,σ) which implies that $g^{-1}(V)$ is ψα-closed in (Y,σ). Since f is ψα-irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψα-closed in (X,τ). Hence g o f is a ψα-continuous function.

Proposition 4.2.34

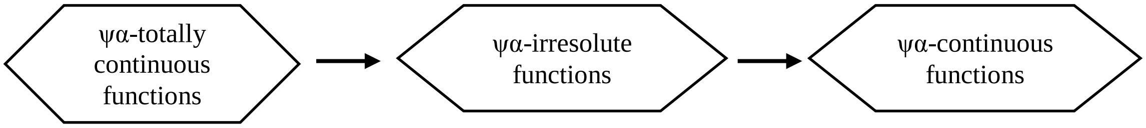
If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a $\psi\alpha$-irresolute function and $g:(Y,\sigma)\rightarrow(Z,\eta)$ is a $\psi\alpha$- totally continuous function, then $g \circ f:(X,\tau)\rightarrow(Z,\eta)$ is a $\psi\alpha$- continuous function.

Proof:

Let V be any closed set in (Z,η). By result 1.11, V is $\psi\alpha$-closed in (Z,η). Since g is $\psi\alpha$-totally continuous, $g^{-1}(V)$ is clopen in (Y,σ). By result 1.11, $g^{-1}(V)$ is $\psi\alpha$-clopen in (Y,σ) which implies that $g^{-1}(V)$ is $\psi\alpha$-closed in (Y,σ). Since f is $\psi\alpha$-irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$-closed in (X,τ). Hence $g \circ f$ is a $\psi\alpha$-continuous function.

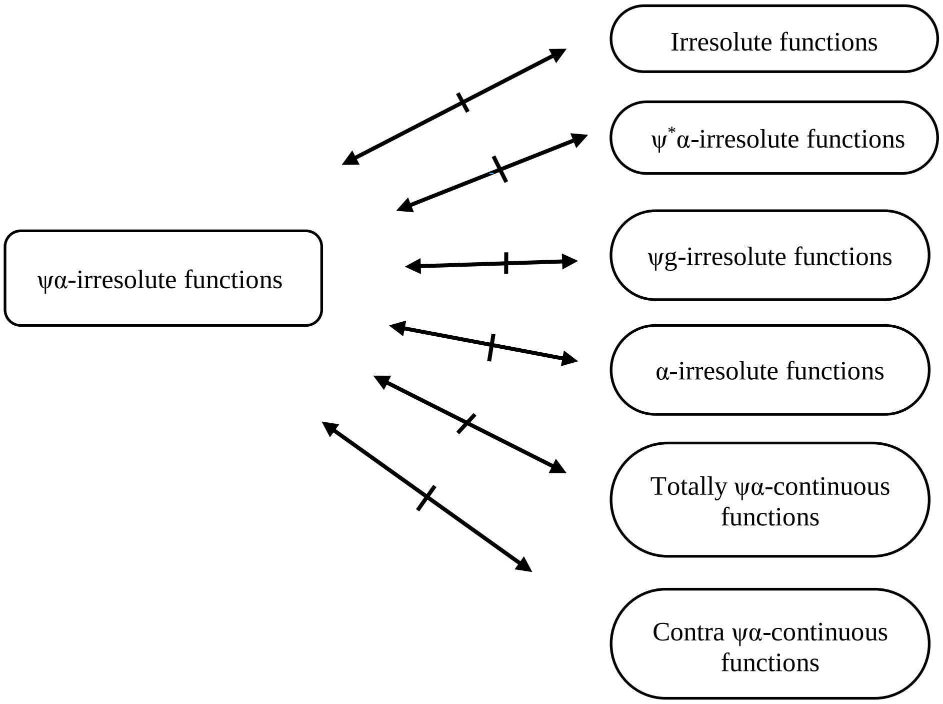
Remark 4.2.35

The above observations are given in the following diagram.



Remark 4.2.36

From the above results we have the following diagram.



Summary and conclusion

Basic definitions in topological spaces are given in chapter one.

In chapter 2, totally ψα-continuous functions and ψα-totally continuous functions in topological spaces are defined and some of their properties and interrelations are analyzed.

Two new types of contra continuous functions called contra ψα-continuous functions and almost contra ψα-continuous functions in topological spaces are introduced and studied their properties in chapter 3.

In chapter 4, a new form of irresolute functions called ψα-irresolute functions in topological spaces are introduced and obtained the interrelations between ψα-irresolute functions with already existing various irresolute functions.

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