



*Review of Literature*

## REVIEW OF LITERATURE

Research on the field of generalized closed sets was developed by many authors in the last two decades. The theory was extensively developed in 1990's several new concepts were studied and investigated.

The initiation of the study of generalized closed sets was done by Levine [42] in 1970 as he considered sets whose closures belong to every open super set. The space in which the concepts of g-closed and closed sets coincide is called  $T_{1/2}$  spaces .

In 1987, Bhattacharya and Lahiri [6] introduced the notion of semi-generalized closed sets by replacing the closure operator in the original Levine's definition by semi-closure operator and by replacing openness of the superset with semi-openness. In 1986 ,Fukutaka introduced and studied generalized closed sets in bitopological spaces . In 1993, Levine's original definition was generalized by Palaniappan and Rao by defining a set  $A$  to be regular generalized closed (briefly rg-closed) if  $\overline{A} \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular open.

In 1989 Ganster and Reilly [30] using the concept of locally closed set they have introduced and studied three different notions of generalized continuity namely LC-irresoluteness, LC-continuity and sub-LC-continuity. A subset  $S$  of a topological space  $X$  is locally closed if it is the intersection of an open and a closed set and also that have discussed some properties and characterizations of these functions. Several examples are provided to illustrate the behaviour of these new classes of functions.

In 1990, Arya and Nour [3] defined the notion of generalized semi-closed sets (gs-closed sets). Although g-closed and sg-closed sets

are independent notions, they both imply gs-closedness and the reverse implications fail to be always true.

In 1991, Sundaram [46] introduced the concept of semi-generalized continuous maps and generalized semi-continuous maps. In 1993, Devi, Maki and Balachandran [49] introduced sg and gs closed maps and studied some of their basic properties. As applications, they showed that under the continuous, gs-closed surjection the image of a normal space is s-normal and that under semi-open continuous, generalized semi-closed surjection the image of a regular space is s-regular. Further, they characterized the class of  $T_{1/2}$ -spaces by using gs-closed sets and semi-closed sets and investigated the relation between the product and  $T_{1/2}$ -spaces.

In 1992, Ganster, Reilly and Vamanamurthy [32] have given a characterization of  $(X, \tau_{\otimes})$ , where  $\tau_{\otimes}$  denotes  $\otimes$  topology of a given topological space  $(X, \tau)$ . Further they discussed the family of locally closed subsets of an arbitrary space.

In 1997, Dunham [25] showed that  $T_{1/2}$ -spaces are precisely the spaces in which singletons are open or closed.

Intensive research in the field of generalized closed sets was done in the past 15 years as the theory was developed by Balachandran, Devi, Maki, Noiri, Ogata, Sundaram, Umehara and Yamamura [21]. Several forms of generalized continuity [5], openness and irresoluteness of functions associated with the generalized closedness in question have been introduced and investigated

In 1998, Julian Dontchev [24] has discussed some recent progress in the study of sg-open sets, sg-compact spaces, N-scattered and related concepts. Using the concept of sg-closed sets the author has introduced

the concept of sg-compact spaces and N-scattered spaces and has generalized some of the related concepts.

In 1998, Gnananbal and Balachandran [33] have introduced a new class of sets called beta locally closed sets and two different notions of generalizations of continuous maps namely beta-LC-continuous and beta-LC-irresoluteness in a topological spaces and discussed some of their properties.

In 1999, Veera Kumar [65] has introduced a new class of sets called semi-pre generalized closed sets, spg-continuity and spg-irresoluteness and discussed their properties. As an application of these sets he has introduced semi pre  $T_{1/4}$  axioms which is strictly weaker than semi pre  $T_{1/2}$  and  $\alpha T_{1/2}$  axioms.

In 2001, Veera Kumar [66] has introduced  $\hat{g}$ -locally closed sets and different notions of generalization of different continuous functions namely  $\hat{G}LC$ -continuity,  $\hat{G}LC^*$ -continuity,  $\hat{G}LC^{**}$ -continuity,  $\hat{G}LC$ -irresoluteness,  $\hat{G}LC^*$ -irresoluteness,  $\hat{G}LC^{**}$ -irresoluteness and sub- $\hat{G}LC^*$ -continuity in topological spaces and obtained characterizations and some interrelation of these maps. These notions fall strictly in between the respective notions of generalization of continuous maps introduced by Gaster and Reilly [24] and Balachandran, Sundaram and Maki [6]. In addition he has defined  $\hat{g}$ -submaximal spaces and proved that pasting lemma holds good for  $\hat{G}LC^{**}$ -continuous functions and  $\hat{G}LC^{**}$ -irresolute functions but not for  $\hat{G}LC^{**}$ -continuous functions. As an application of g-closed sets, he has introduced a new separation axiom  $T_f$ , which is weaker than  $T_b$  and  $T_{1/2}$  axioms.

In 2001 Shafei [59] introduced the notions of  $g$ -regularity and  $g$ -normality in fuzzy topological spaces and obtained some characterizations and several preservation theorems of such spaces.

In 2002, Jiling cao, Maximilian Ganster and Ivan Reilly [36] are studied some lower separation axioms weaker than  $T_1$  using the concept of generalized closed sets due to Levine. Characterization of extremely dis connected spaces and  $sg$ -sub minimal spaces are discussed by using various kinds of generalized cloed sets are discussed.

In 2003, Veera kumar [68] has introduced the concept of  $g^*$ -semi closed sets , $g^*$ s-continuous functions and  $g^*$ s-irresolute functions in topological spaces. As an application of  $g^*$ -semi closed sets he has obtained four new spaces namely  $*T_b$  spaces,  $T_b^*$  spaces, $T_b^{**}$  spaces and  ${}_{\alpha} T_b^*$  spaces and has proved that a space is a  $T_b$  space if and only if it is a  $T_b^*$  and  $*T_b$ .

In 2003, Veera kumar [67] has introduced  $g^{\#}$  - closed sets in topological spaces which is properly placed in between the class of closed sets and the class of  $g$ -closed sets. As applications of  $g^{\#}$ -closed sets, he has introduced  $g^{\#}$ -continuous maps and  $g^{\#}$ -irresolute maps

In 2003, Veera Kumar [69] has introduced  $G^*$ locally closed sets ,  $G^*LC$ -continuity,  $G^*LC^*$ - continuity,  $G^*LC^{**}$ -continuity,  $G^*LC$ -irresoluteness,  $G^*LC^*$ -irresoluteness,  $G^*LC^{**}$ irresoluteness, in topological spaces . Properties and characterization of these functions are analysed.

In 2006, Ravi and Lellis Thivagar [58] have introduced a new type of generalized sets called  $(1, 2)^*$ -semi generalized closed sets and a new class of generalized functions  $(1, 2)^*$ -semi generalized continuous maps. Properties, characterization and the relationship with  $(1, 2)^*$ -g-continuous maps and  $(1, 2)^*$ -g-irresolute map are discussed.

In 2007, Takashi Noiri and Veleriu Popa [54] as a generalization of pre continuous functions have introduced the notion of weakly pre continuous functions in bitopological spaces and obtained several characterizations and studied properties of weakly pre continuous functions.

In 2010, Maragathavalli, Sheik John and Sivaraj [51] have defined  $g_\mu$ -closed sets in generalized topological spaces and  $Tg_\mu$ -closed sets in generalized topological spaces with a hereditary class and discussed characterization and properties of these sets.

In 2010, Navaneeth Krishanan and Sivaraj [52] have defined  $I_{\{rg\}}$ -closed sets and  $I_{\{rg\}}$ -open sets, and discussed the properties and characterization of these sets. Further they have defined  $\vee r$ -sets and  $\wedge r$ -sets and obtained relationship between them. They have also defined a separation axiom stronger than  $T_1$  and obtained various characterizations.

In 2010, Kannan, Narashimhan, Chandrasekhara Rao and Sundaram [38] introduced the concepts of  $(\tau_1, \tau_2)^*$ -semi star generalized closed sets,

$(\tau_1, \tau_2)^*$ -semi star generalized open sets , pairwise semi star generalized  $T_S$ -spaces and a new class of generalized continuous maps and analyzed the basic properties of these concepts and obtained the relationship between  $(1,2)^*$ -s\*g continuous maps with other continuous maps like  $(1,2)^*$ -g continuous maps,  $(1,2)^*$ -sg continuous maps and  $(1,2)^*$ -gs continuous maps.

The concepts discussed can be extended to fuzzy topological spaces and fuzzy bitopological spaces,