Role of Mathematics in Entrepreneurship and Measuring the Effectiveness of Outbound Open Innovation between Large Organizations and Start-Ups

By

Thasnim Nihar I

(23PMA023)

Supervisor

Dr. C. Antony Crispin Sweety

Thesis Submitted to

Avinashilingam Institute for Home Science and Higher Education for Women

Coimbatore-641043

In Partial Fulfillment of the Requirements for the Degree of

Master of Science in Mathematics

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Signature of the Director

C. [signature] 28/04/25

Signature of the Supervisor

DECLARATION

I declare that the thesis "**Role of Mathematics in Entrepreneurship and Measuring the Effectiveness of Outbound Open Innovation Between Large Organizations and Start-ups**" submitted by me for the degree of **Master of Science (M.Sc.)** is the record of work carried out during the period from December 2024 to May 2025 under the guidance of **Dr. C. Antony Crispin Sweety, B.Ed., M.Sc., M.Phil., Ph.D.,** Assistant Professor, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, and has not formed the basis for the award of any Degree, Diploma, Associateship, Fellowship, Titles in this institute or any other University or other similar institution of Higher Learning.

Thasnim Nihas I
29/4/2025
Signature of the Candidate

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ABSTRACT

Mathematics plays a crucial role in entrepreneurship by enabling informed, strategic decision-making, particularly in today’s fast-paced and uncertain innovation landscape. As innovation increasingly relies on collaborative efforts, especially between large organizations and agile startups, the ability to evaluate and manage such partnerships becomes essential. In Outbound Open Innovation collaborations, mathematical tools provide a structured approach to analyzing complex, multidimensional scenarios. Through the application of Multi-Criteria Decision-Making techniques such as Fuzzy Entropy and Fuzzy Weighted Aggregated Sum Product Assessment, it is possible to quantitatively assess and prioritize various qualitative and uncertain factors influencing the success of these collaborations. These techniques integrate expert knowledge with objective data analysis, transforming subjective judgments into actionable insights. By addressing ambiguity and imprecision, they offer a transparent, consistent, and systematic framework for evaluating innovation partnerships, ultimately guiding stakeholders toward more effective and sustainable entrepreneurial outcomes.

CHAPTER 1

CHAPTER 1

INTRODUCTION

Mathematics is often referred to as the mother of all sciences due to its pervasive role across various fields of study, including science, management, commerce, and economics. Its influence extends to numerous disciplines, providing a foundation for analysis, modeling, and decision-making. The significance of mathematics becomes even more apparent in the context of entrepreneurship, marketing, and salesmanship. To understand the importance of mathematics in these domains, it is essential first to grasp the concept of entrepreneurship. Entrepreneurship is the process of identifying, creating, and pursuing opportunities to develop new products, services, or ventures. Entrepreneurs, the individuals who drive this process, are characterized by their willingness to take on risks and uncertainties in bringing innovations to market. These innovations often manifest in the form of new businesses or startups that aim to solve problems, meet unmet needs, or capitalize on market gaps. Entrepreneurs are motivated by the desire to create value, disrupt existing markets, and drive growth. Their activities are rooted in innovation, focusing on business development and market competitiveness.

In the current business landscape, innovation has moved beyond a purely internal activity and is increasingly integrated into a global and collaborative framework. Digital technologies have played a significant role in fostering open, collaborative approaches to innovation across industries. This shift has led to the emergence of open innovation, which encourages the exchange of knowledge and intellectual property across organizational boundaries.

Open innovation can be categorized into three models: Inbound Open Innovation, where external knowledge from suppliers, customers, and partners is integrated to enhance an organization's competitiveness; Outbound Open Innovation, where organizations share their intellectual property (IP) such as patents and technologies with external companies to foster innovation and expand market influence; and Coupled Open Innovation, a hybrid approach that combines both inbound and outbound strategies, enabling organizations to collaborate with external partners for mutual benefit and sustainability.

In the context of entrepreneurship, large organizations and startups play distinct yet complementary roles. While large organizations possess established resources and market influence, startups bring fresh ideas and agility, often driving disruptive innovation. The dynamic interaction between these two entities is critical to the advancement of open innovation, where startups leverage the expertise and infrastructure of large organizations to scale their innovations, while large firms benefit from the agility and novel perspectives startups offer.

Multi-criteria decision-making (MCDM) techniques are methods used to evaluate and make decisions when there are multiple factors or criteria to consider. In the context of outbound open innovation (OOI) between large organizations and startups, MCDM helps analyze and prioritize various factors to identify the best innovation opportunities and make balanced, informed decisions.

This study employs multi-criteria decision-making (MCDM) techniques, specifically Fuzzy Entropy and Fuzzy Weighted Aggregated Sum Product Assessment (WASPAS), to evaluate the factors influencing outbound open innovation between large organizations and startups. These methods integrate both subjective expert judgments and objective data-driven insights, providing a comprehensive analysis of the complexities in innovation collaboration.

Fuzzy Entropy is a method used to determine the importance (weights) of various qualitative criteria, especially when there is uncertainty in their evaluation. It translates subjective expert opinions into precise numerical values, aiding in a clearer understanding of each factor's significance in collaborative innovation efforts. Fuzzy WASPAS is employed to rank the criteria based on the calculated weights, combining the strengths of the Weighted Sum Model (WSM) and the Weighted Product Model (WPM) to ensure robustness. It effectively handles uncertainty in data, making it suitable for evaluating qualitative inputs. The fuzzy component enables the method to manage imprecise or subjective information, allowing for the accurate prioritization of key criteria that align with collaboration objectives.

Together, these methods provide a reliable and structured evaluation framework, helping to identify the most critical factors that enhance collaboration between large organizations and startups. This optimized decision-making process supports successful innovation outcomes and promotes long-term sustainability within open innovation ecosystems.

ABBREVIATIONS

- WSM - Weighted Sum Model
- WPM - Weighted Product Model
- WASPAS - Weighted Aggregated Sum Product Assessment
- MCDM - Multi-criteria decision-making
- OOI - Outbound Open Innovation
- OI – Open Innovation
- OC - Organizational Commitment
- IE – Innovation Ecosystem
- KT – Knowledge Transfer
- TR – Technology Relevance
- OR – Operational Research
- AHP – Analytic Hierarchy Process
- ANP – Analytic Network Process
- TFN – Triangular Fuzzy Number
- SA – Strongly Agree
- A - Agree
- N - Neutral
- D - Disagree
- SD – Strongly Disagree
- NFDM – Normalized Fuzzy Decision Matrix

1.1 LITERATURE REVIEW

The conceptual foundation of entrepreneurship has long been tied to innovation and economic development, with Joseph A. Schumpeter (1934) being one of the earliest scholars to make this connection. His portrayal of the entrepreneur as an innovator who disrupts existing market structures laid the groundwork for incorporating more structured and quantitative approaches to understanding entrepreneurship. As operations research (OR) emerged in the 1950s through the 1970s, its integration into business and industry decision-making led to a growing recognition of mathematics and modeling tools as essential components in strategic entrepreneurial planning.

The concept of open innovation gained prominence in the 2010s, bringing a paradigm shift in how organizations approached collaborative value creation. Nambisan and Baron (2013) introduced the idea of the innovation ecosystem as a critical context for startup growth, emphasizing the importance of knowledge sharing and collective capability building. Almirall et al. (2014) further developed this by advocating for ecosystems that balance community collaboration and competition, ensuring sustainable innovation. Weiblen and Chesbrough (2015) explored the role of large firms in empowering startups, particularly through openness in technology sharing and incubation initiatives. At the same time, Brunswicker and Vanhaverbeke (2015) examined the benefits of external knowledge sourcing for SMEs, showing how openness to external input can lead to more effective innovation. Supporting this, Prashantham and Kumar (2019) discussed how outbound knowledge transfer by multinational companies can fuel startup innovation, especially when building disruptive solutions.

Knowledge transfer remained a key theme throughout the late 2010s into the 2020s. Olaisen and Revang (2017) emphasized that strategic knowledge sharing creates immense value when embedded into organizational routines. As technological innovation became increasingly central to success, Castillo-Vergara (2020) linked technological capacity to overall innovation performance, highlighting the importance of advanced tools and systems. Barbosa et al. (2020) discussed how coordination and communication serve as essential enablers of knowledge flow within innovation ecosystems. Wang et al. (2021) added that technology relevance enhances openness in innovation networks, ultimately improving outcomes. Similarly, Dall-Orsoletta et al. (2022) highlighted how the exchange of emerging technologies reduces innovation costs while improving scalability.

The evolution of innovation ecosystems was further refined by recent scholars. Fallah (2022) proposed a conceptual model for open innovation ecosystems specific to startups, identifying how structural and strategic factors influence their performance. Cirule and Uvarova (2022) described these ecosystems as catalysts for driving startup-led technological transformation. Giglio et al. (2023) underscored that commitment between organizations leads to cultural transformation and is crucial for innovation success, especially when coupled with trust and sustained collaboration.

In decision-making and performance evaluation, the entropy method has gained popularity as a reliable tool for determining objective weights of criteria. Initially developed by Shannon (1948) in information theory to measure uncertainty, entropy was later adopted for decision-making applications. It quantifies variability in data to assign relative importance to criteria without subjective input. De Gruyter (2022) offered methodological refinements to the entropy weight method, addressing challenges like weight distortion when entropy values approach unity. MDPI (2023) emphasized the relevance of objective evaluation methods in the context of open innovation and sustainable business models, aligning with entropy's core principles. Zhu (2020) supported the entropy method's effectiveness in multi-criteria decision-making, showing its adaptability across domains. Coleman, Fronk, and Valentine (2023) used entropy balancing in a study on private value creation in open innovation, particularly analyzing firms on platforms like GitHub, thus validating entropy's role in real-world data contexts.

Another prominent decision-making method is the Weighted Aggregated Sum Product Assessment (WASPAS), introduced by Zavadskas et al. (2012). It combines the weighted sum and product models for comprehensive analysis, offering flexibility and robustness in multi-criteria decision-making (MCDM). Over time, WASPAS has evolved to handle fuzzy environments where data may be imprecise or subjective. Mardani et al. (2017) surveyed various fuzzy MCDM techniques and emphasized their growing relevance in solving real-world decision problems. Agarwal et al. (2020) applied F-WASPAS in humanitarian logistics, demonstrating its utility in evaluating complex alternatives. Lin et al. (2021) and Rudnik et al. (2021) showed how fuzzy WASPAS improves decision accuracy, particularly when expert judgments are necessary. Turskis et al., (2019) and Panpatil et al., (2022) expanded its use into socio-industrial and high-uncertainty contexts, where traditional methods fall short. Most recently, Sumrit (2022) reviewed applications of fuzzy MCDM methods in innovation policy, reinforcing their critical role in guiding entrepreneurship and innovation strategies.

1.2 OUTLINE OF THE THESIS

Chapter 1 outlines the fundamental concepts of fuzzy set theory, including fuzzy sets, support, singletons, height, and crossover points. It covers basic operations like equality, subset, complement, union, and intersection, and introduces special types such as convex fuzzy sets, fuzzy numbers, and triangular fuzzy numbers. These concepts form the foundation for applying fuzzy logic in the decision-making models used in the thesis.

Chapter 2 highlights how entrepreneurship drives innovation and economic growth, especially in countries like India. It covers key traits of entrepreneurs like risk-taking, innovation, and leadership and emphasizes the importance of operational research. This chapter also shows how mathematical fields such as algebra, calculus, statistics, and fuzzy math help entrepreneurs make smarter decisions, manage risks, and optimize business operations.

Chapter 3 outlines the methodology used to assess the effectiveness of Outbound Open Innovation (OOI) collaborations between startups and large firms. It introduces key factors-Organizational Commitment, Innovation Ecosystem, Knowledge Transfer, and Technology Relevance-each comprising four sub-criteria. Expert evaluations were gathered through a structured questionnaire using linguistic terms, which were then transformed into triangular fuzzy numbers. The study employs the Fuzzy Entropy method to objectively determine the weights of each criterion based on the degree of uncertainty and variation in expert responses. This approach enables a transparent, data-driven assessment of the most influential factors contributing to the success of OOI collaborations.

Chapter 4 introduces the Fuzzy Weighted Aggregated Sum Product Assessment (WASPAS) method, which integrates the strengths of the Weighted Sum Model (WSM) and the Weighted Product Model (WPM) under a fuzzy environment using Triangular Fuzzy Numbers (TFNs). This chapter details the process of constructing the fuzzy decision matrix, normalizing the data, and applying both WSM and WPM to calculate fuzzy weighted scores, followed by a defuzzification step to derive crisp rankings. Fuzzy WASPAS is applied to assess and prioritize outbound open innovation alternatives, enabling a comparative analysis of their effectiveness across multiple criteria. Additionally, the chapter presents the comparative analysis of Fuzzy WASPAS and Ensemble outcomes and provides valuable insights into the effectiveness and reliability of each method, enabling a balanced incorporation of both subjective expert judgments and objective performance data.

1.3 BASIC CONCEPTS

This chapter presents an introduction to our study, highlighting the necessity of flexible sets that play a significant role in the later sections of the thesis.

Definition 1.3.1

Let X be a nonempty set. A fuzzy set A drawn from X is defined as

$$A = \{\langle x, \mu_A(x)\rangle | x \in X\}$$

Where $\mu: X \rightarrow [0,1]$ is the membership function of the fuzzy set A.

Definition 1.3.2

The support of Ã is the crisp set (or nonfuzzy set) of all x ∈ X, such that $\mu_{\tilde{A}}(x) > 0$ and is denoted by S(Ã) or Sup(Ã).

Definition 1.3.3

A fuzzy singleton (or fuzzy point) x_α (or $\tilde{x}$) is a fuzzy set whose support is a single point x ∈ X, with membership function:

$$x_\alpha(y) = \begin{cases} \alpha, & if\ x = y \\ 0, & if\ x \neq y \end{cases}$$

Definition 1.3.4

The height of a fuzzy set Ã (denoted by hgt (Ã)) is the supremum of $\mu_{\tilde{A}}(x)$ over all x ∈X. If hgt (Ã) = 1, then Ã is normal, otherwise it is subnormal, and a fuzzy set may be always normalized by defining the scaled membership function:

$$\mu^*_{\tilde{A}}(x) = \frac{\mu_{\tilde{A}}(x)}{Sup\ \mu_{\tilde{A}}(x)}, \forall\ x \in\ X$$

Definition 1.3.5

The crossover point of a fuzzy set Ã is that point in X, whose grade of membership in Ã is 0.5

Definition 1.3.6

$\tilde{A} = \tilde{B}$ if and only if, $\mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x) \forall$x ∈ X.

Definition 1.3.7

$\tilde{A} \subseteq \tilde{B}$if and only if $\mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)$, ∀x ∈ X.

Definition 1.3.8

$\tilde{A}^C$is the complement of $\tilde{A}$ with membership function $\mu_{\tilde{A}^C}(x) \ = \ 1 - \mu_{\tilde{A}}(x)$,∀ x ∈ X.

Definition 1.3.9

The empty fuzzy set $\tilde{\phi}$ and the universal set X, have the membership function $\mu_{\tilde{A}}(x) = 0$ and$\mu_{\tilde{A}}(x) = 1$, respectively, ∀ x ∈ X**.**

Definition 1.3.10

$\tilde{C} = \tilde{A} \cup \tilde{B}$is a fuzzy set with a membership function:

$$\mu_{\tilde{C}}(x) = min\,\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\},$$

More generally, for any index set J, then $\bigcap_{j \in J} \tilde{A}_j$ is also a fuzzy set of X with membership function:

$$\mu_{\bigcap_{j \in J} \tilde{A}_j}(x) = \underset{i \in J}{inf} \ \mu_{\tilde{A}_j}(x), \forall \ x \in X$$

Definition 1.3.11

$\widetilde{D} = \tilde{A} \cup \tilde{B}$is a fuzzy set with membership function:

$$\mu_{\widetilde{D}}(x) = max\,\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}, \forall \ x \ \in \ X$$

More generally, for any index set J, then $\bigcup_{j \epsilon J} \tilde{A}_j$ is also a fuzzy set of X with membership function:

$$\mu_{\bigcup_{j \epsilon J} \tilde{A}_j}(x) = \underset{i \in J}{sup} \ \mu_{\tilde{A}_j}(x), \forall \ x \in X$$

Definition 1.3.12

A fuzzy subset $\tilde{A}$ of ℝ is said to be convex if:

$$\mu_{\tilde{A}}(\lambda x_1 \ + \ (1 - \lambda)x_2\,) \geq \ min\,\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}$$

for all $x_1, x_2 \in \mathbb{R}$, and all $\lambda \ \in \ [0,1]$.

Definition 1.3.13

A Fuzzy number is a generalization of a regular, real number, it refers to a connected set of possible values, where each possible value has its own weight between 0 and 1. A fuzzy number is thus a special case of a convex, normalized fuzzy set of the real line.

Definition 1.3.14

A fuzzy number $\tilde{A}$ = (a, b, c) is called a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a\,; \\ \dfrac{x-a}{b-a}, & a \le x \le b\,; \\ \dfrac{c-x}{c-b}, & b \le x \le c\,; \\ 0, & x > c, \end{cases}$$

CHAPTER 2

Chapter 2

ROLE OF MATHEMATICS IN ENTREPRENEURSHIP

Mathematics is essential in entrepreneurship as it helps entrepreneurs make smart decisions, improve their operations, and manage risks. It is used for things like calculating finances, planning strategies, and analyzing data. By using math, entrepreneurs can predict future trends, solve business problems, and make decisions that can lead to business success.

2.1 EXPERT INSIGHTS ON INNOVATION AND ENTREPRENEURSHIP

Entrepreneurship plays a big role in helping countries grow their economies, create new ideas, and provide jobs. In developing countries like India, starting a business is not just about earning money - it's also a way to bring positive change to society.

Some of the more important identifications of entrepreneurship by distinguished authors:

- Entrepreneurship involves the organization and combination of the factors of production into a productive entity. The entrepreneur is more than a manager. He is an innovator and promoter as well. (Frantz)
- Entrepreneurship is the capacity to take risks, the ability to organize, and the desire to diversify and make innovations in the enterprise. (Stepanek)
- Entrepreneurship is a form of social decision-making performed by economic innovators. (Robert Lamp)
- Entrepreneurship is an innovation function. It is a leadership rather than an ownership. (Schumpeter)
- Entrepreneurship can be described as a creative and innovative response to the environment. (Rao and Mehta)
- It can note innovativeness an urge to take risks in the face of industrialization and intuition, i.e. a capacity to see things in a way that afterward proves to be true. (V. R. Gaikwad)

The ability to organize is the most crucial quality of an entrepreneur, according to our analysis of the definitions of entrepreneurship given above. This organizing skill must be connected to output. An entrepreneur must make sure that the production variables are arranged and combined with a productive entity. Of course, studying operational research is the most effective way to accomplish this. In industry, business, government, and military, operational research is the application of scientific approaches to complex problems that arise in the direction and management of huge systems of people, equipment, materials, and money.

The distinctive approach is to develop a scientific model of the system, incorporating measurements of factors such as chance and risk, with which to predict and compare the outcomes of alternative decisions, strategies, or controls.

2.2 CHARACTERISTICS OF AN ENTREPRENEUR

a. Ability to Take Risks

One of the defining traits of an entrepreneur is the willingness to take calculated risks. Entrepreneurs frequently operate in environments filled with uncertainty, where the outcomes of their decisions are not guaranteed. Despite this, they must make bold and informed choices that could potentially lead to significant gains or losses. Unlike mere innovators, entrepreneurs not only generate ideas but also implement them by organizing resources, including human labor, and bearing the risks involved in the pursuit of new ventures.

b. Ability to Innovate

Innovation is arguably the most vital tool in an entrepreneur's arsenal. An entrepreneur is fundamentally an innovator - someone who brings new ideas, products, or methods into the economic system. Entrepreneurs innovate by creating new products, improving existing ones, finding alternative resources, or exploring new markets, all of which contribute to their success.

c. Managerial Skills and Leadership Qualities

To successfully transform an idea into a viable business, an entrepreneur must possess strong managerial skills and effective leadership qualities. This involves planning, coordinating, guiding teams, and controlling business operations to ensure organizational goals are met. Leadership allows entrepreneurs to inspire and guide their teams, while managerial competence ensures that resources are utilized efficiently.

d. A Strong Desire for High Achievement

Entrepreneurs are typically driven by an intense desire to succeed and accomplish ambitious goals. This desire for high achievement is often reflected in their action plans, where long-term vision and personal ambition take precedence. It builds strength, patience, and the will to keep going despite difficulties, all to achieve high standards and success.

e. Organizational Capability

Another key attribute of an entrepreneur is the capacity to organize and manage various components of a business effectively. Entrepreneurs are natural planners who are adept at bringing together resources, people, and processes to achieve specific objectives. They are skilled in structuring workflows, allocating tasks, and ensuring seamless coordination across departments such as procurement, production, and marketing. Organizational capacity is essential for transforming ideas into functioning businesses that operate efficiently and sustainably.

f. Inquisitiveness and Curiosity

Entrepreneurs are inherently inquisitive and possess a strong desire to learn. This curiosity drives them to continuously seek out new knowledge, explore emerging trends, and understand market dynamics. Their eagerness to learn not only keeps them informed but also enables them to innovate, adapt, and stay ahead in a competitive business environment. This trait ensures that entrepreneurs remain open to new ideas and are always looking for ways to improve and grow.

2.3 APPLICATION OF VARIOUS MATHEMATICAL DOMAINS IN ENTREPRENEURSHIP

Entrepreneurship involves strategic decision-making, risk management, and efficient resource allocation, all of which can be significantly enhanced by the application of various mathematical domains. Here are the few mathematical domains that contribute to entrepreneurship:

1. Algebra
2. Calculus
3. Probability Theory
4. Statistics
5. Fuzzy Mathematics

The following is a comprehensive exploration of how various mathematical fields contribute to entrepreneurship:

ALGEBRA

Algebra forms the foundation of many entrepreneurial calculations, helping entrepreneurs, model financial situations like determining profit margins, calculating costs, and forecasting revenues. It is also used to analyze market equilibrium, set pricing strategies, and understand the relationships between various business parameters. For example, entrepreneurs use algebra to solve profit-loss equations, set price points to achieve target margins, and balance supply and demand equations, ensuring that business decisions are grounded in mathematical precision.

CALCULUS

Calculus, particularly differential calculus, helps entrepreneurs optimize business operations by analyzing rates of change and making informed decisions to maximize profit or minimize costs. By understanding how small changes in pricing, production, or investment affect overall outcomes, entrepreneurs can use calculus to determine the optimal level of production or analyze cost functions to minimize operational expenses. Additionally, integrating functions over time allows for the modeling of cumulative costs, revenues, or profits, ensuring businesses remain efficient and profitable.

PROBABILITY THEORY

Probability theory is essential for entrepreneurs to assess risk and uncertainty, allowing them to make decisions based on the likelihood of various outcomes. By understanding probabilities, entrepreneurs can evaluate investment risks, forecast future trends, and develop strategies to mitigate uncertainty. For example, probability theory can be used to estimate the success probability of a new product launch, predict customer behavior, or assess financial risks in a volatile market, enabling entrepreneurs to make informed, data-driven decisions.

STATISTICS

Statistics provides essential tools for analyzing data, identifying trends, and making informed decisions based on empirical evidence. Entrepreneurs use statistics in market research, customer surveys, and performance analysis to gain insights into customer preferences, optimize marketing strategies, and evaluate overall business performance. For instance, entrepreneurs can analyze sales data to predict demand, run hypothesis tests to assess market assumptions, or segment customers based on purchasing behavior, ensuring that business decisions are grounded in reliable data and statistical analysis.

FUZZY MATHEMATICS

Fuzzy mathematics helps entrepreneurs make decisions when dealing with uncertain or imprecise information. In many business scenarios, data may not be entirely precise, and fuzzy logic allows entrepreneurs to handle this vagueness and make decisions based on approximate values rather than exact numbers. For instance, fuzzy mathematics can be used to evaluate customer satisfaction or market potential where exact values are unknown or mange inventory based on fuzzy demand forecasts.

These mathematical domains, when applied to entrepreneurship, empower entrepreneurs to navigate uncertainty, optimize processes, and make decisions that enhance the success and growth of their ventures. By leveraging these tools, entrepreneurs can improve efficiency, reduce risks, and achieve better outcomes in a highly competitive market.

CHAPTER 3

CHAPTER 3

APPLICATION OF THE FUZZY ENTROPY-BASED WEIGHTING APPROACH IN OUTBOUND OPEN INNOVATION EVALUATION

Entrepreneurship is about recognizing opportunities, taking calculated risks, and creating value through innovation, often in environments filled with uncertainty and competition. Entrepreneurs seek to disrupt existing markets, develop novel products or services, and drive growth. In the context of outbound open innovation (OOI), entrepreneurial spirit plays a crucial role as startups and large organizations come together to collaborate and enhance their innovation capabilities.

Outbound Open Innovation (OOI) is a collaborative strategy where organizations share internal knowledge, technologies, or innovations with external partners to create mutual value. This approach contrasts with traditional, closed innovation models by encouraging openness and external collaboration. In OOI partnerships, startups-young, agile companies known for innovation and adaptability-often team up with large organizations, which possess established structures, resources, and market reach. Such collaborations allow startups to scale their ideas, while large firms can accelerate innovation by leveraging the startups' fresh perspectives and technologies. This collaboration benefits both parties by combining entrepreneurial innovation with the resources and capabilities of larger firms. The success of such collaborations depends on several critical factors:

I. Organizational Commitment (OC):

Organizational Commitment refers to the mutual dedication and willingness of both startups and large organizations to actively engage in collaboration. It involves allocating sufficient time, financial investment, personnel, and strategic focus to ensure the partnership thrives. A high level of commitment builds trust, enhances communication, and builds common goals that support long-term success.

II. Knowledge Transfer (KT):

Effective knowledge transfer is critical for turning collaboration into real-world innovation. It involves the smooth exchange, absorption, and utilization of technical knowledge, business practices, market insights, and innovation strategies between the two parties. Knowledge-sharing mechanisms help avoid misunderstandings, prevent repeated efforts, and improve problem-solving in the partnership.

III. Innovation Ecosystem (IE):

This includes the broader context in which the collaboration takes place, including access to funding, mentorship, regulatory support, technological infrastructure, and innovation networks. A healthy ecosystem helps both startups and corporations adapt quickly, try new ideas, and access the resources needed for innovation to grow. It supports their growth and success together.

IV. Technology Relevance (TR):

The success of the collaboration also depends on how well the technologies being developed or exchanged fit the strategic objectives, operational needs, and future vision of both partners. Relevant technologies must not only be innovative but also applicable and scalable within the existing systems and markets. High technology relevance ensures the partnership delivers real value and supports sustainable growth for both sides.

These factors are crucial in determining the success and sustainability of outbound open innovation collaborations.

This study utilized a purposive sampling approach to select a panel of experts with direct involvement in startup-corporate innovation collaborations. The sample included individuals from academia, startup founders, innovation managers from large corporations, and professionals actively engaged in open innovation practices. These experts were chosen for their specialized knowledge and hands-on experience relevant to evaluating the effectiveness of outbound open innovation (OOI) initiatives. Data collection was conducted through a structured questionnaire developed to capture expert judgments across multiple criteria influencing OOI. Respondents assessed each criterion using a linguistic scale ranging from "Strongly Agree" to "Strongly Disagree," which was subsequently converted into triangular fuzzy numbers to facilitate analysis using the Fuzzy ENTROPY and WASPAS methods.

ANALYTICAL TOOLS AND TECHNIQUES USED:

The main tools used in this research are fuzzy MCDM techniques, specifically:

- Fuzzy Entropy Method: To objectively determine the weights of each decision criterion based on variability in expert evaluations.

- Fuzzy WASPAS Method: To rank the importance and effectiveness of different factors, combining both additive and multiplicative models for more robust decision-making.
- Additionally, triangular fuzzy numbers are used to convert linguistic evaluations into quantifiable inputs.

3.1 MULTIPLE CRITERIA DECISION MAKING (MCDM)

In today's dynamic and interconnected world, decision-makers often face complex problems involving the evaluation of multiple alternatives across several frequently conflicting criteria. Multiple-criteria decision-making (MCDM), a vital domain of operations research, addresses such challenges by providing structured approaches to support rational and informed decision-making in the presence of trade-offs among diverse objectives. At its core, MCDM aims to identify the most appropriate alternative(s) from a feasible set, evaluated against both quantitative (e.g., cost, efficiency) and qualitative (e.g., strategic fit, innovation potential) criteria. Key MCDM components include goals, criteria, alternatives, and decision-making techniques.

Traditional Multi-criteria decision-making (MCDM) models generally rely on crisp, deterministic inputs—a significant limitation in practical scenarios where decision-makers often express preferences using subjective and imprecise linguistic terms. To address this issue, the Fuzzy Set Theory, introduced by Lotfi Zadeh in 1965, has been integrated with MCDM to develop Fuzzy MCDM (F-MCDM) techniques. These methods effectively manage uncertainty and vagueness in expert judgments by translating qualitative linguistic terms into quantitative representations known as Triangular Fuzzy Numbers (TFNs). In this study, the following fuzzy linguistic scale is adopted:

LINGUISTIC TERMS	TFN (l, m, u)	l	m	u
STRONGLY AGREE-SA	(3.5,4,4.5)	3.5	4	4.5
AGREE-A	(2.5,3,3.5)	2.5	3	3.5
NEUTRAL-N	(1.5,2,2.5)	1.5	2	2.5
DISAGREE-D	(0.67,1,1.5)	0.67	1	1.5
STRONGLY DISAGREE-SD	(1,1,1)	1	1	1

TABLE: 3.1

In this research, Fuzzy Entropy (F-ENTROPY) and Fuzzy WASPAS (F-WASPAS) are integrated to evaluate and rank 16 sub-criteria grouped under four major criteria that influence Outbound Open Innovation (OOI) partnerships between large organizations and startups.

Fuzzy Entropy is applied to determine the weights of the sub-criteria by measuring the degree of uncertainty in expert evaluations, making it suitable for handling imprecise and subjective data. Fuzzy WASPAS, a hybrid multi-criteria decision-making method combining the Weighted Sum Model (WSM) and Weighted Product Model (WPM), is then used to rank the sub-criteria based on the calculated weights. By using fuzzy logic, this combined approach clearly reflects expert opinions and offers a simple, organized way to identify and rank the most important factors that support successful OOI partnerships. The 16 sub-criteria are given below:

Organizational Commitment (OC)

OC1: Participation in product design and development

OC2: Provision of training for faster time-to-market

OC3: Support in solving issues with innovative approaches

OC4: Ethical commitment during product development

Innovation Ecosystem (IE)

IE1: Organizing knowledge-sharing events

IE2: Access to a product innovation ecosystem through co-working

IE3: Provision of working infrastructure (labs, instruments)

IE4: Access to technology experts

Knowledge Transfer (KT)

KT1: Help in product definition lifecycle

KT2: Facilitation of collaboration opportunities

KT3: Training in new technologies

KT4: Co-working to foster innovative products

Technology Relevance (TR)

TR1: Relevance of technology to market needs

TR2: Enablement of disruptive innovation

TR3: Market traction of the product under development

TR4: Gaining a competitive edge through collaboration

By applying F-MCDM, this study seeks to provide startup-oriented insights that help large organizations structure their innovation collaborations more effectively.

3.2 CRITERION WEIGHTS BY FUZZY ENTROPY METHOD

The **Fuzzy Entropy Method** is an extension of the traditional entropy approach used in Multi-criteria decision-making (MCDM), adapted to handle uncertain, imprecise, or subjective data by incorporating fuzzy logic. It is used to objectively determine the weights of criteria based on the degree of variability or uncertainty in expert evaluations, especially when those evaluations are expressed using linguistic terms.

We use this method because it combines the objectivity of entropy with the flexibility of fuzzy logic, making it ideal for analyzing complex decision environments where qualitative and vague data are common such as in evaluating innovation partnerships between startups and large organizations. This ensures more accurate, consistent, and data-driven weight assignments for decision-making.

ALGORITHM for ENTROPY Method:

The calculation process of the entropy objective weighting method is presented step-by-step as follows.

STEP 1: Construction of the Initial Decision Matrix.

The first step is to construct the initial decision matrix, which consists of performance scores of various alternatives with respect to a set of criteria. This matrix is denoted as:

$$X = [x_{ij}]_{m \times n} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}; \; i = 1,2,\ldots,m; \; j = 1,2 \ldots n$$

where x_{ij} is the performance of the i^{th} alternative to the j^{th} criterion, m is the number of alternatives and n is the number of criteria.

STEP 2: Normalization of the Decision Matrix.

To ensure comparability among the different units or scales of the data, the decision matrix is normalized. The normalization is carried out column-wise for each criterion using the following formula:

$$v_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}$$

where v_{ij} means the normalized value of alternative A_i about C_j. x_{ij} denotes the crisp value of alternative AA_i with respect to C_j; m is the total number of evaluated alternatives.

This step transforms all performance values into a dimensionless scale, allowing fair comparisons across criteria.

STEP 3: Calculation of Entropy for Each Criterion.

Once the matrix is normalized, the entropy value for each criterion is computed. Entropy measures the uncertainty or the degree of disorder associated with a criterion. A criterion with high entropy carries less useful information, while low entropy indicates more informative dispersion.

$$e_j = -k \sum_{i=1}^{m} v_{ij} ln(v_{ij})$$

$$e_j = -\frac{1}{\ln(m)} \sum_{i=1}^{m} v_{ij} \ln(v_{ij})$$

where ln (m) is the logarithm based on e, m is the number of alternatives and e_j is [0, 1].

STEP 4: Calculation of the Degree of Diversification d_j.

The degree of diversification (or degree of divergence) reflects the amount of useful information each criterion contributes to the decision-making process. It is calculated as:

$$d_j = 1 - e_j \, , j \in [1, \dots, n]$$

Where d_j represents the divergence of the criterion, indicating how much it deviates from uniformity or maximum entropy.

STEP 5: Computation of the Objective Weights w_j.

Finally, the normalized weights of each criterion are obtained by dividing each diversification value by the total diversification across all criteria. The resulting weights represent the objective importance of each criterion:

$$w_j = \frac{d_j}{\sum_{j=i}^{n} d_j}$$

These objectively derived weights will serve as crucial inputs in the next stage of the decision-making process, particularly when applying aggregation-based methods such as the Weighted Aggregated Sum Product Assessment (WASPAS). By using entropy-derived weights, the analysis becomes more data-driven, reliable, and less susceptible to personal biases.

Weight Determination by Fuzzy ENTROPY

TABLE 3.2.1 (Linguistic Terms for Evaluation Criteria)

CRITERIA	SA	A	N	D	SD
OC1	7	5	2	1	1
OC2	5	4	4	1	2
OC3	4	9	2	0	1
OC4	8	7	1	0	0
IE1	4	7	4	1	0
IE2	5	9	1	0	1
IE3	7	6	3	0	0
IE4	3	7	4	2	0
KT1	1	6	7	1	1
KT2	5	5	5	1	0
KT3	2	9	3	1	1
KT4	6	6	3	1	0
TR1	8	7	1	0	0
TR2	7	7	1	1	0
TR3	2	11	3	0	0
TR4	7	3	5	1	0

STEP 1: Construction of the Initial Decision Matrix.

$$X = [x_{ij}]_{m\times n} = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix}; \; i = 1,2,\dots,m; \; j = 1,2 \dots n$$

TABLE 3.2.2. Initial Decision Matrix

CRITERIA	SA	A	N	D	SD	$\sum_{i=1}^{m} x_{ij}$
OC1	28	15	4	1.0567	1	49.0567
OC2	20	12	8	1.0567	2	43.0567
OC3	16	27	4	0	1	48
OC4	32	21	2	0	0	55
IE1	16	21	8	1.0567	0	46.0567
IE2	20	27	2	0	1	50
IE3	28	18	6	0	0	52
IE4	12	21	8	2.1133	0	43.1133
KT1	4	18	14	1.0567	1	38.0567
KT2	20	15	10	1.0567	0	46.0567
KT3	8	27	6	1.0567	1	43.0567
KT4	24	18	6	1.0567	1	49.0567
TR1	32	21	2	0	0	55
TR2	28	21	2	1.0567	0	52.0567
TR3	8	33	6	0	0	47
TR4	28	9	10	1.0567	0	48.0567

STEP 2: Normalization of the Decision Matrix.

$$v_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}$$

TABLE 3.2.3. Normalized Decision Matrix

CRITERIA	SA	A	N	D	SD
OC1	0.5708	0.3058	0.0815	0.0215	0.0204
OC2	0.4645	0.2787	0.1858	0.0245	0.0465
OC3	0.3333	0.5625	0.0833	0.0000	0.0208
OC4	0.5818	0.3818	0.0364	0.0000	0.0000
IE1	0.3474	0.4560	0.1737	0.0229	0.0000
IE2	0.4000	0.5400	0.0400	0.0000	0.0200
IE3	0.5385	0.3462	0.1154	0.0000	0.0000
IE4	0.2783	0.4871	0.1856	0.0490	0.0000
KT1	0.1051	0.4730	0.3679	0.0278	0.0263
KT2	0.4342	0.3257	0.2171	0.0229	0.0000
KT3	0.1858	0.6271	0.1394	0.0245	0.0232
KT4	0.4892	0.3669	0.1223	0.0215	0.0000
TR1	0.5818	0.3818	0.0364	0.0000	0.0000
TR2	0.5379	0.4034	0.0384	0.0203	0.0000
TR3	0.1702	0.7021	0.1277	0.0000	0.0000
TR4	0.5826	0.1873	0.2081	0.0220	0.0000

STEP 3: Calculation of Entropy for Each Criterion.

$$e_j = -k \sum_{i=1}^{m} v_{ij} ln(v_{ij})$$

$$e_j = -\frac{1}{\ln(m)} \sum_{i=1}^{m} v_{ij} \ln(v_{ij})$$

$$k = \frac{1}{\ln(m)} = \frac{1}{\ln(5)} = \mathbf{0.62134}$$

STEP 4: Calculation of the Degree of Diversification d_j.

$$d_j = 1 - e_j \text{ , } j \in [1, \dots, n]$$

TABLE 3.2.4. Calculated Entropy Value and Degree of Diversification

CRITERIA	e_j	d_j
OC1	0.6517	0.3483
OC2	0.7820	0.2180
OC3	0.6074	0.3926
OC4	0.4991	0.5009
IE1	0.6934	0.3066
IE2	0.5631	0.4369
IE3	0.5901	0.4099
IE4	0.7249	0.2751
KT1	0.7170	0.2830
KT2	0.7119	0.2881
KT3	0.6576	0.3424
KT4	0.6569	0.3431
TR1	0.4991	0.5009
TR2	0.5617	0.4383
TR3	0.5048	0.4952
TR4	0.6456	0.3544
TOTAL		$\sum_{j=i}^{n} d_j$**=5.9337**

STEP 5: Computation of the Objective Weights w_j.

$$w_j = \frac{d_j}{\sum_{j=i}^{n} d_j}$$

TABLE 3.2.5 Calculated Objective Weight

CRITERIA	w_j
OC1	0.0587
OC2	0.0367
OC3	0.0662
OC4	0.0844
IE1	0.0517
IE2	0.0736
IE3	0.0691
IE4	0.0464
KT1	0.0477
KT2	0.0485
KT3	0.0577
KT4	0.0578
TR1	0.0844
TR2	0.0739
TR3	0.0835
TR4	0.0597

Table 3.2.6 Average Weights

CRITERIA	w_j	RANK
OC	0.0615	2
IE	0.0602	3
KT	0.0529	4
TR	0.0754	1

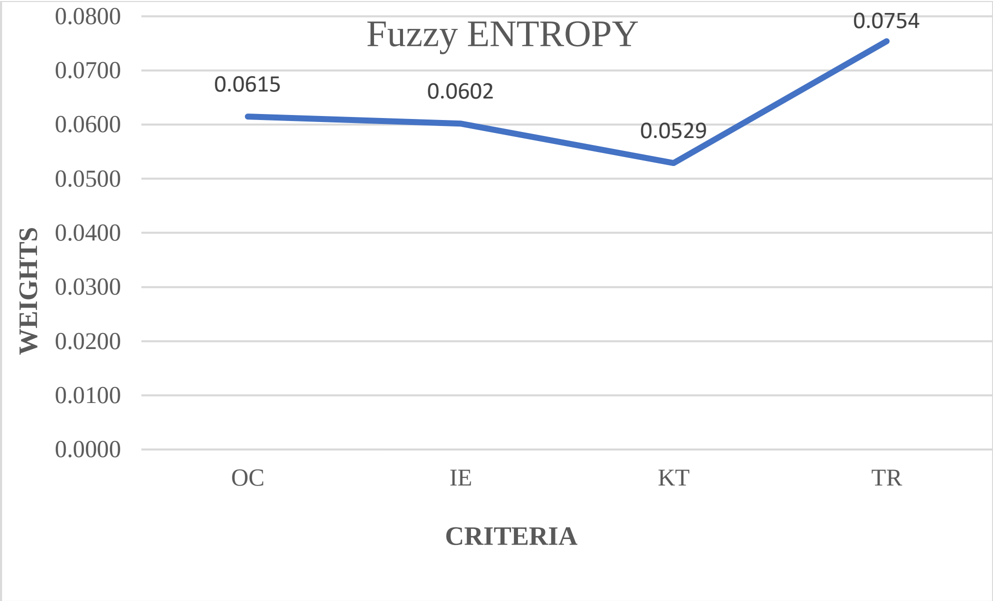

Figure 3.1: F-ENTROPY GRAPH

The F-Entropy method objectively measures the effectiveness of Outbound Open Innovation between big organizations and startups. It highlights **Technology Relevance (TR)** as the most critical factor, reflecting its strong impact on successful collaboration. The fact that **Knowledge Transfer (KT)** came in last suggests that this area has to be improved. The ranking offers precise, fact-based insights into the elements that have the biggest effects on innovative partnerships. This makes it easier for large organizations to prioritize areas where startups need support.

CHAPTER 4

CHAPTER 4

PRIORITIZING OPEN INNOVATION FACTORS THROUGH FUZZY WASPAS METHODOLOGY

4.1 FUZZY WASPAS METHOD

The Fuzzy Weighted Aggregated Sum Product Assessment (Fuzzy WASPAS) method is a powerful hybrid model in Multi-Criteria Decision-Making (MCDM) that combines the strengths of the Weighted Sum Model (WSM) and the Weighted Product Model (WPM). By integrating additive and multiplicative aggregation principles, it offers a balanced and comprehensive evaluation framework. The key advancement in the Fuzzy WASPAS method is its use of fuzzy logic particularly Triangular Fuzzy Numbers (TFNs) to address the uncertainty and imprecision inherent in human judgments. This is especially valuable in scenarios involving strategic innovation, collaboration, and subjective assessments.

TFNs are expressed as a triplet (l, m, u), representing the lower limit, most likely value, and upper limit of an estimate. This fuzzy representation allows for more flexible and accurate modeling of expert opinions. In the Fuzzy WASPAS framework, both the performance ratings of alternatives and the weights of criteria are represented using TFNs. The fuzzy WSM score is calculated by multiplying each criterion's fuzzy weight by the fuzzy rating and summing the results, while the fuzzy WPM score is derived by raising the ratings to the power of their respective fuzzy weights and multiplying the outcomes. These two scores are then integrated using a balancing coefficient (typically $\lambda = 0.5$), ensuring equal emphasis on both components.

To facilitate comparison and decision-making, the aggregated fuzzy scores undergo defuzzification. The centroid method commonly used in fuzzy MCDM converts each fuzzy number into a single crisp value that represents the center of gravity of the triangular distribution. These crisp values serve as the basis for ranking alternatives or criteria in a clear, data-driven manner.

Overall, the Fuzzy WASPAS method demonstrates strong applicability for complex decision-making in uncertain environments. Its integration of expert-driven fuzzy logic with robust mathematical modeling offers a transparent and reliable means of prioritizing strategic criteria, ultimately supporting more informed and effective decision-making in innovation-driven collaborations.

ALGORITHM for Fuzzy WASPAS Method

STEP 1:

Prepare the fuzzy decision matrix using the triangular fuzzy number as shown in Table 3.1. This scale comprises five linguistic expressions ranging from "Strongly Agree" to "Strongly Disagree". Each linguistic variable is paired with corresponding fuzzy numbers to facilitate a nuanced analysis of the solutions.

$$X_{ij} = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix},$$

where *n* = number of criteria (factors), *m* = number of alternatives, and x_{ij}= fuzzy evaluation of the i^{th} alternative against the j^{th} decision criterion.

STEP 2: Normalizing fuzzy decision matrix.

The decision matrix is normalized and the element of the normalized decision matrix is represented by

$$X = [\bar{x}_{ij}]_{m\times n} \,..$$

$$\bar{x}_{ij} = \begin{cases} \dfrac{x_{ij}}{\max x_{ij}} \; if \; \max x_{ij} \; is\, preferable, j = 1, \dots, n; i = 1, \dots, m \\ \dfrac{\min x_{ij}}{x_{ij}} \; if \; \min x_{ij} \; is\, preferable, j = 1, \dots, n; i = 1, \dots, m \end{cases}$$

STEP 3: Compute the weighted normalized fuzzy decision matrix.

(a) Calculate the weighted normalized fuzzy decision matrix $\hat{X}_q$ for the weighted sum model (WSM):

$$\hat{X}_q = \begin{bmatrix} \hat{x}_{11} & \dots & \hat{x}_{1j} & \dots & \hat{x}_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \hat{x}_{i1} & \dots & \hat{x}_{ij} & \dots & \hat{x}_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \hat{x}_{m1} & \dots & \hat{x}_{mj} & \dots & \hat{x}_{mn} \end{bmatrix};$$

$$\hat{x}_{ij} = \bar{x}_{ij} w_j.$$

(b) Calculate the weighted normalized fuzzy decision matrix $\hat{X}_p$ for the weighted product model (WPM):

$$\hat{X}_p = \begin{bmatrix} \bar{x}_{11} & \dots & \bar{x}_{1j} & \dots & \bar{x}_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{x}_{i1} & \dots & \bar{x}_{ij} & \dots & \bar{x}_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{x}_{m1} & \dots & \bar{x}_{mj} & \dots & \bar{x}_{mn} \end{bmatrix};$$

$$\bar{x}_{ij} = (\bar{x}_{ij})^{w_j}.$$

STEP 4: Calculate the values of the optimality function.

(a)The WSM for i^{th} alternative**:**

$$\tilde{Q}_i = \sum_{j=1}^{n} \hat{x}_{ij} \text{ , i=1, …, m}$$

(b)The WPM for i^{th} alternative:

$$\widetilde{P_i} = \prod_{j=1}^{n} \bar{x}_{ij} \text{ , i=1,…,m}$$

Defuzzification of the fuzzy performance evaluation is carried out using the center-of-area method, which is the most practical method.

$$Q_i = \frac{1}{3} (Q_{i\alpha} + Q_{i\beta} + Q_{i\gamma}),$$

$$P_i = \frac{1}{3} (P_{i\alpha} + P_{i\beta} + P_{i\gamma}),$$

STEP 5: The utility function value K_i of the F-WASPAS method is calculated as follows:

$$K_i = \lambda \sum_{j=1}^{n} Q_i + (1-\lambda)\sum_{j=1}^{n} P_i \text{ ; } \lambda = 0, \dots, 1 \text{ ,} 0 \leq K_i \leq 1,$$

where,

$$\lambda = \frac{\sum_{i=1}^{m} P_i}{\sum_{i=1}^{m} Q_i + \sum_{i=1}^{m} P_i}$$

STEP 6: Rank the preference order or select the decision criteria (solution), starting from the highest value, K_i

Ranking the criterion through Fuzzy WASPAS

TABLE 4.1.1. Linguistic Terms for Evaluation Criteria

CRITERIA	SA	A	N	D	SD
OC1	7	5	2	1	1
OC2	5	4	4	1	2
OC3	4	9	2	0	1
OC4	8	7	1	0	0
IE1	4	7	4	1	0
IE2	5	9	1	0	1
IE3	7	6	3	0	0
IE4	3	7	4	2	0
KT1	1	6	7	1	1
KT2	5	5	5	1	0
KT3	2	9	3	1	1
KT4	6	6	3	1	0
TR1	8	7	1	0	0
TR2	7	7	1	1	0
TR3	2	11	3	0	0
TR4	7	3	5	1	0

STEP 1: Preparing the fuzzy decision matrix using the triangular fuzzy number as shown in Table 3.1.

$$X_{ij} = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix},$$

TABLE 4.1.2. Fuzzy Decision Matrix

CRITERIA	SA	A	N	D	SD
OC 1	(24.5,28,31.5)	(12.5,15,17.5)	(3,4,5)	(0.67,1,1.5)	(1,1,1)
OC 2	(17.5,20,22.5)	(10,12,14)	(6,8,10)	(0.67,1,1.5)	(2,2,2)
OC 3	(14,16,18)	(22.5,27,31.5)	(3,4,5)	(0,0,0)	(1,1,1)
OC 4	(28,32,36)	(17.5,21,24.5)	(1.5,2,2.5)	(0,0,0)	(0,0,0)
IE 1	(14,16,18)	(17.5,21,24.5)	(6,8,10)	(0.67,1,1.5)	(0,0,0)
IE 2	(17.5,20,22.5)	(22.5,27,31.5)	(1.5,2,2.5)	(0,0,0)	(1,1,1)
IE 3	(24.5,28,31.5)	(15,18,21)	(4.5,6,7.5)	(0,0,0)	(0,0,0)
IE 4	(10.5,12,13.5)	(17.5,21,24.5)	(6,8,10)	(1.34,2,3)	(0,0,0)
KT 1	(3.5,4,4.5)	(15,18,21)	(10.5,14,17.5)	(0.67,1,1.5)	(1,1,1)
KT 2	(17.5,20,22.5)	(12.5,15,17.5)	(7.5,10,12.5)	(0.67,1,1.5)	(0,0,0)
KT 3	(7,8,9)	(22.5,27,31.5)	(4.5,6,7.5)	(0.67,1,1.5)	(1,1,1)
KT 4	(21,24,27)	(15,18,21)	(4.5,6,7.5)	(0.67,1,1.5)	(0,0,0)
TR 1	(28,32,36)	(17.5,21,24.5)	(1.5,2,2.5)	(0,0,0)	(0,0,0)
TR 2	(24.5,28,31.5)	(17.5,21,24.5)	(1.5,2,2.5)	(0.67,1,1.5)	(0,0,0)
TR 3	(7,8,9)	(27.5,33,38.5)	(4.5,6,7.5)	(0,0,0)	(0,0,0)
TR 4	(24.5,28,31.5)	(7.5,9,10.5)	(7.5,10,12.5)	(0.67,1,1.5)	(0,0,0)

STEP 2: Normalizing fuzzy decision matrix.

$$X = [\bar{x}_{ij}]_{m\times n}\ .$$

$$\bar{x}_{ij} = \begin{cases} \dfrac{x_{ij}}{\max x_{ij}}\ if\ \max x_{ij}\ is\ preferable, j = 1, \dots, n; i = 1, \dots, m \\ \dfrac{\min x_{ij}}{x_{ij}}\ if\ \min x_{ij}\ is\ preferable, j = 1, \dots, n; i = 1, \dots, m \end{cases}$$

TABLE 4.1.3. Normalized Fuzzy Decision Matrix

CRITERIA	SA	A	N	D	SD
OC 1	(0.7778,0.8889,1.0000)	(0.3968,0.4762,0.5556)	(0.0952,0.1270,0.1587)	(0.0213,0.0317,0.0476)	(0.0317,0.0317,0.0317)
OC 2	(0.7778,0.8889,1.0000)	(0.4444,0.5333,0.6222)	(0.2667,0.3556,0.4444)	(0.0298,0.0444,0.0667)	(0.0889,0.0889,0.0889)
OC 3	(0.4444,0.5079,0.5714)	(0.7143,0.8571,1.0000)	(0.0952,0.1270,0.1587)	(0.0000,0.0000,0.0000)	(0.0317,0.0317,0.0317)
OC 4	(0.7778,0.8889,1.0000)	(0.4861,0.5833,0.6806)	(0.0417,0.0556,0.0694)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)
IE 1	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)
IE 2	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)
IE 3	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)
IE 4	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)
KT 1	(0.2857,0.2500,0.2222)	(0.0667,0.0556,0.0476)	(0.0952,0.0714,0.0571)	(1.4925,1.0000,0.6667)	(1.0000,1.0000,1.0000)
KT 2	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)
KT 3	(0.1429,0.1250,0.1111)	(0.0444,0.0370,0.0317)	(0.2222,0.1667,0.1333)	(1.4925,1.0000,0.6667)	(1.0000,1.0000,1.0000)
KT 4	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)
TR 1	(0.7778,0.8889,1.0000)	(0.4861,0.5833,0.6806)	(0.0417,0.0556,0.0694)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)
TR 2	(0.7778,0.8889,1.0000)	(0.5556,0.6667,0.7778)	(0.0476,0.0635,0.0794)	(0.0213,0.0317,0.0476)	(0.0000,0.0000,0.0000)
TR 3	(0.1818,0.2078,0.2338)	(0.7143,0.8571,1.0000)	(0.1169,0.1558,0.1948)	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)
TR 4	(0.7778,0.8889,1.0000)	(0.2381,0.2857,0.3333)	(0.2381,0.3175,0.3968)	(0.0213,0.0317,0.0476)	(0.0000,0.0000,0.0000)

STEP 3: Compute the weighted normalized fuzzy decision matrix.

(a) Calculate the weighted normalized fuzzy decision matrix $\hat{X}_q$ for the Weighted Sum Model (WSM):

$$\hat{X}_q = \begin{bmatrix} \hat{x}_{11} & \cdots & \hat{x}_{1j} & \cdots & \hat{x}_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \hat{x}_{i1} & \cdots & \hat{x}_{ij} & \cdots & \hat{x}_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \hat{x}_{m1} & \cdots & \hat{x}_{mj} & \cdots & \hat{x}_{mn} \end{bmatrix}$$

$$\hat{x}_{ij} = \bar{x}_{ij} w_j.$$

TABLE 4.1.4. Calculated weighted NFDM for WSM

CRITERIA	SA	A	N	D	SD	$\tilde{Q}_i$
OC 1	(0.0457,0.0522, 0.0587)	(0.0233,0.0280, 0.0326)	(0.0056,0.0075, 0.0093)	(0.0012,0.0019, 0.0028)	(0.0019,0.0019, 0.0019)	(0.0777,0.0913, 0.1053)
OC 2	(0.0285,0.0326, 0.0367)	(0.0163,0.0196, 0.0228)	(0.0098,0.0130, 0.0163)	(0.0011,0.0016, 0.0024)	(0.0033,0.0033, 0.0033)	(0.0590,0.0701, 0.0816)
OC 3	(0.0294,0.0336, 0.0378)	(0.0473,0.0567, 0.0662)	(0.0063,0.0084, 0.0105)	(0.0000,0.0000, 0.0000)	(0.0021,0.0021, 0.0021)	(0.0851,0.1009, 0.1166)
OC 4	(0.0656,0.0750, 0.0844)	(0.0410,0.0492, 0.0574)	(0.0035,0.0047, 0.0059)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.1102,0.1289, 0.1477)
IE 1	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)
IE 2	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)
IE 3	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)
IE 4	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)
KT 1	(0.0136,0.0119, 0.0106)	(0.0032,0.0027, 0.0023)	(0.0045,0.0034, 0.0027)	(0.0712,0.0477, 0.0318)	(0.0477,0.0477, 0.0477)	(0.1402,0.1134, 0.0951)
KT 2	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)
KT 3	(0.0082,0.0072, 0.0064)	(0.0026,0.0021, 0.0018)	(0.0128,0.0096, 0.0077)	(0.0861,0.0577, 0.0385)	(0.0577,0.0577, 0.0577)	(0.1674,0.1344, 0.1121)
KT 4	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)
TR 1	(0.0656,0.0750, 0.0844)	(0.0410,0.0492, 0.0574)	(0.0035,0.0047, 0.0059)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.1102,0.1289, 0.1477)
TR 2	(0.0575,0.0657, 0.0739)	(0.0411,0.0493, 0.0575)	(0.0035,0.0047, 0.0059)	(0.0016,0.0023, 0.0035)	(0.0000,0.0000, 0.0000)	(0.1036,0.1220, 0.1408)
TR 3	(0.0152,0.0174, 0.0195)	(0.0596,0.0716, 0.0835)	(0.0098,0.0130, 0.0163)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0846,0.1019, 0.1193)
TR 4	(0.0464,0.0531, 0.0597)	(0.0142,0.0171, 0.0199)	(0.0142,0.0190, 0.0237)	(0.0013,0.0019, 0.0028)	(0.0000,0.0000, 0.0000)	(0.0761,0.0910, 0.1061)

(b) Calculate the weighted normalized fuzzy decision matrix $\hat{X}_p$ for Weighted Product Model (WPM):

$$\hat{X}_p = \begin{bmatrix} \bar{x}_{11} & \cdots & \bar{x}_{1j} & \cdots & \bar{x}_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{x}_{i1} & \cdots & \bar{x}_{ij} & \cdots & \bar{x}_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{x}_{m1} & \cdots & \bar{x}_{mj} & \cdots & \bar{x}_{mn} \end{bmatrix};$$

$$\bar{x}_{ij} = (\bar{x}_{ij})^{w_j}.$$

TABLE 4.1.5. Calculated Weighted NFDM for WPM

CRITERIA	SA	A	N	D	SD	$\widetilde{P_\iota}$
OC 1	(0.9854,0.9931, 1.0000)	(0.9472,0.9574, 0.9661)	(0.8711,0.8859, 0.8976)	(0.7977,0.8167, 0.8363)	(0.8167,0.8167, 0.8167)	(0.5296,0.5618, 0.5923)
OC 2	(0.9908,0.9957, 1.0000)	(0.9707,0.9772, 0.9827)	(0.9526,0.9628, 0.9707)	(0.8790,0.8920, 0.9054)	(0.9150,0.9150, 0.9150)	(0.7369,0.7646, 0.7903)
OC 3	(0.9477,0.9561, 0.9636)	(0.9780,0.9898, 1.0000)	(0.8558,0.8723, 0.8853)	(0.0000,0.0000, 0.0000)	(0.7958,0.7958, 0.7958)	(0.0000,0.0000, 0.0000)
OC 4	(0.9790,0.9901, 1.0000)	(0.9409,0.9555, 0.9680)	(0.7647,0.7835, 0.7984)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)
IE 1	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)
IE 2	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)
IE 3	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)
IE 4	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)
KT 1	(0.9420,0.9360, 0.9308)	(0.8788,0.8712, 0.8648)	(0.8939,0.8817, 0.8724)	(1.0193,1.0000, 0.9808)	(1.0000,1.0000, 1.0000)	(0.7543,0.7190, 0.6888)
KT 2	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)
KT 3	(0.8938,0.8869, 0.8809)	(0.8356,0.8268, 0.8195)	(0.9169,0.9018, 0.8902)	(1.0234,1.0000, 0.9769)	(1.0000,1.0000, 1.0000)	(0.7007,0.6613, 0.6278)
KT 4	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)
TR 1	(0.9790,0.9901, 1.000)	(0.9409,0.9555, 0.9680)	(0.7647,0.7835, 0.7984)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)
TR 2	(0.9816,0.9913, 1.0000)	(0.9575,0.9705, 0.9816)	(0.7985,0.8157, 0.8292)	(0.7524,0.7750, 0.7985)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)
TR 3	(0.8673,0.8770, 0.8857)	(0.9723,0.9872, 1.0000)	(0.8359,0.8562, 0.8723)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)
TR 4	(0.9851,0.9930, 1.0000)	(0.9179,0.9279, 0.9365)	(0.9179,0.9338, 0.9463)	(0.7946,0.8139, 0.8338)	(0.0000,0.0000, 0.0000)	(0.0000,0.0000, 0.0000)

STEP 4: Calculate the values of the optimality function.

(a)The WSM for i^{th} alternative**:**

$$\tilde{Q}_i = \sum_{j=1}^{n} \hat{x}_{ij} \text{ , i=1, …, m}$$

(b)The WPM for i^{th} alternative:

$$\widetilde{P_i} = \prod_{j=1}^{n} \bar{x}_{ij} \text{ , i=1,…,m}$$

TABLE 4.1.6. Optimality value of WSM

CRITERIA	$\tilde{Q}_i$	$\widetilde{P_i}$
OC 1	(0.0777,0.0913,0.1053)	(0.5296,0.5618,0.5923)
OC 2	(0.0590,0.0701,0.0816)	(0.7369,0.7646,0.7903)
OC 3	(0.0851,0.1009,0.1166)	(0.0000,0.0000,0.0000)
OC 4	(0.1102,0.1289,0.1477)	(0.0000,0.0000,0.0000)
IE 1	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)
IE 2	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)
IE 3	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)
IE 4	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)
KT 1	(0.1402,0.1134,0.0951)	(0.7543,0.7190,0.6888)
KT 2	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)
KT 3	(0.1674,0.1344,0.1121)	(0.7007,0.6613,0.6278)
KT 4	(0.0000,0.0000,0.0000)	(0.0000,0.0000,0.0000)
TR 1	(0.1102,0.1289,0.1477)	(0.0000,0.0000,0.0000)
TR 2	(0.1036,0.1220,0.1408)	(0.0000,0.0000,0.0000)
TR 3	(0.0846,0.1019,0.1193)	(0.0000,0.0000,0.0000)
TR 4	(0.0761,0.0910,0.1061)	(0.0000,0.0000,0.0000)

Defuzzification of the fuzzy performance evaluation is carried out using the center-of-area method, which is the most practical method.

$$Q_i = \frac{1}{3} (Q_{i\alpha} + Q_{i\beta} + Q_{i\gamma}),$$

$$P_i = \frac{1}{3} (P_{i\alpha} + P_{i\beta} + P_{i\gamma}),$$

TABLE 4.1.7. Defuzzified Value

CRITERIA	Q_i	P_i
OC1	0.0914	0.5612
OC2	0.0702	0.7639
OC3	0.1009	0.0000
OC4	0.1289	0.0000
IE1	0.0000	0.0000
IE2	0.0000	0.0000
IE3	0.0000	0.0000
IE4	0.0000	0.0000
KT1	0.1162	0.7207
KT2	0.0000	0.0000
KT3	0.1380	0.6633
KT4	0.0000	0.0000
TR1	0.1289	0.0000
TR2	0.1221	0.0000
TR3	0.1019	0.0000
TR4	0.0911	0.0000

STEP 5: The utility function value K_i of the F-WASPAS method is calculated as follows:

$$K_i = \lambda \sum_{j=1}^{n} Q_i + (1-\lambda) \sum_{j=1}^{n} P_i \; ; \lambda = 0, \dots, 1, 0 \leq \mathrm{K_i} \leq 1,$$

where,

$$\lambda = \frac{\sum_{i=1}^{m} P_i}{\sum_{i=1}^{m} Q_i + \sum_{i=1}^{m} P_i}$$

TABLE 4.1.8. Calculated Utility Function

$\boldsymbol{\lambda = 0.5}$

CRITERIA	K_i
OC1	0.3263
OC2	0.4171
OC3	0.0504
OC4	0.0645
IE1	0.0000
IE2	0.0000
IE3	0.0000
IE4	0.0000
KT1	0.4185
KT2	0.0000
KT3	0.4006
KT4	0.0000
TR1	0.0645
TR2	0.0611
TR3	0.0510
TR4	0.0455

STEP 6: Rank the preference order or select the decision criteria (solution), starting from the highest value, K_i

TABLE 4.1.9. Ranking the Decision Criteria

CRITERIA	K_i	RANK
OC	0.2264	4
IE	0.0000	1
KT	0.2023	3
TR	0.0630	2

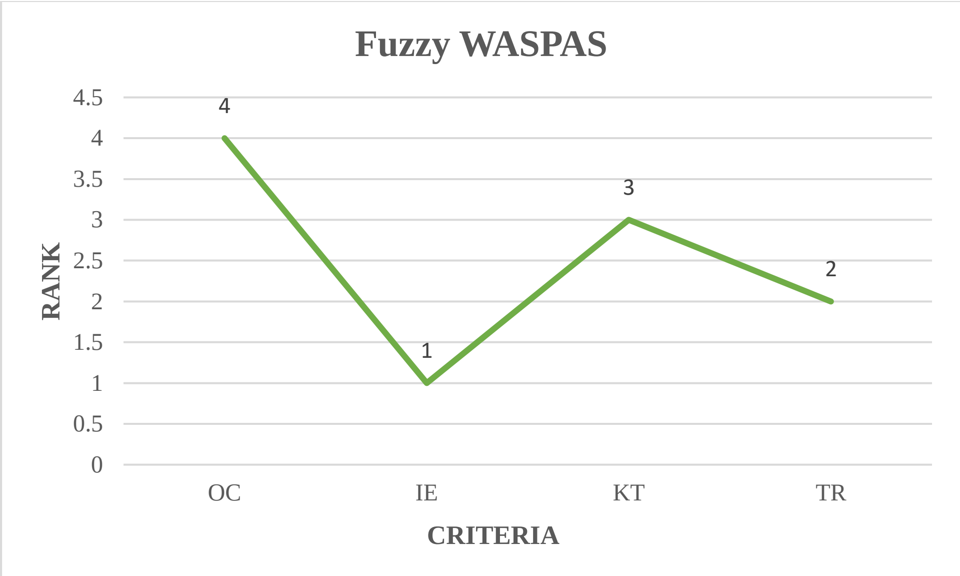

Figure 4.1: F-WASPAS GRAPH

The WASPAS ranking method objectively measures the effectiveness of Outbound Open Innovation between big organizations and startups. It highlights the Innovation Ecosystem (IE) as the most critical factor, reflecting its strong impact on successful collaboration. The fact that Organisational Commitment (OC) ranked lowest suggests that this area requires further strengthening. The ranking offers precise, fact-based insights into the elements that have the greatest effects on innovative partnerships, helping large organizations prioritize support for startups in areas that most influence success.

4.2 RANKING OUTCOMES

In this study, I have built upon previous research that evaluated the effectiveness of outbound open innovation (OOI) between large organizations and startups using fuzzy Multi-Criteria Decision Making (MCDM) methods. Specifically, I referred to a paper that employed fuzzy MCDM techniques to assess the effectiveness of OOI collaboration, utilizing expert evaluations to derive criteria weights and rankings.

While the referenced study used a different fuzzy MCDM approach, I have adopted the same expert-derived linguistic evaluations for consistency and comparability. However, in my analysis, I implemented the Entropy method to determine the objective weights of the evaluation criteria, followed by the WASPAS (Weighted Aggregated Sum Product Assessment) method to rank the effectiveness of OOI collaboration between large organizations and startups.

This methodological shift introduces an objective and integrative perspective to the evaluation framework. The application of the Entropy method enables data-driven determination of criteria importance based on the inherent information diversity in expert responses, eliminating subjective bias. Meanwhile, WASPAS combines the strengths of both the weighted sum and weighted product models, offering a comprehensive and robust mechanism for ranking alternatives across multiple criteria. By utilizing the same expert input values, the study maintains consistency with the original dataset while providing new analytical depth into the comparative performance of OOI collaborations.

The overall weight comparison among Fuzzy AHP, Fuzzy DEMATEL, Fuzzy ANP, and Fuzzy ENTROPY is illustrated.

TABLE 4.2.1. Overall Weight Comparison

CRITERIA	F-AHP	F-DEMATEL	F-ANP	F-ENTROPY
OC	3	3	2	2
IE	1	2	3	3
KT	4	4	4	4
TR	2	1	1	1

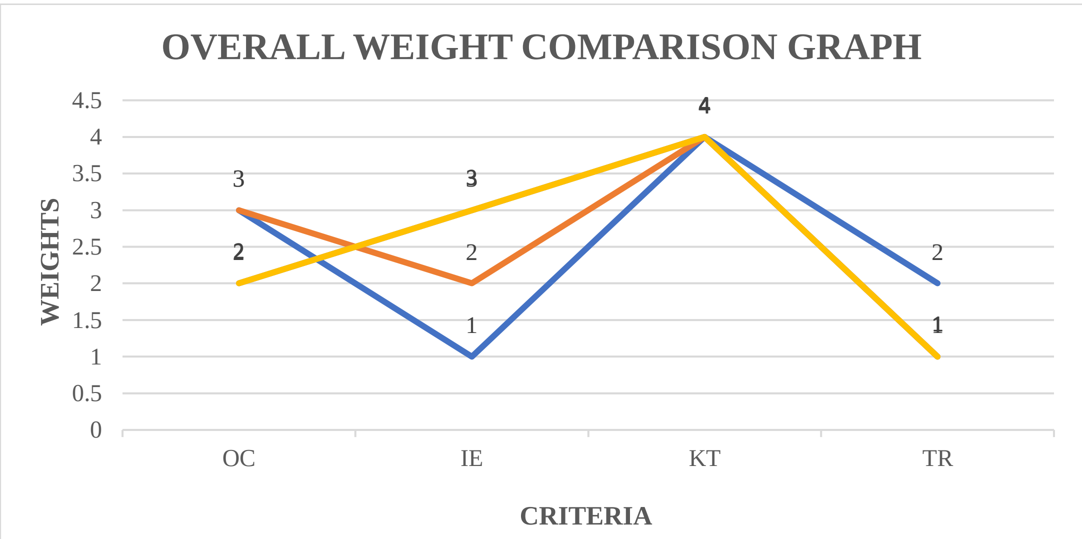

Figure 4.3: Overall Weight Comparison Graph

The comparative ranking analysis across four Fuzzy MCDM methods—F-AHP, F-DEMATEL, F-ANP, and F-ENTROPY—offers varied but converging insights into the effectiveness of outbound open innovation (OOI) between large organizations and startups.

Organizational Commitment (OC) consistently ranks in the mid-range across all methods, indicating its steady relevance as a strategic enabler of collaboration. While it is not identified as the most critical factor, it remains an essential supporting component in fostering effective partnerships.

Innovation Ecosystem (IE) emerges as the most critical factor in the F-AHP method, highlighting expert-driven recognition of its strategic importance. However, in the other methods, its influence is considered moderately critical, pointing to its context-dependent role and limited interdependence with other factors.

Knowledge Transfer (KT) is consistently identified as the least critical factor across all four methods. This suggests that relative to other criteria, KT is either underutilized in current OOI practices or its potential impact is not fully realized within existing collaboration models.

Technology Relevance (TR) stands out as the most critical factor across all methods—both subjective and objective. This strong consensus confirms TR as a universally significant driver of successful innovation partnerships, underscoring its foundational importance in aligning innovation with real-world needs and market demands.

Overall, the comparison identifies **Technology Relevance** as the top priority, with Organizational Commitment and Innovation Ecosystem playing vital but variable roles depending on the analytical method used. **Knowledge Transfer**, while still essential, appears to require greater emphasis and strategic investment. This convergence across multiple MCDM techniques reinforces the robustness and reliability of TR as the central focus for enhancing outbound open innovation effectiveness.

4.2.2. Ranking Methods Comparison

CRITERIA	ENSEMBLE	F-WASPAS
OC	3	4
IE	2	1
KT	4	3
TR	1	2

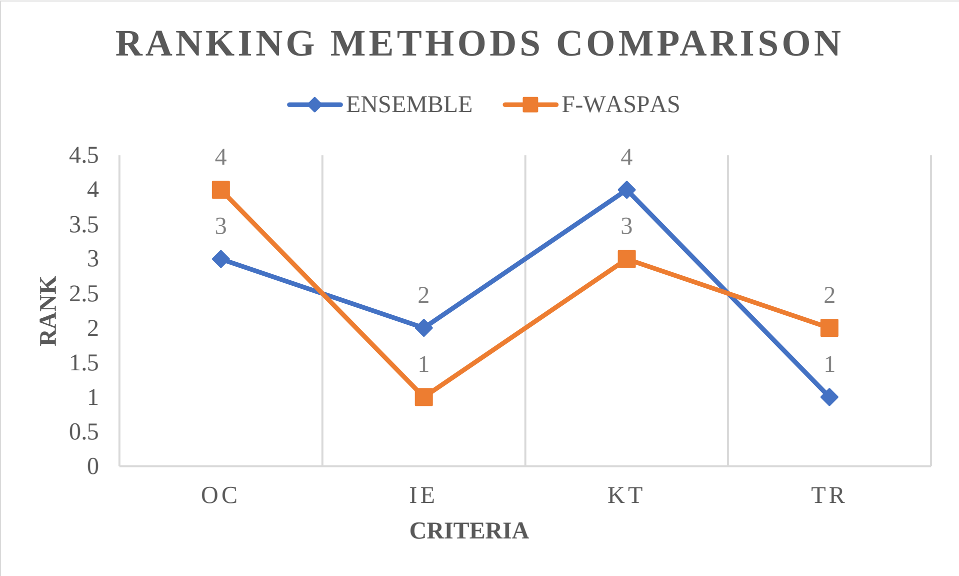

Figure 4.4: Ranking Methods Comparison Graph

The comparative analysis between Ensemble and Fuzzy WASPAS methods reveals important differences in the prioritization of outbound open innovation criteria. **Innovation Ecosystem (IE)** emerges as the most critical factor in F-WASPAS, reflecting its strong influence on driving innovation partnerships, while Ensemble also acknowledges IE's significance by ranking it second, indicating general agreement on its importance.

Technology Relevance (TR) is considered the most vital in the Ensemble method but receives slightly less emphasis in F-WASPAS, where it ranks second, suggesting that TR's strategic role is better captured through a broader multi-perspective evaluation.

Organizational Commitment (OC) is seen as moderately important in Ensemble but is ranked lowest by F-WASPAS, implying that operational performance-based evaluation perceives OC as less immediately impactful compared to other factors.

Knowledge Transfer (KT) holds a low priority in both methods but is rated slightly better in F-WASPAS than in Ensemble, highlighting F-WASPAS's recognition of the practical importance of knowledge exchange.

Overall, the divergence between Ensemble and F-WASPAS outcomes reflects the difference between a multi-perspective consensus approach (Ensemble) and a performance-focused hybrid aggregation model (F-WASPAS). This highlights the necessity of integrating both strategic and operational perspectives to fully assess and enhance innovation collaborations between large organizations and startups.

SUMMARY & CONCLUSION

SUMMARY AND CONCLUSION

The results of this research reaffirm the significant role of mathematics in advancing entrepreneurial success and innovation-driven collaboration, especially within the context of startup and large organization partnerships. By applying multi-criteria decision-making techniques like the Fuzzy Entropy method for weight determination and the Fuzzy WASPAS method for ranking, the study provides a structured framework to evaluate the effectiveness of outbound open innovation (OOI).

The combination of fuzzy logic with objective mathematical methods enabled the accurate modeling of expert uncertainty and subjectivity, which are inherent in strategic decision-making. Among the sixteen sub-criteria evaluated under four broad categories- Organizational Commitment (OC), Innovation Ecosystem (IE), Knowledge Transfer (KT), and Technology Relevance (TR) - Technology Relevance and Innovation Ecosystem emerged as top contributors to successful OOI partnerships.

The Fuzzy Entropy method highlighted Technology Relevance as the most critical factor, reflecting its dominant role in ensuring innovation aligns with market needs. Knowledge Transfer, on the other hand, received the lowest weight, indicating a potential area for capacity-building within collaborative innovation models.

The WASPAS method objectively measures outbound open innovation, ranking Innovation Ecosystem (IE) as the most critical factor for successful collaboration. Organizational Commitment (OC) ranks lowest, indicating a need for improvement. The ranking provides clear, fact-based insights to help organizations prioritize support for startups where it matters most.

The comparison between Ensemble and Fuzzy WASPAS methods reveals key insights into OOI prioritization. Both rank Innovation Ecosystem (IE) and Technology Relevance (TR) highest, with Ensemble stressing TR's strategic value and Fuzzy WASPAS focusing on IE's operational impact. The Ensemble offers stability and consensus, while Fuzzy WASPAS sharpens performance differentiation. Organizational Commitment (OC) and Knowledge Transfer (KT) rank lower. Together, they provide complementary perspectives that enhance strategic decision-making. This study not only advances the application of mathematics in entrepreneurship but also proposes a replicable decision-support framework for fostering impactful startup-corporate collaborations in dynamic economic landscapes.

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