



Avinashilingam Institute for Home Science and Higher Education for Women

(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)

Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B

Coimbatore - 641 043, Tamil Nadu, India

Bachelor's Degree Examination – March 2021

I Semester

Class : I UG
Major : Mathematics

Time : 3 Hours
Max. Marks: 100

18BMAC03 Differential Calculus

Part A

10 x 1 = 10

Choose the Correct Answer

- If $y = e^{ax}$ then $\frac{d^n y}{dx^n} =$ _____.
 - e^{ax}
 - ae^{ax}
 - $a^n e^{ax}$
 - $n^a e^{ax}$
- Which of the following differential equations satisfy $xy = ae^x + be^{-x}$?
 - $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx}$
 - $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + xy = 0$
 - $\frac{d^2 y}{dx^2} - xy = 0$
 - $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} - xy = 0$
- If $u = \frac{xy}{x+y}$ then $\frac{\partial u}{\partial y}$ is
 - $\frac{x^2}{(x+y)^2}$
 - $\frac{y^2}{(x+y)^2}$
 - $\frac{x}{(x+y)^2}$
 - $\frac{y}{(x+y)^2}$
- If $u = \sin(ax + by + cz)$ then $\frac{\partial^2 u}{\partial x^2} =$
 - $-a^2 \sin(ax + by + cz)$
 - $\cos(ax + by + cz)$
 - $a^2 \cos(ax + by + cz)$
 - $-a^2 \sin(ax + by + cz)$
- If $rt - s^2 < 0$, then $f(x, y)$ has
 - Maxima
 - Minima
 - Saddle points
 - None of the above
- The maximum product of three numbers, whose sum is _____
 - $3a$
 - a^3
 - $3a^3$
 - None of the above
- The envelope of the family of circles $(x - a)^2 + y^2 = r^2$ with parameter r is
 - $y = \pm r$
 - $y = 0$
 - $y = r$
 - $y = 1$
- Curvature of the circle is the reciprocal of its _____.
 - centre
 - diameter
 - radius
 - area
- Which one of the following is (p-r) equation of $r = a \sin \theta$?
 - $pr = a^2$
 - $pa = r^2$
 - $pr^2 = a^2$
 - $pr = a$
- The chord of the circle of curvature drawn through the point in any direction is called as _____.
 - chord of curvature
 - curvature
 - radius of curvature
 - centre of curvature

Part B**5 x 6 = 30****Answer ALL questions****Each answer should not exceed 400 words or two pages**11.a. Prove that if $y = \sin^{-1}(m \sin^{-1} x)$ then $(1 - x^2)y_2 - xy_1 + m^2y = 0$.

(or)

11.b. Find the n^{th} differential coefficient of $x^2 \log x$.12.a. If $z = f(x^2 + y^2)$, then show that $x \frac{\partial z}{\partial y} - y \frac{\partial z}{\partial x} = 0$.

(or)

12.b. Illustrate the theorem that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ when $u = \log \frac{x^2 + y^2}{xy}$.13.a. Investigate the maximum and minimum values of $4x^2 + 6xy + 9y^2 - 8x - 24y + 4$.

(or)

13.b. Find the minimum sum of three positive numbers whose product is a^3 .14.a. Find the envelope of the family of straight lines $y + tx = 2at + at^3$ the parameter being t .

(or)

14.b. Find the co-ordinates of the centre of curvature of the curve $xy = 2$ at the point $(2, 1)$.15.a. From the polar equation of the parabola, show that $p^2 = ar$.

(or)

15.b. Find radius of curvature of the curve $r^2 = a^2 \sin 2\theta$.**Part C****5 x 12 = 60****Answer ALL questions****Each answer should not exceed 800 words or fourpages**16.a. Find the n^{th} differential coefficient of $\cos^5 \theta \sin^7 \theta$.

(or)

16.b. If $y = x^2 e^x$ then show that $y_n = \frac{1}{2}n(n-1)y_2 - n(n-2)y_1 + \frac{1}{2}(n-1)(n-2)y$,where y_n stands for $\frac{d^n y}{dx^n}$.17.a. If $V = (x^2 + y^2 + z^2)^{-1/2}$, show that $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$.

(or)

17.b. If $u = \sin^{-1} \frac{x}{y} + \tan^{-1} \frac{x}{y}$, then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$.18.a. Discuss the maximum and minimum of the function $f(x, y) = x^3 y^2 (6 - x - y)$.

(or)

18.b. Show that, if the perimeter of a triangle is constant, the triangle has a minimum area when it is equilateral.

19.a. Find the envelope of the circles drawn on the radius vectors of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
 as diameter.

(or)

19.b. Prove that the radius of curvature at any point of the cycloid $x = a(\theta + \sin \theta)$ and

$$y = a(1 - \cos \theta)$$
 is $4a \cos \frac{\theta}{2}$.

20.a. Show that the radius of curvature of the curve $r^n = a^n \cos n\theta$ is $\frac{a^n r^{-n+1}}{n+1}$ and discuss the cases $n = -2, 2, \frac{1}{2}$ and $-1/2$.

(or)

20.b. Show that the chord of curvature through the focus of a parabola is four times the focal distance of the point and the chord of curvature parallel to the axis has the same length.
