

Avinashilingam Institute for Home Science and Higher Education for Women

Coimbatore -641043

Master's Degree Examination – November 2017

Semester III

Class: II P.G

Max. Marks : 60

Major: Mathematics

Time : 3 Hours

12MMAC14 Stochastic Processes and Queuing Theory

Part - A

10 x ½ = 5

Choose the correct Answer

- The state i is said to be periodic with period $d(i)$ if -----
a. $d(i) > 1$ b. $d(i) < 1$ c. $d(i) \leq 1$ d. $d(i) \geq 1$
- Markov chain is said to be primitive if it is -----
a. aperiodic b. periodic c. imprimitive d. irreducible
- The random variable $Y(t) = S_{N(t)+1} - t$ is called -----
a. random time b. residual life time c. life time d. renewal time
- The Markov chain $\{X_n, n \geq 0\}$ is called -----
a. embedded chain b. markov chain c. embedded markov chain d. markovian property
- In any queuing system in which arrivals and departures occur one by one and that has reached equilibrium state ----- for all $n \geq 0$.
a. $a_n = d_n$ b. $a_n \neq d_n$ c. $a_n > d_n$ d. $a_n \leq d_n$
- For M/M/1 model $E(N_Q) =$ -----
a. $\frac{\rho^2}{(1-\rho)}$ b. $\frac{\rho^2}{(1+\rho)}$ c. $\frac{\rho}{(1-\rho)}$ d. $\frac{\rho}{(1+\rho)}$
- The average number of customers diverted from the system or lost to the system in unit time when $a = 1$, then $\lambda p_k =$ -----
a. $\lambda / (k+1)$ b. λ / k c. $\lambda / (k-1)$ d. $\lambda / 2k$
- Expected number of idle servers for M/M/c models is given by $E(I) =$ -----
a. $c(1+\rho)$ b. $c(1-\rho)$ c. $c\rho$ d. $c / (\rho + 1)$
- The interval time between two transition is called as -----
a. service time b. arrival time c. departure time d. inter arrival time
- The LST $w_s^*(s)$ of w_s the response time is given by $w_s^*(s) =$ -----
a. $w^*(s) B^*(s)$ b. $w_q(s) B^*(s)$ c. $w(s) B(s)$ d. $w_q(s) B(s)$

Part B
Answer all questions

5 x 4 = 20

- 11.a. Explain ergodic property
(Or)
11.b. Derive the Chapman-Kolmogorov equation
- 12.a. Explain the compound Poisson Process
(Or)
12.b. Explain memoryless property of exponential distribution
- 13.a. Derive the transient and steady state behavior
(Or)
13.b. Derive $E(N)$ and $E(w)$ for M/M/1 model
- 14.a. Derive p_n for M/M/1/k model
(Or)
14.b. Derive $E(n)$ for M/M/C model
- 15.a. Find the moments of the busy period for M/G/1 model
(Or)
15.b. Derive the waiting time for single arrival case.

Part C
Answer all questions

5 x 7 = 35

- 16.a. Consider the Markov chain having state space $s = \{0,1,2\}$ and
TPM $p = \begin{pmatrix} 0 & 1/3 & 2/3 \\ 1/2 & 0 & 1/2 \\ 3/4 & 1/4 & 0 \end{pmatrix}$ find $\lim p_{11}(n)$, $\lim p_{12}(n)$ and $\lim p_{13}(n)$
(Or)
16.b. By using Kolmogorov equation for Birth – Death process find $p_j(t)$
- 17.a. Derive the fundamental equation of renewal theory
(Or)
17.b. Derive p_j for the Birth – Death process
- 18.a. Explain Eilon's proof of $L = \lambda w$
(Or)
18.b. Derive p_k by semi-Markov process for M/M/1 model
- 19.a. Derive p_n and p_0 for M/M/C
(Or)
19.b. Derive waiting time distribution for M/M/c model
- 20.a. Derive Pollaczek – Khinchin (p-k) formulae for M/G/1 model
(Or)
20.b. Prove that the LST of the busy period can be expressed as $G^*(s) = B^*(S + \lambda - \lambda G^*(s))$ as functional equation
