

Avinashilingam Institute for Home Science and Higher Education for Women

(Deemed to be University), Coimbatore-641 043

Bachelor's Degree Examination- November 2018

I-Semester

Class : I UG

Time: 3 hours

Major: Mathematics

Max. Marks: 100

18BMAC02 – Classical Algebra and Theory of numbers

Part-A

10 x 1=10

Circle the correct answer

- Every equation $f(x) = 0$ of the n th degree has _____
(a) n roots (b) n roots and no more
(c) $n-1$ roots (d) n roots and more
- If the roots of the equation $x^3 + \frac{x^2}{4} - \frac{x}{16} + \frac{1}{72} = 0$ are multiplied by _____
Then the fractional coefficients of the equation will be removed.
(a) 24 (b) 12 (c) 72 (d) 16
- The inverse of an orthogonal matrix is _____
(a) Orthogonal (b) not orthogonal
(c) unit matrix (d) null matrix
- If $|A| = 0$, then the matrix A is called _____
(a) null matrix (b) Singular matrix
(c) non-singular matrix (d) unit matrix
- By Euler's function $\phi(N)$, the value of $\phi(5) =$ _____
(a) 1 (b) 2 (c) 3 (d) 4
- The integral part of $\left[\frac{2}{5} \right] =$ _____
(a) 0 (b) 1 (c) 3 (d) 4
- If $a \equiv b \pmod{m}; a_1 \equiv b_1 \pmod{m}$, then
(a) $a + a_1 \equiv b + b_1 \pmod{m}$ (b) $aa_1 \equiv bb_1 \pmod{m}$
(c) $a \equiv bb_1 \pmod{m}$ (d) $aa_1 \equiv b \pmod{m}$
- The least positive integer having remainders 2,3,2 when divided by 3,5,7 is _____
(a) 24 (b) 124 (c) 28 (d) 128
- If p is an odd prime and a is prime to p , then _____ is divisible by p .
(a) $a^{p-1} - 1$ (b) $a^{p-1} + 1$ (c) $a^{\frac{1}{2}(p-1)} \pm 1$ (d) $a^{(p-1)} \pm 1$
- The p is a prime greater than 3, then A_{p-2} is a multiple of _____
(a) P (b) P^2 (c) 3 (d) greater than 3

Part B

(5 X 6=30)

Answer all the questions

11.a Solve the equation $x^4 + 3x^3 - 3x - 1 = 0$

(or)

11.b. Determine completely the nature of the roots of the equation
 $x^5 - 6x^2 - 4x + 5 = 0$

12.a. Show that

$$A = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & \frac{1}{3} \end{pmatrix} \text{ is orthogonal} \quad (\text{or})$$

12.b. Calculate A^4 when $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$

13. a. Find the smallest number with 18 divisors
(or)

13. b. Find the number of integers less than n and prime to it when $n=729$ and 720

14.a Show that $13^{2n+1} + 9^{2n+1}$ is divisible by 22.
(or)

14.b. Show that $x^5 - x$ is divisible by 30

15.a .Show that the 8^{th} power of any number is of the form $17m$ (or) $17m \pm 1$.
(or)

15. b. Show that $(18)! + 1$ is divisible by 437.

Part C

5 x 12=60

Answer all the questions

16.a Solve the equation $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$
(or)

16. b. Show that the equation $x^4 - 3x^3 + 4x^2 - 2x + 1 = 0$ can be transformed into a reciprocal equation by diminishing the roots by unity. Hence solve the equation

17.a. Diagonalise the matrix

$$\begin{pmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & +3 & -1 \end{pmatrix} \quad (\text{or})$$

17. b) Find the characteristic equation of the matrix $A =$ and hence determine its inverse.

$$\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$$

18.a. If $d_1, d_2, d_3, \dots, d_r$ (including 1 and N) are the divisors of N , then show that $\phi(d_1) + \phi(d_2) + \dots + \phi(d_r) = N$
(or)

b. (i) Find the highest power of 3 dividing $1000!$
(ii) Show that $n(2n+1)(n+1)$ is divisible by 6

19.a. (i) Find the remainder obtained in dividing 2^{46} by 47
(ii) Find the remainder when 2^{1000} is divisible by 17
(or)

19. b. (i) Show that every integer which is a perfect cube is of the form $7P$ (or) $7P \pm 1$
(ii) If x, y, z be three consecutive integers, show that $(\sum x)^3 - 3\sum x^3$ is divisible by 108.

20.a. State and prove Fermat's theorem
(or)
b. State and prove Lagrange's theorem