Nanimmuk.

Avinashilingam Institute for Home Science and Higher Education for Women
(Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD)
Re-accredited with 'A++' Grade by NAAC.Recognised by UGC Under Section 12B
Coimbatore-641 043, Tamil Nadu, India

Continuous Internal Assessment Test I -August 2024

Semester-III

Class : II PG **Time : 2 Hours**

Branch : Mathematics **Max.Marks : 60**

23MMAC13 Topology I

Course Outcomes:

CO1: Understand the properties of various topologies on a general set.

CO2: Construct continuous functions in topological spaces.

CO3: Analyze the relation between metric spaces and topological spaces.

CO4: Relate the concept of continuity to connected space.

CO5: Demonstrate the properties of compact spaces

Part A **6 x 1 = 6**

Choose the Correct Answer

1. Let X= {a, b, c} then which of the following is topology on X?CO1K2

a. $\tau_1 = \{\varphi, \{a\}, \{b\}, X\}$b. $\tau_2 = \{\varphi, \{a\}, \{b\}, \{c\}, X\}$

c. $\tau_1 = \{\varphi, \{a\}, X\}$d. $\tau_1 = \{\varphi, \{a\}, \{b, c\}, \{a, b\}, X\}$

2. A basis element for the standard topology on real line R is CO1K1

a. (0, 1) b. (0, 1] c. [0, 1) d. [0, 1]

3. If A = [1, 2) ∪ {3} in R then $\bar{A}$isCO2K2

a. [1, 2) ∪ {3} b.[1, 2] c. [1,2] ∪ {3} d. (1,2)∪ {3}

4. If A = [1, 2) ∪ {3} in R then limit point of AisCO2K2

a. [1, 2) ∪ {3} b.[1, 2] c. [1,2] ∪ {3} d. (1,2)∪ {3}

5. Let X and Y be topological spaces. A function f : X→ Y is continuous if CO2K1

a. for each subset V of Y, the set f^{-1} (V) is closed subset of X

b. for each closed subset V of Y the set f^{-1} (V) is an open subset of X

c. for each open subset V of Y, the set f^{-1}(V) is an open subset of X

d.for each subset V of Y, the set f^{-1} (V) is an open subset of X

6. Let $X_1, X_2, X_3, \ldots, X_n$ be topological spaces. .Then the basis for $X_1 x X_2 x \ldots x X_n$ is

CO3K1

a. {U_1 x.....xU_n/ where each $U_i$is an open set of $X_i$for each i}

b. {U_1 x.....xU_n/ where each $U_i$is a closed set of $X_i$for each i}

c. {U_i/ where each $U_i$is an open set of $X_i$for each i}

d.{U_i/ where each $U_i$is a closed set of $X_i$for each i}

Part B **3 x 6 = 18**

Answer ALL questions

7. a.Define basis and subbasis for the topology on X with example.CO1K3

(or)

7. b.Prove that if $\mathbb{B} and \mathcal{C}$ are the bases for topologies X and Y respectively, then the collection $\mathfrak{D} = \{B \times C / B \in \mathbb{B} and c \in \mathcal{C}\}$ is a basis for the topology on X × Y. CO1K3

8.a. If $\mathcal{B}$is a basis for the topology on X, then prove that the collection $\mathcal{B}_Y = \{B \cap Y/B \in \mathcal{B}\}$is a basis for the subspace topology on $Y \subseteq X$.CO2K3

(or)

8. b.Prove that the product of two Hausdorff spaces is a Hausdorff space. CO2K4

9. a.State and prove pasting lemma. CO2K4

(or)

9. b.Define box and product topologies for the family of topological spaces.CO3K3

Part C **3 x 12 = 36**

Answer ALL questions

10. a. Prove that the collection of open sets generated by a basis $\mathcal{B}$is a topology. CO1K4

(or)

10. b. (i) Let$\mathcal{B}$be a basis for a topology τ on X. Then show that τ equals the collection of all unions of elements of $\mathcal{B}$.CO1K4

(ii) Prove that open rays in X are open sets in the order topology.

11. a.Prove that if Y is a subspace of a topological space X, then a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y. CO2K4

(or)

11.b. Let X and Y be topological spaces, let $f: X \to Y$ then prove that following are equivalent:CO2K3

(i) f is continuous.

(ii) For every subset A of X, $f(\bar{A}) \subset \overline{f(A)}$.

(iii) For every closed set B of Y, the set $f^{-1}(B)$is closed in X.

12. a. (i)Let A be a subset of the topological space (X,τ). Then prove that$x \in \bar{A}$ if and only if every open set U containing x intersects A. CO2K4

(ii) Show that every finite point set in a Hausdorff space is closed

(or)

12. b. (i)State and prove the rules for constructing continuous function. CO2K4

(ii) What are the bases for box and product topologies?CO3K3

No.of copies: ~~10~~ 30