INTRODUCTION

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"God used beautiful mathematics in creating the world"

-PAUL DIRAC

Continuous functions and irresolute functions stand among the most important and most researched points in the whole of Mathematical Science. In 1972, Gene Crossly. S., and Hildebrand. S.K., [22] introduced the concept of irresolute functions. In 1989, Mukherjee. M.N., and Sinha. S.P., [29] introduced the concept of fuzzy irresolute functions. Since then various generalizations of irresolute functions and fuzzy irresolute functions have been introduced and studied by many researchers.

Some interesting generalizations of fuzzy irresolute functions are

1) Fuzzy irresolute functions, Yalvac. T. H., 1987 [44].
2) Fuzzy irresolute functions , Mukherjee, M. N., and Sinha .S.P., 1989 [29].
3) Fuzzy semi-irresolute and strongly irresolute functions, Malakar, S., 1992 [26].
4) Fuzzy completely irresolute and fuzzy weakly completely irresolute functions, Bhaumik , R. N. , et al , 1993 [13].
5) Fuzzy SP- irresolute continuous functions, Biljana Krsteska ., 1999 [14].
6) Fuzzy SP-irresolute open and SP-irresolute closed functions, Biljana Krsteska ., 1999 [14].
7) Fuzzy approximately irresolute and contra-fuzzy irresolute functions, Miguel Caldas Cueva., and Ratnesh Kumar Saraf., 2001 [28].

8) Faα- irresolute functions , R. K. Saraf et al ., 2001 [34].

9) Fuzzy super irresolute functions , S. E. Abbas ., 2002 [1].

10) Fuzzy semi irresolute and fuzzy strongly irresolute functions, Seong Hoon Cho and Jae Keun Park., 2003 [38].

11) Fuzzy semi irresolute and strongly irresolute functions, Seong Hoon Cho., and Jae Keun Park., 2003 [38].

12) Fuzzy β-irresolute functions , Abbas.S.E., 2004 [2].

13) Fuzzy β-irresolute open (closed) functions , Abbas.S.E., 2004 [2].

14) Fuzzy strongly semi β-irresolute functions , Abbas.S.E., 2004 [2].

15) Fuzzy SP-irresolute functions , Abbas. S.E., 2004 [3].

16) Fuzzy strongly semi β-irresolute functions , Anjan Mukherjee., and Shymal Debnath., 2006 [7].

17) Fuzzy θ- generalized irresolute functions, El-Shafei. M.E and Zakari. A., 2006 [17].

18) Fuzzy semi-generalized irresolute and fuzzy generalized semi-irresolute functions, El-Shafei. M. E., and Zakari. A., 2007 [18].

19) Fuzzy gαc- irresolute functions , Bayaz Daraby and Nimse. S.B., (2007) [12].

20) Fuzzy δ- semi irresolute functions, Anjan Mukherjee., and Rupak Sarkar., 2008 [8].

This thesis is an attempt to study some interesting generalizations of the notions of fuzzy irresolute functions in fuzzy topological spaces.

Following are the generalizations discussed in this thesis :

1) Fuzzy irresolute functions
2) Fuzzy semi-irresolute and fuzzy strongly irresolute functions
3) Fuzzy completely irresolute and fuzzy weakly completely irresolute functions
4) Fuzzy δ-semi irresolute functions
5) Fuzzy semi-generalized irresolute and fuzzy generalized semi-irresolute functions
6) Fuzzy approximately irresolute and contra fuzzy irresolute functions
7) Fuzzy SP- irresolute functions
8) Fuzzy semi-pre-generalized irresolute functions

The first chapter deals with preliminary definitions and notations.

Chapter II deals with fuzzy irresolute functions. In this chapter, the concept of fuzzy irresolute functions which is stronger than the concept of semi-continuous functions are introduced and studied. An interesting characterization of fuzzy irresolute functions is established in theorem 2.9 .

Chapter III deals with fuzzy semi-irresolute and strongly irresolute functions. Several characterizations of these functions along with their relationships with certain other types of functions are investigated. Characterizations of these functions are given in theorems 3.14, 3.16, 3.17 and 3.18. Mutual correlations among fuzzy irresolute, fuzzy strongly irresolute, fuzzy continuous and fuzzy semi-continuous functions are discussed with examples.

In chapter IV, two new classes of functions, called fuzzy completely irresolute and fuzzy weakly completely irresolute functions are introduced. Their characterizations, examples, compositions and their relationships with other functions are studied.

In chapter V is devoted to the study of fuzzy δ-semi irresolute functions. Interesting characterizations of fuzzy δ–semi irresolute functions are given in theorems 5.8 and 5.11. The concept of fuzzy semi-compact is introduced and the following theorem is proved:

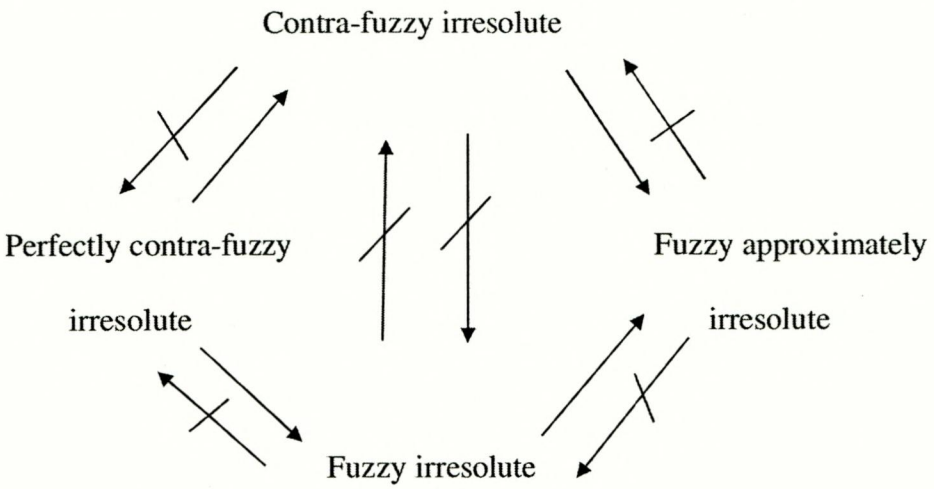
"If f: X → Y be a fuzzy δ-semi irresolute function and X be a fuzzy δ semi-compact space then f (X) is also a fuzzy δ-semi compact space".

Fuzzy semi-generalized irresolute functions and fuzzy generalized semi-irresolute functions are studied in chapter VI. In this chapter fuzzy semi-generalized continuous and fuzzy generalized-semi continuous mappings are also defined and it is proved that, "Fuzzy semi-generalized irresolute (resp. Fuzzy generalized semi irresolute) ⟹ Fuzzy semi-generalized continuous (resp. Fuzzy generalized semi continuous)".

An interesting example (example 6.20) is given to disprove the converse of the above statement. Also it is proved that the composition of two Fuzzy semi-generalized irresolute (Fuzzy generalized semi irresolute) maps is Fuzzy semi-generalized irresolute (Fuzzy generalized semi irresolute).

Chapter VII deals with fuzzy approximately irresolute and contra fuzzy irresolute functions. Fuzzy approximately irresolute functions and fuzzy

approximately semi-closed sets are introduced using fuzzy semi-generalized closed sets and some of their basic properties are studied. The conditions under which maps and inverse maps preserve fuzzy semi-generalized closed sets is obtained. A new generalization of fuzzy irresoluteness called contra-fuzzy irresoluteness is introduced. This notion is a stronger form of fuzzy approximately irresoluteness. The concepts contra-fuzzy irresoluteness and fuzzy irresoluteness are independent of each other. Perfectly contra irresolute maps are also introduced. The following diagram holds and none of its implications is reversible.



Following characterization of a class of topological space called fuzzy semi-$T_{1/2}$ space by using the concepts of Fuzzy approximately irresolute maps and Fuzzy approximately semi-closed maps is established :

"For a fuzzy topological space X, the following statements are equivalent :

(i) X is a Fuzzy semi- $T_{1/2}$ space,

(ii) For every space Y and every map f: X→ Y, f is Fuzzy approximately irresolute".

Chapter VIII deals with fuzzy SP- irresolute functions. The concepts of fuzzy SP-irresolute continuous, fuzzy SP- irresolute open, fuzzy SP-irresolute closed mappings are introduced and studied. Several characteristic properties of these functions are proved and are given in theorems 8.9, 8.10, 8.12, 8.14, 8.15, 8.22,8.23,8.24,8.26,8.28,8.30,8.31,8.32and8.33.

Chapter IX deals with fuzzy semi-pre-generalized irresolute functions .These functions are introduced using the notion of fuzzy semi-pre-generalized closed sets. The characterizations, examples, composition and their relationships with other functions are studied in detail.