Global exponential stability for stochastic Cohen-Grossberg neural networks with multiple time-varying delays

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Abstract: In this paper, together with some Lyapunov functionals and effective mathematical techniques, sufficient conditions are derived to guarantee a class of stochastic Cohen-Grossberg neural networks with multiple time-varying delays to be globally exponential stability by using linear matrix inequality (LMI) approach. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed method by using MATLAB LMI toolbox.

Keywords: global exponential stability; linear matrix inequality; LMI; Lyapunov functional; multiple time-varying delays; stochastic Cohen-Grossberg neural networks.

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1 Introduction

The past two decades have witnessed tremendous developments in the research field of neural networks, cellular neural networks, bidirectional associative neural networks and Cohen-Grossberg neural networks. Cohen-Grossberg models have been widely studied due to their extensive applications in classification of patterns, associative memories, image processing and optimisation (Cohen and Grossberg, 1983; Grossberg, 1988). The time delays are commonly encountered in various engineering systems such as chemical processes, hydraulic and rolling mill systems, etc. These effects are unavoidably encountered in the implementation of neural networks and may cause undesirable dynamic network behaviours such as oscillation and instability. Therefore, it is important to investigate the stability of delayed neural networks and that was explicitly introduced in Marcus and Westervelt (1989). Besides time delays, in the applications and designs of networks, some unavoidable uncertainties, which result from using an approximate system model for simplicity, parameter fluctuations, data errors and so on, must be integrated into the system model. Such time delays, parametric uncertainties and stochastic disturbances may significantly influence the overall properties of a dynamic system. Thus, it is of practical importance to study the stochastic effects on the stability property of delayed Cohen-Grossberg neural networks (see for example, Cao and Li, 2005; Cohen and Grossberg, 1983; Huanget al., 2003; Zheng et al., 2012; Li et al., 2009; Fu and Li, 2011; Wang et al., 2012; Zhu and Cao, 2011; Rong, 2005; Sun and Wan, 2005; Wang and Zou, 2002; Wu et al., 2007a). As a special class of neural networks, Cohen-Grossberg neural networks are of practical and theoretical interest and have attracted considerable attention for many years due to their potential applications in biology, economics, electronics, etc.

The global asymptotic stability results for different classes of delayed neural networks have been discussed in Cao and Li (2005), Rong (2005), Balasubramaniam et al. (2012), and Wu et al. (2007a, 2007b). However, these results are concerned only with the asymptotic stability of such networks, without providing any conditions for exponential stability and any information about the decay rates of the delayed neural networks. Therefore, it is particularly important to consider the problems to determine the speed of neural computations using the exponential convergence rate. Considering this, many researchers have studied the exponential stability analysis problem for delayed neural networks and a great number of results on this topic have been reported in the literature (Huang et al., 2003; Sun and Wan, 2005; Wang and Zou, 2002).

Recently, linear matrix inequality (LMI)-based techniques have been successfully used to tackle various stability problems for neural networks with time delays (Cao and Li, 2005; Rong, 2005; Wu et al., 2007a, 2007b). In Chen and Rong (2003), the asymptotic stability of delayed Cohen-Grossberg neural networks was considered. In Balasubramaniam and Lakshmanan (2009b), the authors investigated stability analysis problem for neural networks with Markovian jumping parameters. However, the criteria were expressed in terms of algebraic inequalities and do not take into account the signs of entries of the connection matrices, hence it might ignore the difference between the neuronal excitatory and inhibitory effects.

It is worth noting that so far there are only a few papers that have taken stochastic phenomenon into account in neural networks (see for example, Blythe et al., 2001; Liao and Mao, 1996; Wan and Sun, 2005; Wang et al., 2007; Balasubramaniam and

Lakshmanan, 2009a; Lakshmanan and Balasubramaniam, 2011). Practically, such phenomenon always appears in the electrical circuit design of neural networks. The results in Liao and Mao (1996), and Mao (1997) suggested that a neural network could be stabilised or destabilised by some stochastic inputs. This implies that it is of practical significance to study the stability for delayed stochastic neural networks. However, the stability analysis of stochastic neural networks is more difficult than that of traditional neural networks. Wang et al. (2006b, 2007) studied the exponential stability of uncertain stochastic neural networks with discrete and distributed delays and robust stability for stochastic Hopfield neural networks with time delays. Huang and Cao (2007) studied the exponential stability of uncertain stochastic neural networks with multiple time-varying delays in terms of LMI. Recently, Wang et al. (2006a) studied the stability analysis of stochastic Cohen-Grossberg neural networks with discrete and distributed delays.

Based on the above discussions, a class of stochastic Cohen-Grossberg neural networks with multiple time-varying delays are considered in this paper. The main aim of this paper is to study the global exponential stability in the mean square for stochastic Cohen-Grossberg neural networks with multiple time-varying delays. By using the Lyapunov functional technique, global exponential stability conditions for the considered stochastic Cohen-Grossberg neural networks are given in terms of LMIs, which can be easily calculated by MATLAB LMI toolbox. We also provide a numerical example to demonstrate the effectiveness of the proposed stability results.

2 Global stability results

Throughout this paper, we will use the notation $A > 0$ (or $A < 0$) denotes matrix A is a symmetric and positive definite (or negative definite). The notation A^T and A^{-1} denotes the transpose of A and the inverse of a square matrix A. For square matrices A and B, the notation $A > (<)\ B$ denotes $A - B$ is positive definite (negative definite) matrix.

Consider the following Cohen-Grossberg neural networks with multiple time-varying delays described by

$$x_i'(t) = -a_i\left(x_i(t)\right)\left[d_i\left(x_i(t)\right) - \sum_{j=1}^{n} a_{ij} f_j\left(x_j(t)\right) - \sum_{j=1}^{n} b_{ij} f_j\left(x_j\left(t-\tau_{ij}(t)\right)\right) + I_i\right], \quad (1)$$

$$i = 1, 2, \ldots, n$$

where x_i is the state variable of the i^{th} neuron and $a_i(\cdot)$ represents amplification function and is assumed to be positive, bounded and locally Lipschitz continuous, $d_i(\cdot)$ is the behaved function. The matrices a_{ij} and b_{ij} are the connection weight and delayed connection weight coefficients, respectively, f_j is the neuron activation function, I_i is the constant input from outside the system and the delay $\tau_{ij}(t)$ is non-negative, bounded and differentiable with $0 \leq \tau_{ij}(t) \leq \tau$ and $\tau_{ij}'(t) \leq \eta_{ij} < 1$, $i, j = 1, 2, \ldots, n$.

(A_1) Each function $d_i : \mathbb{R} \to \mathbb{R}$ is locally Lipschitz and there exists γ_i such that $d_i'(x) \geq \gamma_i$ for all $x \in \mathbb{R}$ at which $d_i(\cdot)$ is differentiable. Let us denote $\Gamma = \text{diag}\{\gamma_1, \gamma_2, \ldots, \gamma_n\}$.

Let $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ be an equilibrium point of neural networks (1), one can derive from (1) that the transformation $y_i(t) = x_i(t) - x_i^*$ transforms system (1) into the following system:

$$y_i'(t) = -\alpha_i\left(y_i(t)\right)\left[\beta_i\left(y_i(t)\right) - \sum_{j=1}^{n} a_{ij} g_j\left(y_j(t)\right) - \sum_{j=1}^{n} b_{ij} g_j\left(y_j\left(t-\tau_{ij}(t)\right)\right)\right], \tag{2}$$

$i = 1, 2, \ldots, n$

where

$$y = \left[y_1, y_2, \ldots, y_n\right]^T \in \mathbb{R}^n,$$

$$\alpha\left(y(t)\right) = diag\left[\alpha_1\left(y_1(t)\right), \alpha_2\left(y_2(t)\right), \ldots, \alpha_n\left(y_n(t)\right)\right] \in \mathbb{R}^{n\times n},$$

$$\beta\left(y(t)\right) = \left[\beta_1\left(y_1(t)\right), \beta_2\left(y_2(t)\right), \ldots, \beta_n\left(y_n(t)\right)\right] \in \mathbb{R}^n,$$

$$g\left(y(t)\right) = \left[g_1\left(y_1(t)\right), g_2\left(y_2(t)\right), \ldots, g_n\left(y_n(t)\right)\right]^T \in \mathbb{R}^n,$$

$$\alpha_i\left(y_i(t)\right) = a_i\left(y_i(t) + x_i^*\right),$$

$$\beta_i\left(y_i(t)\right) = d_i\left(y_i(t) + x_i^*\right) - d_i\left(x_i^*\right),$$

$$g_j\left(y_j(t)\right) = f_j\left(y_j(t) + x_j^*\right) - f_j\left(x_j^*\right).$$

In this paper, consider the following stochastic Cohen-Grossberg neural networks with multiple time-varying delays described by

$$\begin{aligned} dy_i(t) = &-\alpha_i\left(y_i(t)\right)\left[\beta_i\left(y_i(t)\right) - \sum_{j=1}^{n} a_{ij} g_j\left(y_j(t)\right) - \sum_{j=1}^{n} b_{ij} g_j\left(y_j\left(t-\tau_{ij}(t)\right)\right)\right] dt \\ &+ \sum_{k=1}^{m} \sigma_{ik}\left(t, g_j\left(y_j(t)\right), g_j\left(y_j\left(t-\tau_{ij}(t)\right)\right)\right) dw_k(t), \end{aligned} \tag{3}$$

where $w_k(t) = (w_1(t), w_2(t), \ldots, w_m(t))^T$ is an m-dimensional Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, P)$ with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$. Let $\sigma(t, x, y)$: $\mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times m}$ is locally Lipschitz continuous and satisfies the linear growth condition as well.

We can write the system (3) as

$$\begin{aligned} dy(t) = &-\alpha\left(y(t)\right)\left[\beta\left(y(t)\right) - Ag\left(y(t)\right) - Bg\left(y\left(t-\tau(t)\right)\right)\right] dt \\ &+\sigma\left(t, g\left(y(t)\right), g\left(y\left(t-\tau(t)\right)\right)\right) dw(t) \end{aligned} \tag{4}$$

where $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$, $\tau(t) = \tau_{ij}(t)$ for $i, j = 1, 2, \ldots, n$.

Now, let $y(t; \xi)$ denote the state trajectory of the neural networks (4) from the initial data $y(\theta) = \xi(\theta)$ on $-\tau \leq \theta \leq 0$ in $L^2_{\mathcal{F}_0}([-\tau, 0], \mathbb{R}^n)$. It can be easily seen that the system (4) admits a trivial solution $y(t; 0) \equiv 0$ corresponding to the initial data $\xi = 0$ (see Burton, 1985; Hale, 1977).

The main purpose of this paper is to establish LMI-based stability criteria, which can be readily checked by using the MATLAB LMI toolbox, such that the global exponential stability is guaranteed for the stochastic Cohen-Grossberg neural networks (4) with multiple time-varying delays.

3 Main results

Let $C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ : \mathbb{R}_+)$ denote the family of all non-negative functions $V(y, t)$ on $\mathbb{R}^n \times \mathbb{R}_+$ which are continuously twice differentiable in y and once differentiable in t. For each $V \in C^{2,1}([-\tau, \infty] \times \mathbb{R}^n, \mathbb{R}_+)$, define an operator $\mathcal{L}V$ associated with stochastic delayed neural networks (4) from $\mathbb{R}^n \times \mathbb{R}_+$ to $\mathbb{R}$ by

$$\mathcal{L}V(y(t), t) = V_t(y, t) + V_j(y, t)\left[-\alpha(y(t))\left[\beta(y(t)) - Ag(y(t)) - Bg(y(t-\tau(t)))\right]\right] + \frac{1}{2} trace\left[\sigma^T V_{yy}(y, t)\sigma\right]$$

where

$$V_t(y, t) = \frac{\partial V(y, t)}{\partial t}, V_y(y, t) = \left(\frac{\partial V(y, t)}{\partial y_1}, \frac{\partial V(y, t)}{\partial y_2}, \ldots, \frac{\partial V(y, t)}{\partial y_n}\right),$$

and

$$V_{yy}(y, t) = \left(\frac{\partial^2 V(y, t)}{\partial y_i \partial y_j}\right)_{n \times n}$$

where $i, j = 1, 2, \ldots, n$. Applying generalised Ito's formula, we get

$$\mathbb{E}V(y(t), t) = V(y(0), 0) + \mathbb{E}\int_0^t \mathcal{L}V(y(s), s)\, ds.$$

Definition 3.1: For every $\phi \in C^b_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$, the trivial solution of system (4) is said to be globally exponentially stable in the mean square if there exist positive scalars $\alpha > 0$ and $\beta > 0$ such that

$$\mathbb{E}|y(\phi, t)|^2 \leq \alpha e^{-\beta t} \mathbb{E}\| \phi \|^2 .$$

Now we will prove the following theorem on global exponential stability of equation (4).

Theorem 3.2: Assume $\gamma - c_2 > 0$ where γ is the rate of convergence and $c_2 = \frac{\lambda_{\max}(\Omega)}{\lambda_{\min}(P)}$. Assume that there exist matrices $P > 0$, $D_0 > 0$ and $D_i \geq 0$ $(i = 1, 2, \ldots, n)$ such that

$$trace\left[\sigma^T\left(t, g(y(t)), g(y(t-\tau_{ij}(t)))\right) p_0 I \sigma\left(t, g(y(t)), g(y(t-\tau_{ij}(t)))\right)\right] \leq \left(g(y(t))\right)^T D_0 \left(g(y(t))\right) + \sum_{i=1}^n \sum_{j=1}^n \left(g(y(t-\tau_{ij}(t)))\right)^T D_i \left(g(y(t-\tau_{ij}(t)))\right),$$

where

$$p_0 = \sum_{i=1}^n p_i, \bar{D}_i = diag(d_{i1}, d_{i2}, \ldots, d_{in})$$

and $(g(y(t-\overline{\tau}_i(t)))) = (g(y_1(t-\overline{\tau}_{i1}(t)))), \quad (g(y_2(t-\overline{\tau}_{i2}(t)))), \ldots, (g(y_n(t-\overline{\tau}_{in}(t))))$. In addition, system (4) is globally exponentially stable in the mean square, if there exist positive diagonal matrices $Q_i = diag(q_{i1}, q_{i2}, \ldots, q_{in})$, such that the following inequality holds:

$$\begin{aligned}\Omega = 2\gamma P + PA + A^T P - 2P\Gamma + D_0 + e^{\gamma\tau}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\eta_{ij}}D_i \\ +\sum_{i=1}^{n}\left(e^{\gamma\tau}Q_i + PW_iQ_i^{-1}W_i^T P\right) < 0,\end{aligned} \tag{5}$$

where $\Gamma = diag(\gamma_1, \gamma_2, \ldots, \gamma_n)$, $A = (a_{ij})_{n\times n}$, and W_i is an $n \times n$ square matrix, whose i^{th} row is composed of $(b_{i1}, b_{i2}, \ldots, b_{in})$ and other rows are all zeros, $i, j = 1, 2, \ldots, n$.

Proof: We use the following Lyapunov functional to derive the stability result

$$\begin{aligned}V(y,t) &= 2e^{\gamma t}\sum_{i=1}^{n} p_i \int_{x_i^*}^{y_i(t)} \frac{g_i(s)}{\alpha_i(s)}ds \\ &+\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t-\tau_{ij}(t)} e^{\gamma(s+\tau)}\left(g_j\left(y_j(s)\right)\right)^T q_{ij}\left(g_j\left(y_j(s)\right)\right)ds \\ &+\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\eta_{ij}}\int_{t-\tau_{ij}(t)} e^{\gamma(s+\tau)}\left(g_j\left(y_j(s)\right)\right)^T d_{ij}\left(g_j\left(y_j(s)\right)\right)ds.\end{aligned}$$

By Ito's formula, we can calculate $\mathcal{L}V$ along the trajectories of the system (4) and then we have

$$\begin{aligned}&\mathcal{L}V\left(y(t),t\right) \le 2\gamma e^{\gamma\tau} g^T\left(y(t)\right)Py(t) \\ &+2e^{\gamma t}\sum_{i=1}^{n} p_i\left(g_i\left(y_i(t)\right)\right)\left[-\beta_i\left(y_i(t)\right)+\sum_{j=1}^{n}a_{ij}g_j\left(y_j(t)\right)+\sum_{j=1}^{n}b_{ij}g_j\left(y_j\left(t-\tau_{ij}(t)\right)\right)\right] \\ &+\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}\left(e^{\gamma(t+\tau)}\left(g_j\left(y_j(t)\right)\right)^2 - e^{\gamma t}\left(g_j\left(y_j\left(t-\tau_{ij}(t)\right)\right)^2\right)\right) \\ &+\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\eta_{ij}}e^{\gamma(t+\tau)}\left(g_j\left(y_j(t)\right)\right)d_{ij}\left(g_j\left(y_j(t)\right)\right) \\ &-\sum_{i=1}^{n}\sum_{j=1}^{n} e^{\gamma t}\left(g_j\left(y_j\left(t-\tau_{ij}(t)\right)\right)\right)d_{ij}\left(g_j\left(y_j\left(t-\tau_{ij}(t)\right)\right)\right) \\ &+trace\left[\sigma^T\left(t, g1\left(y(t)\right), g\left(y\left(t-\tau_{ij}(t)\right)\right)\right)p_0 I\sigma\left(t, g\left(y(t)\right), g\left(y\left(t-\tau_{ij}(t)\right)\right)\right)\right]\end{aligned}$$

$$
\begin{aligned}
&\leq e^{\gamma t}\Big[2\big(f(x(t))-f(x^*)\big)^T \gamma P\big(x(t)-x^*\big)-2\big(f(x(t))-f(x^*)\big)^T P\Gamma\big(x(t)-x^*\big)\\
&+2\big(f(x(t))-f(x^*)\big)^T PA\big(f(x(t))-f(x^*)\big)\\
&+2\sum_{i=1}^{n} p_i\big(f(x(t))-f(x^*)\big)[b_{i1}, b_{i2}, \ldots, b_{in}]\\
&\times\big(f(x(t-\bar{\tau}_i(t)))-f(x^*)\big)+\sum_{i=1}^{n}\Big[e^{\gamma\tau}\big(f(x(t))-f(x^*)\big)^T Q_i\big(f(x(t))-f(x^*)\big)\\
&-\big(f(x(t-\bar{\tau}_i(t)))-f(x^*)\big)^T Q_i\big(f(x(t-\bar{\tau}_i(t)))-f(x^*)\big)\Big]\\
&+\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\eta_{ij}}e^{\gamma\tau}\big(f(x(t)-f(x^*))\big)^T D_i\big(f(x(t)-f(x^*))\big)\\
&-\sum_{i=1}^{n}\big(f(x(t-\bar{\tau}_i(t)))-f(x^*)\big)^T D_i\big(f(x(t-\bar{\tau}_i(t)))-f(x^*)\big)\\
&+\big(f(x(t))-f(x^*)\big)^T D_0\big(f(x(t))-f(x^*)\big)\\
&+\sum_{i=1}^{n}\big(f(x(t-\bar{\tau}_i(t)))-f(x^*)\big)^T D_i\big(f(x(t-\bar{\tau}_i(t)))-f(x^*)\big)\Big]
\end{aligned}
\tag{6}
$$

where $f(x(t-\bar{\tau}_i(t)))=(f(x_1(t-\tau_{i1}t)), f(x_2(t-\tau_{i2}(t))), \ldots, f(x_n(t-\tau_{in}(t))))^T$, $D_i = diag(d_{i1}, d_{i2}, \ldots, d_{in})$ and $Q_i = diag(q_{i1}, q_{i2}, \ldots, q_{in})$, $i = 1, 2, \ldots, n.$

It can be easily shown that the following conditions hold:

$$
\begin{aligned}
&2\sum_{i=1}^{n} p_i\big(f(x_i(t))-f(x^*)\big)[b_{i1}, b_{i2}, \ldots, b_{in}]\big(f(x(t-\bar{\tau}_i(t)))-f(x^*)\big)\\
&=\sum_{i=1}^{n} 2\big(f(x(t))-f(x^*)\big)^T PW_i\big(f(x(t-\bar{\tau}_i(t)))-f(x^*)\big)\\
&\leq\big(f(x(t))-f(x^*)\big)^T\left(\sum_{i=1}^{n} PW_iQ_i^{-1}W_i^T P\right)\big(f(x(t))-f(x^*)\big)\\
&+\sum_{i=1}^{n}\big(f(x(t-\bar{\tau}_i(t)))-f(x^*)\big)^T Q_i\big(f(x(t-\bar{\tau}_i(t)))-f(x^*)\big).
\end{aligned}
\tag{7}
$$

Using (7) in (6), we have

$$\mathcal{L}V(y(t),t) \le e^{\gamma t}\Big[2\big(f(x(t))-f(x^*)\big)^T \gamma P\big(x(t)-x^*\big)$$
$$-2\big(f(x(t))-f(x^*)\big)^T P\Gamma\big(x(t)-x^*\big)+2\big(f(x(t))-f(x^*)\big)^T PA\big(f(x(t))-f(x^*)\big)$$
$$+\big(f(x(t))-f(x^*)\big)^T\left[\sum_{i=1}^{n} PW_iQ_i^{-1}W_i^T P\right]\big(f(x(t))-f(x^*)\big)$$
$$+\sum_{i=1}^{n}\big(f(x(t-\overline{\tau}_i(t)))-f(x^*)\big)^T Q_i\big(f(x(t-\overline{\tau}_i(t)))-f(x^*)\big)$$
$$+\sum_{i=1}^{n}\Big[e^{\gamma\tau}\big(f(x(t))-f(x^*)\big)^T Q_i\big(f(x(t))-f(x^*)\big)$$
$$-\big(f(x(t-\overline{\tau}_i(t)))-f(x^*)\big)^T Q_i\big(f(x(t-\overline{\tau}_i(t)))-f(x^*)\big)\Big]$$
$$+\big(f(x(t))-f(x^*)\big)^T D_0\big(f(x(t))-f(x^*)\big)$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\eta_{ij}}e^{\gamma\tau}\big(f(x(t)-f(x^*))\big)^T D_i\big(f(x(t)-f(x^*))\big)\Big]$$
$$\le e^{\gamma t}\Big[2\big(f(x(t))-f(x^*)\big)^T \gamma P\big(x(t)-x^*\big)-2\big(f(x(t))-f(x^*)\big)^T P\Gamma\big(x(t)-x^*\big) \qquad (8)$$
$$+2\big(f(x(t))-f(x^*)\big)^T PA\big(f(x(t))-f(x^*)\big)$$
$$+f\big(x(t)-f(x^*)\big)^T\left[\sum_{i=1}^{n} PW_iQ_i^{-1}W_i^T P+e^{\gamma\tau}Q_i\right]\big(f(x(t))-f(x^*)\big)$$
$$+2\big(f(x(t))-f(x^*)\big)^T \gamma P\big(f(x(t))-f(x^*)\big)$$
$$-2\big(f(x(t))-f(x^*)\big)^T \gamma P\big(f(x(t))-f(x^*)\big)$$
$$+2\big(f(x(t))-f(x^*)\big)^T P\Gamma\big(f(x(t))-f(x^*)\big)$$
$$-2\big(f(x(t))-f(x^*)\big)^T P\Gamma\big(f(x(t))-f(x^*)\big)$$
$$+\big(f(x(t))-f(x^*)\big)^T D_0\big(f(x(t))-f(x^*)\big)$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\eta_{ij}}e^{\gamma\tau}\big(f(x(t)-f(x^*))\big)^T D_i\big(f(x(t)-f(x^*))\big)\Big].$$

According to Liao et al. (2005) (Lemma 2), the following inequality holds:

$$-2\big(f(x(t))-f(x^*)\big)^T PC\big(x(t)-x^*\big)$$
$$+2\big(f(x(t))-f(x^*)\big)^T PC\big(f(x(t))-f(x^*)\big)\le 0, \qquad (9)$$

$$-2\big(f(x(t))-f(x^*)\big)^T \gamma P\big(x(t)-x^*\big)$$
$$+2\big(f(x(t))-f(x^*)\big)^T \gamma P\big(f(x(t))-f(x^*)\big)\le 0. \qquad (10)$$

We can rewrite equation (8) using (9), then

$$\mathcal{L}V(y(t),t) \le \big(f(x(t)) - f(x^*)\big)^T e^{\gamma t}\Bigg[2\gamma P - 2P\Gamma + PA + A^T P$$

$$+ \sum_{i=1}^{n}\big(PW_iQ_i^{-1}W_i^T P + e^{\gamma\tau}Q_i\big) + D_0 + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\eta_{ij}}e^{\gamma\tau}D_i\Bigg]\big(f(x(t)) - f(x^*)\big).$$

By the generalised Ito's formula, it yields

$$\mathbb{E}V(y(t),t) \le \mathbb{E}V(y(0),0) + \lambda_{\max}(\Omega)\mathbb{E}\int_0^t e^{\gamma s}\|g(y(s))\|^2\,ds$$

$$\le \mathbb{E}V(y(0),0) + \lambda_{\max}(\Omega)\mathbb{E}\int_0^t e^{\gamma s}\|y(s)\|^2\,ds.$$

On the other hand, from the definition of $V(y(t), t)$,

$$\mathbb{E}V(y(0),0) \le \lambda_{\max}(P)\mathbb{E}\|\phi\|^2 + \lambda_{\max}(Q_i)\tau e^{\gamma\tau}\mathbb{E}\|\phi\|^2 + \lambda_{\max}(\bar{D}_i)\tau e^{\gamma\tau}\mathbb{E}\|\phi\|^2$$

$$= \psi_1\mathbb{E}\|\phi\|^2,$$

$$\mathbb{E}V(y(t),t) \ge e^{\gamma t}\lambda_{\min}(P)\mathbb{E}\|y(t)\|^2.$$

Therefore, we have

$$e^{\gamma t}\mathbb{E}\|y(t)\|^2 \le \frac{\psi_1}{\lambda_{\min}(P)}\mathbb{E}\|\phi\|^2 + \frac{\lambda_{\max}(\Omega)}{\lambda_{\min}(P)}\mathbb{E}\int_0^t e^{\gamma s}\|y(s)\|^2\,ds.$$

By the Gronwall integral inequality, we have

$$\mathbb{E}\|y(t)\|^2 \le c_1 e^{-(\gamma - c_2)t},$$

where $c_1 = \frac{\psi_1}{\lambda_{\min}(P)}\mathbb{E}\|\phi\|^2$ and $c_2 = \frac{\lambda_{\max}(\Omega)}{\lambda_{\min}(P)}$.

Therefore if (3) holds, then $\mathcal{L}V(y(t),t) < 0$ for any $(f(x(t)) - f(x^*)) \neq 0$. $\mathcal{L}V(y(t),t) = 0$ if and only if $(f(x(t)) - f(x^*)) = 0$. This completes the proof.

Remark 3.3: If the time delay $\tau_{ij}(t) = \tau_j(t)$ in (1), $i, j = 1, 2, \ldots, n$, we will have the following result.

Corollary 3.4: The equilibrium point of (4) with $\tau_{ij} = \tau_j$ is globally exponentially stable if there exist positive diagonal matrix $Q = diag(q_1, q_2, \ldots, q_n)$ such that the following inequality holds:

$$2\gamma P - 2P\Gamma + PA + A^T P + PBQ^{-1}B^T P + e^{\gamma\tau}Q + D_0 + \sum_{i=1}^{n}\frac{1}{1-\eta_i}e^{\gamma\tau}D_i < 0 \tag{11}$$

where $B = (b_{ij})_{n\times n}$ and others are the same as those defined in Theorem 3.2.

Proof: We use the following Lyapunov functional to derive the stability result,

$$V(y,t)=\sum_{i=1}^{n}2p_i\int_{x_i^*}^{y_i(t)}\frac{g_i(s)}{\alpha_i(s)}ds+\int_{t-\tau_j(t)}g_j\left(y_j(s)\right)q_jg_j\left(y_j(s)\right)ds$$
$$+\sum_{j=1}^{n}\frac{1}{1-\eta_j}\int_{t-\tau_j(t)}g_j\left(y_j(s)\right)d_jg_j\left(y_j(s)\right)ds$$

where $q_j > 0$ and $d_j > 0$ $i = 1, 2, \ldots, n$. In a similar manner to the proof of Theorem 3.1, condition (11) is easy to be derived. The details are omitted.

Remark 3.5: Theorem 3.2 and Corollary 3.4 can be expressed in the form of LMI as

$$\begin{bmatrix}\Psi & PW_1 & \cdots & PW_n\\ W_1^TP & -Q_1 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ W_n^TP & 0 & \cdots & -Q_n\end{bmatrix}<0$$

and

$$\begin{bmatrix}2\gamma P-2P\Gamma+PA+A^TP+e^{\gamma\tau}Q+D_0+e^{\gamma\tau}\sum_{i=1}^{n}D_i & PB\\ B^TP & -Q\end{bmatrix}<0$$

where $\Psi=2\gamma P-2P\Gamma+PA+A^TP+e^{\gamma\tau}\sum_{i=}^{n}Q_i+D_0+e^{\gamma\tau}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\eta_{ij}}D_i$. Therefore, the conditions in Theorem 3.2 and Corollary 3.4 are easy to verify.

Remark 3.6: Theorem 3.2 gives a novel condition to check the global exponential stability in the sense of mean square for system (4). Recently, the stability problem was discussed in Blythe et al. (2001), and Liao and Mao (1996) for stochastic Hopfield neural networks with constant delay. In addition, compared with Blythe et al. (2001), Liao and Mao (1996), and Wan and Sun (2005), multiple time-varying delays is taken into account in this paper. This paper differs from Huang and Cao (2007) in which the sign of entries in connection weight matrix and the non-linear diffusion term involved with the general activation function. Therefore, the result proposed, here, is less conservative and less restrictive than some early results.

Remark 3.7: In this paper, a new class of stochastic Cohen-Grossberg neural networks with multiple time-varying delays has been studied. The advantage of this approach is that sufficient conditions have been derived to obtain less conservative results. In Wang et al. (2006a), the authors studied the stability analysis for a class of stochastic Cohen-Grossberg neural networks with discrete and distributed delays. It should be noted that the stochastic neural networks studied in this paper has the time-varying delays as in Huang and Cao (2007). Therefore, our results and those established in Huang and Cao (2007), and Wang et al. (2006a) are complementary to each other.

Remark 3.8: In Cao and Li (2005), Rong (2005), and Wu et al. (2007a, 2007b), the Cohen-Grossberg neural networks with time delays were investigated, and several LMI-based conditions were proposed to guarantee the stability of equilibrium point of Cohen-Grossberg neural networks. However, the stochastic term was not taken into account in the models. Therefore, our developed results presented in this paper are more general than those reported in Cao and Li (2005), Rong (2005), and Wu et al. (2007a, 2007b).

Remark 3.9: The results in Chen and Rong (2003), Huang et al. (2003), Sun and Wan (2005), and Wang and Zou (2002) did not consider the sign of entries in connection weight matrix, therefore, the neuron's excitatory and inhibitory effect on neural networks was not considered. In contrast, the delayed connection weight matrix could be decomposed into sum of the singular matrices in which the sign entries in connection weight matrix and the neuron's excitatory and inhibitory effect on neural networks is considered.

4 Numerical example

In this section, one numerical example is provided to demonstrate the effectiveness and applicability of the proposed method.

Consider the two-neuron stochastic neural networks (1) with the following parameters:

$$\Gamma = \begin{bmatrix} 7 & 0 \\ 0 & 6.5 \end{bmatrix} \quad A = \begin{bmatrix} 0.5330 & 1.0328 \\ 1.0328 & 0.3621 \end{bmatrix} \quad B = \begin{bmatrix} -0.0184 & -0.1375 \\ -0.6138 & -0.0802 \end{bmatrix}$$

We assume that $\tau = 0.5$, $\tau_{11}(t) = \tau_{12}(t) = 0.5 + 0.5 \sin t$, $\tau_{21}(t) = \tau_{22}(t) = 0.5 - 0.5 \cos t$ and

$$\sigma\left(t, y(t), g\left(y(t)\right), g\left(y\left(t-\tau(t)\right)\right)\right) = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix},$$

where

$$\sigma_{11} = \sigma_{22} = 0.5 \tanh y_1(t) + 0.5 \tanh y_1\left(t-\tau_{11}(t)\right) + 0.5 \tanh y_1\left(t-\tau_{12}(t)\right)$$

$$\sigma_{12} = \sigma_{21} = 0.4 \tanh y_2(t) + 0.4 \tanh y_3\left(t-\tau_{21}(t)\right) + 0.4 \tanh y_2\left(t-\tau_{22}(t)\right).$$

Now the matrix B can be expressed in the following two matrices

$$W_1 = \begin{bmatrix} -0.0184 & -0.1375 \\ 0 & 0 \end{bmatrix} \text{ and } W_2 = \begin{bmatrix} 0 & 0 \\ -0.6137 & -0.0802 \end{bmatrix}.$$

By setting $P = I$, $D_0 = D_1 = D_2 = diag(1.0, 0.64)$ and taking $\gamma = 0.324$ and by using the MATLAB LMI toolbox, we can obtain the following feasible solution for the LMIs in Theorem 3.2:

$$Q_1 = \begin{bmatrix} 0.0802 & 0 \\ 0 & 0.2755 \end{bmatrix} \text{ and } Q_2 = \begin{bmatrix} 0.3591 & 0 \\ 0 & 0.1996 \end{bmatrix}.$$

Therefore, the concerned stochastic Cohen-Grossberg neural networks with multiple time-varying delays is globally exponentially stable in the mean square sense. Since the time delays are variant in t, the stability criteria in Wang et al. (2006b, 2007) cannot be applicable to the example.

5 Conclusions

In this paper, a new sufficient condition is derived to guarantee the global exponential stability of the equilibrium point for stochastic Cohen-Grossberg neural networks with multiple time-varying delays, which is different from the existing ones and has wider applications. The obtained result can be expressed in the form of LMI and could be verified easily. Finally, a numerical example is provided to demonstrate the effectiveness of the main results described in this paper. In future, it is possible to extend our main results for robust stability analysis for stochastic Cohen-Grossberg neural networks with unbounded time-varying delays.

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