

**Avinashilingam Institute for Home Science and Higher Education for Women**  
Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD (now MoE)  
Re-accredited with A++ Grade by NAAC. CGPA 3.65/4, Category I by UGC  
Coimbatore - 641 043, Tamil Nadu, India

**Continuous Internal Assessment Test II – April 2025**  
**VI Semester**

**Class : III UG**  
**Branch : Mathematics**

**Time : 2 Hours**  
**Max. Marks : 60**

**21BMAC26 – Graph Theory**

**Course Outcomes:**

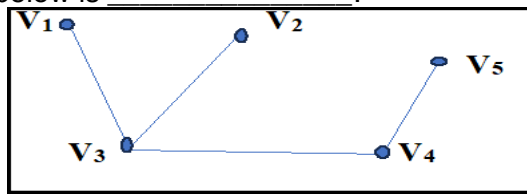
- CO1: Understand the basic definitions and properties of graphs.  
CO2: Identify different kinds of special graphs.  
CO3: Apply the concept of graph theory to relevant fields.  
CO4: Develop mathematical models using graph theory.  
CO5: Solve real world problems using graph theory.

**Part A**

**6 x 1 = 6**

**Choose the Correct Answer**

- The length of Hamiltonian path in a connected graph of  $n$  vertices is \_\_\_\_\_. CO3K1  
a.  $n-1$                       b.  $n$                       c.  $1$                       d.  $0$
- A \_\_\_\_\_ graph cannot have a circuit. CO4K1  
a. simple                      b. minimally connected                      c. connected                      d. complete
- A vertex of degree one is called as a \_\_\_\_\_. CO4K2  
a. cut vertex                      b. pedant vertex                      c. isolated vertex                      d. connectivity vertex
- The center of the graph below is \_\_\_\_\_. CO4K1



- a.  $V_3, V_5$     b.  $V_3, V_4$     c.  $V_1, V_2$     d.  $V_1, V_5$                       CO4K1
- The spanning tree is also referred to as \_\_\_\_\_. CO5K2  
a. Skeleton                      b. Scaffolding                      c. both a. & b.                      d. neither a. nor b.
- The one connected graph is same as the \_\_\_\_\_. CO5K1  
a. separable graph                      b. standard graph  
c. edge connected graph                      d. vertex connected graph

**Part B**

**3 x 6 = 18**

**Answer ALL questions**

- a. Prove that in a complete graph with  $n$  vertices there are  $(n-1)/2$  edge disjoint Hamiltonian circuit if  $n$  is an odd number  $n \geq 3$ . CO3K3  
(or)
- b. Prove that a tree with  $n$  vertices has  $n-1$  edges. CO4K3
- a. A graph  $G$  with  $n$  vertices,  $n-1$  edges and no circuit is connected. CO4K3  
(or)
- b. Prove that every tree has either one or more centres. CO4K2
- a. Prove that a vertex  $V$  in a connected graph  $G$  is a cut vertex if and only if there exist two vertices  $x$  and  $y$  in  $G$  such that every path between  $x$  and  $y$  passes through  $V$ . CO5K3  
(or)
- b. Show that vertex connectivity of any graph  $G$  never exceed the edge connectivity. CO5K3

**Part C**

**3 x 12 = 36**

**Answer ALL questions**

- a. State and prove DIRAC's theorem. CO3K3  
(or)
- b. State and prove Ore theorem. CO4K3
- a. (i) Prove that the distance between the vertices of a connected graph is a metric. CO4K3  
(ii) Show that every graph has one spanning tree. CO4K3  
(or)
- b. (i) Show that if the intersection of two paths is a disconnected graph, then the union of the two paths has at least one circuit.  
(ii) Prove that with respect to any of its spanning trees, a connected graph of  $n$  vertices and  $e$  edges has  $n-1$  tree branches and  $e-n+1$  chords. CO4K3
- a. (i) Prove that a ring sum of any two cut set in a graph is either a third cut set or an edge disjoint union of cut set.  
(ii) Prove that on minimum vertex connectivity one can get the graph with  $G$  of  $n$  vertices and  $e$  edges ( $e \geq n-1$ ), if the integral part of the number is  $(2e)/n$ . CO5K3  
(or)
- b. Prove that with respect to a given spanning tree  $T$ , a branch  $b_i$  that determines a fundamental cut set  $S$  is contained in every fundamental circuit associated with the chords in  $S$  and is no other. CO5K3

No. of copies: 27