14-15

Avinashilingam Institute for Home Science and Higher Education for Women
Deemed to be University Estd. u/s 3 of UGC Act 1956, Category 'A' by MHRD (now MoE)
Re-accredited with 'A++' Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore - 641 043, Tamil Nadu, India

Master's Degree Examination - May 2025
IV Semester

Class : II PG **Time: 3 Hours**
Major : Mathematics **Max. Marks: 100**

23MMAC22 Functional Analysis

Course Outcomes:
CO1: Explain the fundamental concepts of functional analysis in applied contexts.
CO2: Understand the properties of Banach space.
CO3: Use the Hilbert space to construct orthonormal sets.
CO4: Identify normal, self adjoint and unitary operators.
CO5: Communicate the spectrum of bounded linear operator.

Part A **10 x 1 = 10**
Choose the Correct Answer

1. A _____ is a complete normed linear space. CO1K1
   a. Compact b. Connected c. Hausdroff d. Banach space

2. In a Banach algebra B (N), which of the following is true?. CO1K2
   a. $\|TT'\| = \|T\|\|T'\|$ b. $\|TT'\| = \leq \|T\|\|T'\|$ c. $\|TT'\| \geq \|T\|\|T'\|$ d. $\|TT'\| > \|T\|\|T'\|$

3. If B and B′ are Banach spaces and if T is a continuous linear transformation of B onto B′, then T is _____ CO2K2
   a. open b. closed c. complete d. projection

4. A nonempty subset X of normed linear space N is _____ CO2K1
   a. bounded b. closed c. linear space d. open sphere

5. If x and y are any two vectors in a Hilbert space then |(x,y)| CO3K2
   a. $\leq \|x\|$ b. $\leq \|x\|\,\|y\|$ c. $\geq \|x\|\,\|y\|$ d. $\geq \|y\|$

6. Every non zero Hilbert space contains a complete_____ CO3K1
   a. orthogonal b. Hilbert space c. closed linear d. orthonormal set

7. If T is an operator on H for which (Tx, x) =0, for all x, then_____ CO4K2
   a. $T \neq 0$ b. T=0 c. x=0 d. T<0

8. An operator T on H is_____ CO4K2
   a. normal b. self adjoint c. real d. imaginary

9. If P is the projection on a closed linear subspace M of H, then M reduces an operator T If_____ CO5K1
   a. TP= PT b. T*P=P*T c. PT= TM d. T*M=MT

10. If T is normal, then M_i's are_____ CO5K1
    a. pair wise orthogonal b. span c. reduces T d. all

Part B 5 x 6 = 30

Answer ALL questions

Each answer should not exceed 400 words or two pages

11. a. Let N be a normed linear space then prove that (i) Norm is a continuous function (ii) Addition and scalar multiplication are jointly continuous. CO1K3

(or)

11. b. Let M be a linear subspace of a normed linear space N and let f be a functional defined on M. If x_0 is a vector not in M and if $M_0 = M+[x_0]$ is the linear subspace spanned by M and x_0, then prove that f can be extended to a functional f_0 and M_0 such that $\|f_0\| = \|f\|$. CO1K4

12. a. If P is a projection on a Banach space B and if M and N are its range null spaces then prove that M and N are closed linear subspaces of B such that $B = M \oplus N$. CO2K4

(or)

12. b. If B and B' are Banach spaces and if T is linear transformation of B into B' then prove that T is continuous iff its graph is closed. CO2K5

13. a. If x and y are any two vectors of a Hilbert space then prove that $|(x, y)| \leq \|x\| \, \|y\|$ CO3K4

(or)

13. b. Prove that a closed convex subject C of a Hilbert space H contains a unique vector of smallest norm. CO3K4

14. a. Let y be a fixed vector in a Hilbert space H and let f_y be a scalar valued function on H defined by the formula $f_y(x) = (x, y), \forall x \in H$. Then show that f_y is a functional in H. CO4K2

(or)

14. b. If A_1 and A_2 are self adjoint operator on a Hilbert space H then prove that their product A_1A_2 is also self adjoint iff $A_1A_2 = A_2A_1$. CO4K3

15. a. Prove that a closed linear subspace M of H is invariant under on operator T iff $M^{\perp}$ is invariant under T CO5K4

(or)

15. b. If T is normal then prove that the M_i' s are pair wise orthogonal. CO5K1

Part C 5 x 12 = 60

Answer ALL questions

Each answer should not exceed 800 words or four pages

16. a. State and Prove Hahn Banach theorem. CO1K1

(or)

16. b. Let N, N' be two normed linear spaces as let $T:N \to N$ be a linear transformation then Prove that the following statements are equivalent. (i) T is continuous (ii) T is continuous at the origin in the sense that $x_n \to 0 \Rightarrow T(x_n) \to 0$. (iii) there exist a real number k such that $\|T(K)\| \leq K\|x\|, \forall x \in N$ (iv) If $S = \{x \in N / \|x\| \leq T$ is the closed unit sphere in N. Then T(S) is bounded in N. CO1K4

17. a. State and Prove Open mapping theorem. CO2K1

(or)

17. b. State and Prove Uniform Boundedness theorem. CO2K1

18. a. Let M be a closed linear subspace of a Hilbert space H. Let x be a vector not in M and let d be the distance from x to M then prove that there exist a unique vector y_0 in M such that $\|x - y_0\| = d$. CO3K3

(or)

18. b. If M is a proper closed linear subspace of a Hilbert space H then prove that there exist a non-zero vector Z_0 in H such that $Z_0 \perp M$. CO3K4

19. a. If N_1 and N_2 are normal operators on H with the property that either commutes with the adjoint of the other then prove that N_1+N_2 and N_1N_2 are also normal. CO4K2

(or)

19. b. If T is a operator on H then prove that T is normal iff its real and imaginary parts are commute. CO4K5

20. a. Prove that if P is a projection on a closed linear subspace M of H reduces on operator T iff TP=PT. CO5K2

(or)

20. b. If T is normal, then prove that the M_i's span H. CO5K4

****