CHAPTER II

CHAPTER II

BASIC DEFINITIONS AND PROPERTIES OF FUZZY COGNITIVE MAPS, FUZZY COGNITIVE BIMAPS, FUZZY COGNITIVE INTERVAL MAPS, SUPER FUZZY COGNITIVE MAPS AND NEUTROSOPHIC COGNITIVE MAPS

Definition 2.1

A **Fuzzy Cognitive Map (FCM)** is a directed graph with concepts like policies, events etc., as nodes and causalities as edges or arcs. It represents causal relationship between concepts. Each arc is accompanied by a weight that defines the type of causal relation between the two nodes.

Definition 2.2

When the nodes of the FCM are fuzzy sets then they are called as **fuzzy nodes.**

Definition 2.3

Consider the nodes / concepts C_1, C_2, …, C_n of the FCM. Suppose the directed graph is drawn using edge weight $e_{ij} \in \{0, 1, -1\}$ or $e_{ij} \in [0,1]$ .The matrix E defined by $E = (e_{ij})$ where e_{ij} is the weight of the directed edge $C_i C_j$ is called the **adjacency matrix** of the FCM, also known as the **connection matrix** of the FCM. Simple FCMs have edge values from the set { -1, 0, 1 }.

Definition 2.4

Let C_1, C_2, …, C_n be the nodes of an FCM. $A = (a_1, a_2, \ldots, a_n)$ where

$a_i \in \{0, 1\}$ is called the **instantaneous state vector** and it denotes the on-off position of the node at an instant.

$a_i = 0$ if a_i is off and

$a_i = 1$ if a_i is on for $i = 1, 2, \ldots, n$.

Definition 2.5

Let $C_1, C_2, \ldots, C_n$ be the nodes of an FCM. Let $\overrightarrow{C_1C_2}$, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$ ….., $\overrightarrow{C_iC_j}$ be the edges of the FCM ($i \neq j$). Then the edges form a **directed cycle**. An FCM is said to be **cyclic** if it possesses a directed cycle. An FCM is said to be **acyclic** if it does not possess any directed cycle.

Definition 2.6

An FCM with cycles is said to have a **feedback.**

Definition 2.7

When there is a feedback in an FCM, i.e., when the causal relations flow through a cycle in a revolutionary way, the FCM is called a **dynamical system.**

Definition 2.8

Let $\overrightarrow{C_1C_2}$, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$,…, $\overrightarrow{C_{n-1}C_n}$ be a cycle. When C_i is switched on and if the causality flows through the edges of a cycle and if it again causes C_i, we say that the dynamical system goes round and round. This is true for any node C_i, for $i = 1, 2, \ldots, n$. The equilibrium state for this dynamical system is called the **hidden pattern.**

Definition 2.9

If the equilibrium state of a dynamical system is a unique state vector, then it is called a **fixed point.**

Definition 2.10

If the FCM settles down with a state vector repeating in the form $A_1 \rightarrow A_2 \rightarrow \ldots \rightarrow A_i \rightarrow A_1$ then this equilibrium is called a **limit cycle**.

Methods of Finding the Hidden Pattern

Definition 2.11

Finite number of FCMs can be combined together to produce the joint effect of all the FCMs. Let $E_1, E_2, \ldots, E_p$ be the adjacency matrices of the FCMs with nodes $C_1, C_2, \ldots, C_n$ then the **combined FCM** is got by adding all the adjacency matrices $E_1, E_2, \ldots, E_p$.

We denote the combined FCM adjacency matrix by $E = E_1 + E_2 + \ldots + E_p$.

Notation 2.12

Suppose $A = (a_1, \ldots, a_n)$ is a vector which is passed into a dynamical system E. Then $AE = (a'_1, \ldots, a'_n)$ after thresholding and updating the vector suppose we get $(b_1, \ldots, b_n)$ we denote that by $(a'_1, a'_2, \ldots, a'_n) \rightarrow (b_1, b_2, \ldots, b_n)$. Thus the symbol '→' means the resultant vector has been thresholded and updated.

Definition 2.13

Fuzzy Cognitive bimaps (FCBMs) are fuzzy signed directed bigraphs with feedback ($G = G_1 \cup G_2$ is said to be a bigraph if G_1 and G_2 are two graphs such that G_1 is not a subgraph of G_2 or G_2 is not a subgraph of G_1 i.e., they have either distinct vertices or edges). The directed edge e_{ij}^{p} from causal concept c_i^p concept c_j^p measures how much c_i^p causes c_j^p, (p=1,2).The edge values e_{ij}^{p} takes values in the fuzzy causal interval [-1, 1], $e_{ij}^{p} = 0$ indicates no causality, $e_{ij}^{p} > 0$ indicates causal increase, c_j^p increases as c_i^p increases (or c_j^p decreases as c_i^p decreases); $e_{ij}^{p} < 0$ indicates causal decrease or negative causality c_i^p decreases as c_j^p increases (and or c_i^p increases or c_j^p decreases) (p=1, 2).Simple FCBMs have edge values $e_{ij}^{p} \in \{-1, 0, 1\}$, (p=1, 2).

Definition 2.14

Fuzzy Cognitive Interval Maps

Suppose that p number of experts spells out their opinion on n number of nodes. Using the directed graph suppose we get a fuzzy matrix P_i given by the i^{th} expert and further the FCM is not a simple FCM, $1 \leq i \leq p$.

The entries in the n×n matrices will be from the fuzzy interval [–1, 1]. Now in general if we take the collection of all n×n fuzzy matrices with entries from the fuzzy interval [–1, 1] we have the interval of fuzzy n×n matrix. [A, B] would be an infinite collection in general. Further this fuzzy interval matrices associated with any FCM model will satisfy the following condition:

1. The fuzzy interval matrix will always be a square matrix.
2. The fuzzy interval matrix will always have the main diagonal entries to be zero.
3. The number of fuzzy matrices in the fuzzy interval n × n matrices is though infinite by all means for us in our system we can have only a finite number of fuzzy matrices associated with a FCM and with its associated fuzzy interval n×n square matrices.

This fuzzy interval of n×n square matrix associated with the FCMs of p-experts will be called as the **Fuzzy Cognitive Interval Maps (FCIMs)** of the multi experts dynamical system, as the related connection matrices forms an interval of n×n square matrices.

This fuzzy interval of matrices satisfying the conditions 1, 2 and 3 has lots of advantage over the infinite collection. Suppose we have some p experts who have given their opinion on n concepts. Then we will have only p, n×n square fuzzy matrices with entries from the fuzzy interval [–1, 1]. Now we using these p, n×n square fuzzy matrices form an associated interval of square n×n matrices by the following method.

One can know if they are the fuzzy connection matrices all the main diagonal terms are zero. Now in order to obtain the fuzzy interval n×n matrix [A,B] using the p-matrices one have to construct A and B for A and B may not exist in general. Now call A the minimal matrix (element) of [A, B] and B the maximal (element) of [A, B]. One can give the method by which A is built using the p matrices. Suppose $A = (a_{ij})$, $a_{ii} = 0$ for $1 \leq i \leq n$; $1 \leq i, j \leq n$. So all the diagonal elements are zero. One can make the observation of the element a_{12} in all the p matrices $P_1, \ldots, P_p$ where $P^t = (a^t_{ij})$, $1 \leq t \leq p$; we take the minimum value from the p entries $a^1_{12}, a^2_{12}, \ldots, a^p_{12}$ and put in the new matrix as the value of a_{12} likewise for every a_{ij}, $1 \leq i, j \leq n$.

This newly formed matrix may not in general be any of the matrices given by the p experts. Call this matrix the minimal element A of the fuzzy interval of the n×n matrices got from the p experts.

Like wise one can form the maximal matrix $B = (b_{ij})$ by taking the maximal element. If B is the maximal matrix, the elements of the fuzzy interval of matrices, in general B need not be a connection matrix given by any of the p experts. Now having obtained the minimal and maximal fuzzy matrices as A and B; form the fuzzy interval matrix [A, B]. Clearly by the very construction of A and B, all the p connection fuzzy matrices given by the p experts will lie in the fuzzy interval of matrices [A, B].

Now having constructed the minimal and maximal element on the fuzzy interval of matrices we construct the optimal fuzzy matrix O as follows;

$$O = \frac{A+B}{2} = \frac{(a_{ij}+b_{ij})}{2} = (o_{ij})$$

may be in [A, B] if O is not in [A, B] we include O in the fuzzy interval of matrices [A, B] and call it as the optimal fuzzy matrix of the interval of fuzzy matrices and the associated weighted directed graph will be called as the optimal weighted directed graph.

Now one can work with the minimal matrix A, maximal matrix B and optimal matrix O and compare our results. One can also adjoin the matrix $\overline{A}$ which will be the average matrix of the combined matrices of the p experts excluding the minimal, optimal and the maximal matrices provided they are not the opinion given by any of the p experts.

These four matrices A, B, O and $\overline{A}$ may or may not in general be some of the p experts opinion, The fuzzy interval matrix which is formed will have A to be minimal and B to be the maximal element [A, B], the fuzzy interval matrix i.e., if $M \in [A, B]$ and if $M = (m_{ij})$ then $(a_{ij}) \leq (m_{ij}) \leq (b_{ij})$, $1 \leq i, j \leq n$ also for $O = (o_{ij})$ then, $a_{ij} \leq o_{ij} \leq b_{ij}$, $1 \leq i, j \leq n$. Further for $\overline{A} = (\overline{a_{ij}})$.We have $a_{ij} \leq \overline{a_{ij}} \leq b_{ij}$ for $1 \leq i, j \leq n$.

Definition 2.15

Super Fuzzy Cognitive Maps

Suppose n experts want to work with a problem P using a FCM model, then how to form an integrated dynamical system which can function simultaneously using the n experts opinion. Suppose the first expert spells out the attributes of a problem as $x_1^1, x_2^1, \ldots, x_{t_1}^1$, the second expert gives the attributes as $x_1^2, x_2^2, \ldots, x_{t_2}^2$ and so on. Thus the i^{th} expert gives the attributes with which he wishes to work as $x_1^i, x_2^i, \ldots, x_{t_i}^i$; i = 1, 2, 3, … n. The problem is modeled using super fuzzy cognitive maps and the special diagonal super fuzzy matrix; this super matrix will be called as the super connection matrix of the Super Fuzzy Cognitive Map. The special feature of this special super diagonal fuzzy matrix would be all the diagonal matrices are square matrices and the main diagonal of each of these sub matrices of the special fuzzy super diagonal matrix is zero. The special diagonal super fuzzy matrix for the problem takes the following form and is denoted by M_D.

$$M_D = \begin{array}{c} x_1^1 \\ x_2^1 \\ \vdots \\ x_{t_1}^1 \\ x_1^2 \\ x_2^2 \\ \vdots \\ x_{t_2}^2 \\ \vdots \\ x_1^n \\ x_2^n \\ \vdots \\ x_{t_n}^n \end{array} \overset{\begin{array}{cccc} x_1^1\ x_2^1 \ldots x_{t_1}^1 & x_1^2\ x_2^2 \ldots x_{t_2}^2 & \ldots & x_1^n\ x_2^n \ldots x_{t_n}^n \end{array}}{\left(\begin{array}{c|c|c|c} M_1^1 & (0) & \ldots & (0) \\ \hline (0) & M_2^2 & & (0) \\ \hline & & & \\ \hline (0) & (0) & & M_n^n \end{array}\right)}$$

M_i^i is a fuzzy matrix with main diagonal elements to be zero i.e.,

$$M_i^i = \begin{bmatrix} 0 & m_{12}^i & \dots & m_{1ti}^i \\ m_{21}^i & 0 & \dots & m_{2ti}^i \\ \vdots & \vdots & & \vdots \\ m_{t_i^2}^i & m_{t_i^2}^i & \dots & 0 \end{bmatrix}$$

for i = 1, 2, …, n.

Definition 2.16

A **neutrosophic directed graph** is a directed graph which has at least one edge to be an indeterminacy.

Definition 2.17

A **Neutrosophic Cognitive Map** (NCM) is a neutrosophic directed graph with concepts like policies, events etc., as nodes and causalities or indeterminates as edges. It represents the causal relationship between concepts.

Let $C_1, C_2, \ldots, C_n$ denote n nodes, further each node is a neutrosophic vector from neutrosophic vector space V. So a node C_i will be represented by $(x_1, x_2, \ldots, x_n)$ where x_k's are zero or one or I (I is the indeterminate) and $x_k = 1$ means that the node C_k is in the on state and $x_k = 0$ means that the node is in the off state and $x_k = I$ means the nodes state is an indeterminate at that time or in that situation.

Let C_i and C_j denote the two nodes of the NCM. The directed edge from C_i to C_j denotes the causality of C_i on C_j called connections. Every edge in the NCM is weighted with a number in the set {-1, 0, 1, I}. Let e_{ij} be the weight of the directed edge C_iC_j, $e_{ij} \in \{-1, 0, 1, I\}$. $e_{ij} = 0$ if C_i does not have any effect on C_j,

$e_{ij} = 1$ if increase (or decrease) in C_i causes increase (or decrease) in C_j , $e_{ij} = -1$ if increase (or decrease) in C_i causes decrease (or increase) in C_j. $e_{ij} = I$ if the relation or effect of C_i on C_j is an indeterminate.

Definition 2.17

NCMs with edge weight from $\{-1, 0, 1, I\}$ are called **simple NCMs.**

Definition 2.18

Let $C_1, C_2, \ldots, C_n$ be nodes of a NCM. Let the neutrosophic matrix N(E) be defined as $N(E) = (e_{ij})$ where e_{ij} is the weight of the directed edge C_iC_j, where $e_{ij} \in \{-1, 0, 1, I\}$. N(E) is called the **neutrosophic adjacency matrix** of the NCM.

Definition 2.19

Let $C_1, C_2, \ldots, C_n$ be the nodes of the NCM. Let $A = (a_1, a_2, \ldots, a_n)$ where $a_i \in \{0, 1, I\}$. **A** is called the **instantaneous state neutrosophic vector** and it denotes the on-off-indeterminate state position of the node at an instant.

$a_i = 0$ if a_i is off (no effect)

$a_i = 1$ if a_i is on (has effect)

$a_i = I$ if a_i is indeterminate (effect cannot be determined)

for $i = 1, 2, \ldots, n$.

Definition 2.20

Let $C_1, C_2, \ldots, C_n$ be the nodes of the FCM. Let $\overrightarrow{C_1C_2}, \overrightarrow{C_2C_3}, \overrightarrow{C_3C_4}, \ldots, \overrightarrow{C_iC_j}$ be the edges of the NCM. Then the edges form a directed cycle. An NCM is said to be **cyclic** if it possesses a directed cyclic. An NCM is said to be **acyclic** if it does not possess any directed cycle.

Definition 2.21

An NCM with cycles is said to have a **feedback**. When there is a feedback in the NCM i.e. when the causal relations flow through a cycle in a revolutionary manner the NCM is called a **dynamical system.**

Definition 2.22

Let $\overrightarrow{C_1C_2}$, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$,…, $\overrightarrow{C_{n-1}C_n}$ be cycle, when C_i is switched on and if the causality flow through the edges of a cycle and if it again causes C_i, we say that the dynamical system goes round and round. This is true for any node C_i, for $i = 1, 2,\ldots, n$. the equilibrium state for this dynamical system is called the **hidden pattern.**

Definition 2.23

If the equilibrium state of a dynamical system is a unique state vector, then it is called a fixed point. Consider the NCM with $C_1, C_2,\ldots, C_n$ as nodes. For example let us start the dynamical system by switching on C_1. Let us assume that the NCM settles down with C_1 and C_n on, i.e. the state vector remain as (1, 0,…, 1) this neutrosophic state vector (1,0,…, 0, 1) is called the **fixed point.**

Definition 2.24

If the NCM settles with a neutrosophic state vector repeating in the form

$$A_1 \rightarrow A_2 \rightarrow \ldots \rightarrow A_i \rightarrow A_1,$$

then this equilibrium is called a **limit cycle** of the NCM.

Methods of determining the hidden pattern:

Let $C_1, C_2,\ldots, C_n$ be the nodes of an NCM, with feedback. Let E be the associated adjacency matrix. Let us find the hidden pattern when C_1 is switched on when an input is given as the vector $A_1 = (1, 0, 0, \ldots, 0)$, the data should pass through the neutrosophic matrix N(E), this is done by multiplying A_1 by the matrix N(E). Let $A_1N(E) = (a_1, a_2, \ldots, a_n)$ with the threshold operation that is by replacing a_i by 1 if $a_i \geq k$ and a_i by 0 if $a_i < k$ (k – a suitable positive integer) and a_i by I if a_i is not a integer. We update the resulting concept, the concept C_1 is included in the updated vector by making the first coordinate as 1 in the resulting vector. Suppose $A_1N(E) \rightarrow A_2$ then consider $A_2N(E)$ and repeat the same procedure. This procedure is repeated till we get a limit cycle or a fixed point.