Avinashilingam Institute for Home Science and Higher Education for Women
Deemed to be University Estd. u/s 3 of UGC Act 1956, Category 'A' by MHRD (now MoE)
Re-accredited with 'A++' Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore - 641 043, Tamil Nadu, India

Master's Degree Examination - May 2025
IV Semester

Class : II PG **Time: 3 Hours**
Major : Mathematics **Max. Marks: 100**

23MMAC23 Mathematical Methods

Course Outcomes:
CO1: Demonstrate first fundamental theorems of Fredholm.
CO2: Apply Fourier transform in physical sciences
CO3: Evaluate integral equations of various types.
CO4: Apply integral equations in boundary value problems.
CO5: Identify problems in calculus of variation.

Part A **10 x 1 = 10**
Choose the Correct Answer

1. Find the Fourier Cosine transform of e^{-x} _____ CO1K4
   a. b/a^2+b^2 b. ab/a^2+b^2 c. ab^2/a^2+b^2 d. a/a^2+b^2

2. Let f(x) be a function satisfying _____ conditions in every finite interval. CO1K1
   a. Neumann b. Dirichlet c. Fourier cosine d. Fourier sine

3. Laplace transform of a convolution is the..... of the Laplace transforms of the individual functions. CO2K3
   a. product b.sum c. difference d. differential

4. The Laplace transform is often used to solve _____ equations CO2K2
   a. multiple b. differential c. integral d. two or more

5. In the Fredholm integral equation of the first kind _____ CO3K4
   a. h(s)=0 b. h(s)=1 c.f(s)=0 d. k(s,t)=0

6. The inhomogeneous Fredholm equation, the functions f(s) and g(s) are_____ CO3K1
   a. integrable b. differentiable c. unique solution d. two solutions

7. Boundary value problems in ordinary differential equation leads to_____ CO4K2
   a. Fredholm type integral equation b. Volterra type integral equation
   c. Integral equation d. Homogeneous equation

8. An initial value problem has been formulated in terms of an _____ CO4K3
   a. Integral equation b. Differential equation
   c. Volterra equation d. Fredholm equation

9. If the absolute values of the difference y(x)-y1(x) is small it is called as_____ CO5K4

a. zero order proximity b. first order proximity

c. second order proximity d. proximity .

10. The variation of a functional is the principal part of the increment of the functional, which part is linear in_____. CO5K1

a. δx b. $\delta^2 y$ c. δy d. $\delta^2 x$

Part B **5 x 6 = 30**

Answer ALL questions

Each answer should not exceed 400 words or two pages

11. a. Find the Fourier cosine transform of $f(x)=\begin{cases}\cos x, 0<x<a\\ 0, x\geq a\end{cases}$. CO1K4

(or)

11. b. Find the Fourier transform of $f(x)=\begin{cases}1, |x|<a\\ 0, x>a\end{cases}$. CO1K5

12 .a. State and Prove the convolution theorem. CO2K1

(or)

12. b. Using Transforms method, evaluate $\int_0^\infty \frac{x^2}{(x^2+a^2)^2}dx$, where a>0 CO2K5

13. a. Solve the homogeneous Fredholm integral equation $g(s)=\lambda\int_0^1 e^s e^t g(t)dt$ CO3K2

(or)

13. b. State and Prove Fredholm's first fundamental theorem. CO3K1

14. a. Reduce the initial value problem $y''(s)+\lambda y(s)=F(s), y(0)=1, y'(0)=0$ to Volterra integral equation. CO4K3

(or)

14. b. State and prove the Abel's integral equation. CO4K1

15. a. Derive the Euler's equation. CO5K3

(or)

15.b. Obtain the Euler's equation for solving the extremal $\int_{x_1}^{x_2} y'(1+x^2y')dx$ CO5K4

Part C

5 x 12 = 60

Answer ALL questions

Each answer should not exceed 800 words or four pages

16.a. Find the Fourier transform of $f(x)=\begin{Bmatrix}1-x^2,|x|<1\\0,|x|>1\end{Bmatrix}$ and hence evaluate $\int_0^{\infty}\left(\frac{x\cos x-\sin x}{x^3}\right)\cos\frac{x}{2}dx$ CO1K5

(or)

16.b. Find the Fourier sine transform of $f(x)=\frac{e^{-ax}}{x}$ and hence use it to evaluate $\int_0^{\infty}\tan^{-1}\left(\frac{x}{a}\right)\sin x dx$. CO1K4

17.a. Solve the Laplace's equation in the half plane. CO2K1

(or)

17.b. Find the Fourier transform of $f(x)=\begin{Bmatrix}1-|x|,|x|<1\\0,|x|>1\end{Bmatrix}$. Hence deduce that $\int_0^{\infty}\left(\frac{\sin t}{t}\right)^4 dt=\frac{\pi}{3}$ CO2K3

18.a. Evaluate the resolvant for the integral equation $g(s)=f(s)+\lambda\int_0^1(s+t)g(t)dt$. CO3K2

(or)

18.b. Solve the integral equation $g(s)=f(s)+\lambda\int_0^s e^{s-t}g(t)dt$ and evaluate the resolvant kernel. CO3K4

19.a. Solve the initial value problem $y''+y=F(s),0<s<1,y(0)=y'(0)=0$. CO4K1

(or)

19.b. Reduce the boundary value problem $y''(s)+\lambda p(s)y=Q(s),y(a)=y(b)=0$ to a Fredholm integral equation. CO4K3

20.a. Find the extremals of the functional $I[y(x)]=\int_a^b[(y'')^2-2(y')^2+y^2]dx$ CO5K4

(or)

20.b. Find the extremals of the functional $I[y(x)]=\int_{x_0}^{x_1}[(2xy+y''^2]dx$ CO5K5

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