Chapter VI

CHAPTER VI

APPLICATION OF FUZZY INTERVAL MATRICES AND FUZZY INTERVAL BIMATRICES TO FUZZY MODELS

In this section, a new model called fuzzy cognitive interval maps model is introduced as an application of fuzzy interval matrices. A rough sketch of this model is given and a symptom disease model in children taking five attributes using three experts is illustrated.

ROUGH SKETCH OF FUZZY COGNITIVE INTERVAL MAPS MODEL:

Suppose that p number of experts spells out their opinion on n number of nodes. Using the directed graph suppose we get a fuzzy matrix P_i given by the i^{th} expert and further the FCM is not a simple FCM, $1 \leq i \leq p$.

The entries in the n×n matrices will be from the fuzzy interval [–1, 1]. Now in general if we take the collection of all n×n fuzzy matrices with entries from the fuzzy interval [–1, 1] we have the interval of fuzzy n×n matrix. [A, B] would be an infinite collection in general. Further this fuzzy interval matrices associated with any FCM model will satisfy the following condition.

1. The fuzzy interval matrix will always be a square matrix.
2. The fuzzy interval matrix will always have the main diagonal entries to be zero.
3. The number of fuzzy matrices in the fuzzy interval n × n matrices is though infinite by all means for us in our system we can have only a finite number of fuzzy matrices associated with a FCM and with its associated fuzzy interval n×n square matrices.

This fuzzy interval of n×n square matrix associated with the FCMs of p-experts will be called as the Fuzzy Cognitive Interval Maps (FCIMs) of the multi experts dynamical system, as the related connection matrices forms an interval of n×n square matrices.

This fuzzy interval of matrices satisfying the conditions 1, 2 and 3 has lots of advantage over the infinite collection. Suppose we have some p experts who have given their opinion on n concepts. Then we will have only p, n×n square fuzzy matrices with entries from the fuzzy interval [–1, 1]. Now we using these p, n×n square fuzzy matrices form an associated interval of square n×n matrices by the following method.

Now we known if they are the fuzzy connection matrices all the main diagonal terms are zero. Now in order to obtain the fuzzy interval n×n matrix [A,B] using the p-matrices we have to construct A and B for A and B may not exist in general. Now we call A the minimal matrix (element) of [A, B] and B the maximal (element) of [A, B]. We give the method by which A is built using the p matrices. Suppose $A = (a_{ij})$, $a_{ii} = 0$ for $1 \leq i \leq n$; $1 \leq i, j \leq n$. So all the diagonal elements are zero. We make the observation of the element a_{12} in all the p matrices $P_1, \ldots, P_p$ where $P^t = (a^t_{ij})$, $1 \leq t \leq p$; we take the minimum value from the p entries $a^1_{12}, a^2_{12}, \ldots, a^p_{12}$ and put in the new matrix as the value of a_{12} likewise for every a_{ij}, $1 \leq i, j \leq n$.

This newly formed matrix may not in general be any of the matrices given by the p experts. We call this matrix the minimal element A of the fuzzy interval of the n×n matrices got from the p experts.

Like wise we form the maximal matrix $B = (b_{ij})$ by taking the maximal element. If B is the maximal matrix, the elements of the fuzzy interval of matrices, in general B need not be a connection matrix given by any of the p experts. Now having obtained the minimal and maximal fuzzy matrices as A and B; we form the fuzzy interval matrix [A, B]. Clearly by the very construction of A and B, all the p connection fuzzy matrices given by the p experts will lie in the fuzzy interval of matrices [A, B].

Now having constructed the minimal and maximal element on the fuzzy interval of matrices we construct the optimal fuzzy matrix O as follows;

$$O = \frac{A+B}{2} = \frac{(a_{ij}+b_{ij})}{2} = (o_{ij})$$

may be in [A, B] if O is not in [A, B] we include O in the fuzzy interval of matrices [A, B] and call it as the optimal fuzzy matrix of the interval of fuzzy matrices and the associated weighted directed graph will be called as the optimal weighted directed graph.

Now we can work with the minimal matrix A, maximal matrix B and optimal matrix O and compare our results. We would also adjoin the matrix $\overline{A}$ which will be the average matrix of the combined matrices of the p experts excluding the minimal, optimal and the maximal matrices provided they are not the opinion given by any of the p experts. We can find also the resultant of state vectors using the average connection matrix $\overline{A}$ if $\overline{A} \cup$ [A, B], well other wise we will adjoin $\overline{A}$ also to the interval of fuzzy matrices as our interval contains only the matrices related with the p experts opinion.

These four matrices A, B, O and A may or may not in general be some of the p experts opinion, The fuzzy interval matrix which is formed will have $\overline{A}$ to be minimal and B to be the maximal element [A, B], the fuzzy interval matrix i.e., if $M \in [A, B]$ and if $M = (m_{ij})$ then $(a_{ij}) \leq (m_{ij}) \leq (b_{ij})$, $1 \leq i, j \leq n$ also for $O = (o_{ij})$ then, $a_{ij} \leq o_{ij} \leq b_{ij}$, $1 \leq i, j \leq n$. Further for $\overline{A} = (\overline{a_{ij}})$.We have $a_{ij} \leq \overline{a_{ij}} \leq b_{ij}$ for $1 \leq i, j \leq n$.

ILLUSTRATION OF FUZZY COGNITIVE INTERVAL MAPS MODEL :

A model called the symptom disease model in children taking 5 major attributes using three experts is given.

Let us take the 5 attributes which the child shows as symptoms of the disease as $C_1, C_2, \ldots, C_5$:

C_1 – Fever with cold / cough

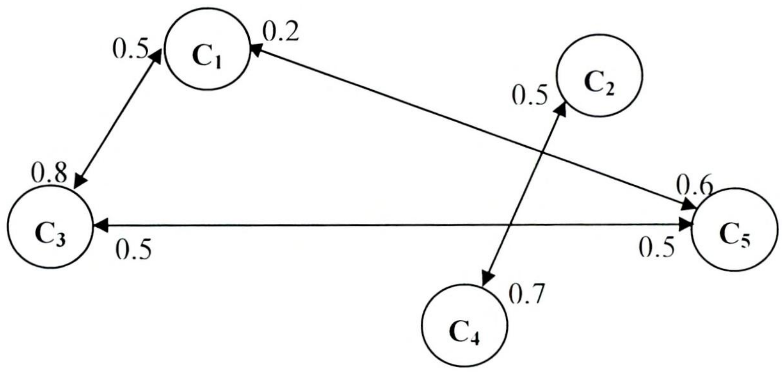
C_2 – Fever with vomiting/ loose motion / loss of appetite

C_3 – Respiratory diseases

C_4 – Gastroenteritis

C_5 – Tuberculosis.

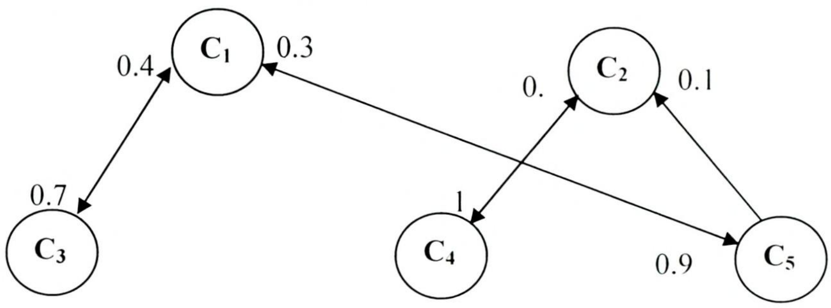
The directed graph given by the doctor who is used as an expert. The directed graph is a weighted one; given by the figure.



The fuzzy matrix M_1 (which is not a simple FCM) associated with the directed weighted graph is given below:

$$M_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0.8 & 0 & 0.6 \\ 0 & 0 & 0 & 0.7 & 0 \\ 0.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0.2 & 0 & 0.5 & 0 & 0 \end{bmatrix} \end{array}.$$

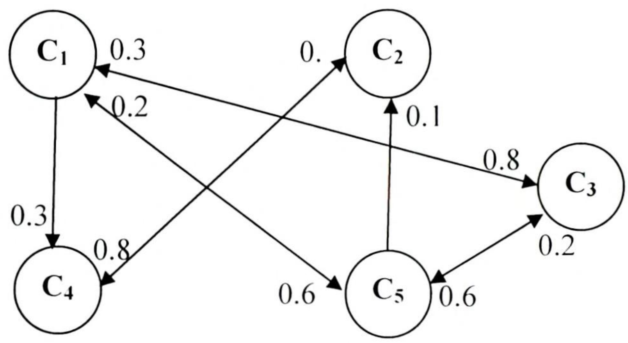
The opinion given by the second expert who is also a doctor and the weighted directed graph given by him is as follows:



The fuzzy matrix M_2 from the directed graph given by the second expert is as follows:

$$
M_2 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array}
\begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\
\begin{bmatrix} 0 & 0 & 0.4 & 0 & 0.9 \\ 0 & 0 & 0 & 1 & 0 \\ 0.4 & 0 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 & 0 \\ 0.3 & 0.1 & 0 & 0 & 0 \end{bmatrix} \end{array}.
$$

The weighted directed graph given by the 3rd expert who is also a doctor is given by the figure :



The related fuzzy matrix M_3 given by the 3rd expert is as follows:

$$
M_3 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array}
\begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\
\begin{bmatrix} 0 & 0 & 0.8 & 0.3 & 0.6 \\ 0 & 0 & 0 & 0.8 & 0 \\ 0.3 & 0 & 0 & 0 & 0.6 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0.2 & 0.1 & 0.2 & 0 & 0 \end{bmatrix} \end{array}
$$

Using the three matrices M_1, M_2 and M_3 we obtain the minimal matrix A and the maximal matrix B so that we can form the fuzzy interval matrix of this FCIM model.

$$
A = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array}
\begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\
\begin{bmatrix} 0 & 0 & 0.4 & 0 & 0.6 \\ 0 & 0 & 0 & 0.7 & 0 \\ 0.3 & 0 & 0 & 0 & 0.5 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0 & 0 \end{bmatrix} \end{array}
$$

The maximal matrix B of the fuzzy interval matrix is given below:

$$B = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0.8 & 0.3 & 0.9 \\ 0 & 0 & 0 & 1 & 0 \\ 0.5 & 0 & 0 & 0 & 0.6 \\ 0 & 0.6 & 0 & 0 & 0 \\ 0.2 & 0.1 & 0.5 & 0 & 0 \end{bmatrix} \end{array}$$

Thus we have the fuzzy interval matrix associated with the FCIM given by [A, B], clearly M_1, M_2, $M_3 \in$ [A, B]. Now the optimal matrix of the FCIM is given by O where

$$O = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0.6 & 0.15 & 0.75 \\ 0 & 0 & 0 & 0.85 & 0 \\ 0.4 & 0 & 0 & 0 & 0.55 \\ 0 & 0.55 & 0 & 0 & 0 \\ 0.1 & 0.05 & 0.35 & 0 & 0 \end{bmatrix} \end{array}$$

$O \in$ [A, B].

Now we find the CFCM using. M_1, M_2 and M_3 and the average fuzzy matrix $\overline{A}$; $\overline{A} \in$ [A, B].

$$\overline{A} = \frac{M_1+M_2+M_3}{3}$$

$$\overline{A} = \begin{bmatrix} 0 & 0 & 0.66 & 0.1 & 0.7 \\ 0 & 0 & 0 & 0.83 & 0 \\ 0.4 & 0 & 0 & 0 & 0.53 \\ 0 & 0.53 & 0 & 0 & 0 \\ 0.13 & 0.06 & 0.33 & 0 & 0 \end{bmatrix}.$$

Now we will study the effect of the state vector, symptom suffered is C_1 = fever with cold and cough alone in the on state and all other nodes are in the off state i.e., X = (1, 0, 0, 0, 0). We shall study the hidden pattern of X on the dynamical system FCIM given by the 3 expert doctors, the minimal A, maximal B, optimal O and the CFCM average matrix A using the max-min operation.

Hidden pattern of X given by the dynamical system M_1;

$$XM_1 = (0, 0, 0.8, 0, 0.6)$$

after updating and thresholding we get

$$XM_1 \rightarrow X_1 = (1, 0, 0.8, 0, 0.6),$$

('→ ' this symbol denotes the vector has been updated and thresholded).

Now the effect of X_1 on M_1 is given by

$$X_1M_1 \rightarrow (1, 0, 0.8, 0, 0.6)$$

$$= X_2 \text{ (say) } (= X_1).$$

Thus we arrive at the fixed point. According to the first expert who is doctor the symptom fever together with the symptom of cold and cough implies the child will have the maximum probability of suffering with some respiratory disease i.e., 0.8 degree it suffers from some respiratory disease and 0.6 degree it suffers from TB. However the difference is only 0.2.

Now we find the hidden pattern for X = (1, 0, 0, 0, 0) the same state vector using the second experts opinion.

$$XM_2 = (0, 0, 0.4, 0, 0.9)$$

$$\rightarrow (1, 0, 0.4, 0, 0.9)$$

$$= X_1 \text{ (say).}$$

$$X_1M_2 \rightarrow (1, 0.1, 0.4, 0, 0.9)$$

$$= X_2 \text{ (say)}$$

$$X_2M_2 \rightarrow (1, 0.1, 0.4, 0.1, 0.9)$$

$$= X_3 \text{ (say)}$$

$$X_3M_2 \rightarrow (1, 0.1, 0.4, 0.1, 0.9)$$

$$= X_4 \quad (= X_3).$$

The hidden pattern of X is a fixed point given by the resultant vector X_4 = (1, 0.1, 0.4, 0.1, 0.9). According to this expert who is also a doctor, we see that if a child suffers the symptom of fever with cold /cough it has the maximum possibility it suffers from tuberculosis / primary complexes and gives only 0.4 less than half of the possibility that it suffers from respiratory diseases. However it does not rule out that it may also suffer the symptom of fever with vomiting and gastroenteritis.

His argument that most of the infants born in India suffer from malnutrition and many suffer from the primary complexes (if the tuberculosis systems suffered by the child) he further adds when the child has cold/cough with fever many a time it will vomit due to cough and cold / cough with fever also may make the child suffer indigestion due to fever, he says.

Now we proceed on to work with the 3rd experts opinion on the same state vector

$$X = (1\ 0\ 0\ 0\ 0).$$

The effect of X on M_3 is given by

$$XM_3 \rightarrow (1, 0, 0.8, 0.3, 0.6) = X_1 \text{ (say)}$$

$$X_1M_3 \rightarrow (1, 0.3, 0.8, 0.3, 0.6) = X_2 \text{ (say)}$$

$$X_2M_3 \rightarrow (1, 0.3, 0.8, 0.3, 0.6) = X_3 \quad (= X_2 \text{ say}).$$

We see the hidden pattern of the resultant state vector is a fixed point given by

$$X_3 = (1, 0.3, 0.8, 0.3, 0.6).$$

This doctor also feels like the first doctor the symptoms suffered by the child may be mainly due to respiratory disease. However he does not rule out the factor that the child may suffer from 0.6 degree T.B (Tuberculosis or primary complexes). He gives 0.3 degree to the factor that the child may have vomiting with fever and also 0.3 degree it may be due to gastroenteritis.

Now we study the effect of the same state vector X = (1, 0, 0, 0, 0) on the minimal fuzzy matrix A of the fuzzy interval matrix.

$$XA \rightarrow (1, 0, 0.4, 0, 0.6) = X_1 \text{ (say)}$$

$X_1A \rightarrow (1, 0, 0.4, 0, 0.6)$

$= X_2 \quad (= X_1 \text{ say}).$

Thus we see the resultant is a fixed point. The minimal matrix expresses; 0.6 degree, the child's symptom (say) may be due to Tuberculosis and 0.4 degree the symptom the child show is due to it may suffer due to respiratory diseases and totally rules out the possibility the child may suffer the symptom from vomiting or gastroenteritis.

Now we proceed on to study the effect of the state vector X = (1, 0, 0, 0, 0) on the maximal fuzzy matrix B of the fuzzy interval matrix [A, B].

$XB \rightarrow (1, 0, 0.8, 0.3, 0.9)$

$= X_1 \text{ (say)}$

$X_1B \rightarrow (1, 0.3, 0.8, 0.3, 0.9)$

$= X_2 \text{ (say)}$

$X_2B = (1, 0.3, 0.8, 0.3, 0.9)$

$= X_3 \quad (= X_2).$

Thus we see the resultant is more or less the same as the 3rd expert but the maximal matrix gives maximal degree to T. B or primary complex disease i.e., according to this maximal system the symptom with fever and cold and cough is the main cause for T.B followed by respiratory diseases, however does not rule out the possibility that the child may have 0.3 degree of suffering from the symptom of vomiting and gastroenteritis.

Now we see the predictions of the optimal matrix O from the fuzzy interval of matrices for the same state vector X = (1, 0, 0, 0, 0)

$XO = (1, 0, 0.6, 0.15, 0.75)$

$= X_1 \text{ (say)}$

$X_1O = (1, 0.15, 0.6, 0.15, 0.75)$

$= X_2 \text{ (say)}$

$X_2O = (1, 0.15, 0.6, 0.15, 0.75)$

$= X_3 \text{ (say)} = X_2.$

Thus we see from the optimal matrix, if a child suffers with a symptom of fever with cold or cough alone there is 0.75 degree probability it suffers from TB (Tuberculosis) and also it may suffer from the respiratory disease upto 0.6 degree but the optimal matrix gives a very moderate degree (say) 0.15 to the symptom it may be have vomiting and also symptom of gastroenteritis.

Now finally we see the resultant of the state vector X = (1, 0, 0, 0, 0) on the CFCM average $\overline{A}$;

$$\text{i.e., } \overline{A} = \frac{M_1+M_2+M_3}{3};$$

$$X\overline{A} \rightarrow (1, 0, 0.66, 0.1, 0.7)$$

$$= X_1 \text{ (say)}$$

$$X_1\overline{A} \rightarrow (1, 0.06, 0.66, 0.1, 0.7)$$

$$= X_2 \text{ (say).}$$

Now $$X_2\overline{A} \rightarrow (1, 0.6, 0.66, 0.1, 0.7)$$

$$= X_3 \ (= X_2).$$

The hidden pattern of the dynamical system is a fixed point given by X_3 = (1, 0.06, 0.66, 0.1, 0.7). Thus we can say the average value of the CFCM happens to be the best prediction for the symptom fever with cold and cough for the doubt whether the child suffers from Tuberculosis or respiratory disease is very small amounting only to 0.04 degree. Mostly they feel due to cold; fever and cough the child may also suffer a very mild symptom of vomiting followed by a mild symptom of gastroenteritis. Thus the adaptation of fuzzy interval model can give an optimal solution.

Suppose there are several experts analyzing a problem and if each one of them accept to work with two sets of concept with different numbers say m and n, $n \neq m$ and some opt to give opinion on m concepts and others opt to give opinion on n concepts, a new model called the fuzzy interval bimatrix model, which has the capacity to work on two sets of experts opinions taking the two sets of concepts is introduced. In the second part of this chapter, this fuzzy interval bimatrix model is constructed and demonstrated.

consider a problem which has m attributes/concepts chosen by a set of t experts and n attributes / concepts chosen by another set of p experts ($n \neq m$), we may have overlaps of concepts, both the sets of workers work on the same problem. We first take the 't' experts opinion on the m concepts and form the $m \times m$ connection FCM matrices. We take only non simple FCM models. Using these t number of $m \times m$ matrices we construct the minimal $m \times m$ fuzzy matrix A_1, maximal $m \times m$ fuzzy matrix B_1, the optimal matrix O_1, where

$$O_1 = \frac{A_1 + B_1}{2} = \left(\frac{(a^1_{ij}) + (b^1_{ij})}{2} \right).$$

The average of all the CFCM matrices $M^1_1 + M^1_2 + \ldots + M^1_t = \overline{M}^1$

and $$\overline{M}^1 = \frac{M^1_1 + M^1_2 + \ldots + M^1_t}{t}.$$

Now the fuzzy interval $m \times m$ matrix $[A_1, B_1]$ contains all the t number of $m \times m$ matrices together with O_1 and $\overline{M}^1$.

In a similar way now using the p experts opinion and using the n concepts we work and obtain the $[A_2, B_2]$ fuzzy interval $n \times n$ matrices. Thus $[A_2, B_2]$ will be a fuzzy interval $n \times n$ matrices having $A_2 = (a^2_{ij})$ to be the minimal fuzzy matrix of the fuzzy interval matrix $[A_2, B_2]$ and $B_2 = (b^2_{ij})$ to be the maximal fuzzy matrix, constructed using the method given in the earlier section.

$$O_2 = \frac{A_2 + B_2}{2} = \left(\frac{(a^2_{ij}) + (b^2_{ij})}{2} \right) \text{ and } \overline{M}^2 = \frac{M^2_1 + M^2_2 + \ldots + M^2_p}{p}$$

the fuzzy interval n×n matrix. $[A_2, B_2]$ will contain O_2 and M_2 together with the p number of n×n matrices $M_1^2, M_2^2, \ldots, M_p^2$.

Now set $[A, B] = [A_1, B_1] \cup [A_2, B_2]$; [A, B] will be a fuzzy interval mixed square bimatrix. [A, B] is defined as the fuzzy interval mixed square bimatrix of the FCIBM or associated with it. We will just show how the model functions, suppose $[A, B] = [A_1, B_1] \cup [A_2, B_2]$ be a fuzzy interval square bimatrix associated with the FCIBM, suppose $[A_1, B_1]$ is a fuzzy interval 4×4 square matrix associated with an FCIM on some problem and $[A_2, B_2]$ is the fuzzy interval 5×5 square matrix associated with an FCIM for the same problem.

Thus $[A, B] = [A_1, B_1] \cup [A_2, B_2]$ is a fuzzy interval mixed square bimatrix associated with the FCIBM model.

Let $M = M_1 \cup M_2$

$$= \begin{bmatrix} 0 & 0.6 & 0.4 & 0 \\ 0.7 & 0 & 0.2 & 0.1 \\ 0 & 0.6 & 0 & 0 \\ 0.2 & 0.1 & 0 & 0 \end{bmatrix} \cup \begin{bmatrix} 0 & 0.7 & 0.2 & 0.1 & 0 \\ 0.5 & 0 & 0 & 0 & 0.3 \\ 0.7 & 0 & 0 & 0 & 0.6 \\ 0.4 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.7 & 0.5 & 0 \end{bmatrix} \in [A,B].$$

Let M_1 and M_2 be the opinion of two experts one who works with 4 concepts and other with 5 concepts on the same problem.

One wishes to study the problem when the node C_1 is in the on state in the first fuzzy dynamical system M_1 and the node D_2 is in the on state in the second fuzzy dynamical system M_2.

Let $X = (1\ 0\ 0\ 0) \cup (0\ 1\ 0\ 0\ 0)$ be the state bivector where all other nodes are in the off state except the nodes C_1 and D_2 in the bivector $X = X_1 \cup X_2$.

Now to find the hidden bipattern of the state bivector $X = X_1 \cup X_2$ on the dynamical bisystem $M = M_1 \cup M_2$.

$$\begin{aligned} XM &= X_1M_1 \cup X_2M_2 \\ &= (1, 0.6, 0.4, 0) \cup (0.5, 1, 0, 0, 0.3) \\ &= Y_1 \cup Y_2 \text{ (say)}. \end{aligned}$$

Let $\quad Y = Y_1 \cup Y_2.$

$$
\begin{aligned}
YM &= (Y_1 \cup Y_2)(M_1 \cup M_2) \\
&= Y_1M_1 \cup Y_2M_2 \\
&= (1, 0.6, 0.4, 0.1) \cup (0.5, 1, 0.3, 0.3, 0.3) \\
&= Z_1 \cup Z_2 \\
&= Z \text{ (say)}. \\
ZM &= Z_1M_1 \cup Z_2M_2 \\
&= (1, 0.6, 0.4, 0.1) \cup (0.5, 1, 0.3, 0.3, 0.3).
\end{aligned}
$$

Thus the hidden bipattern is a fixed bipoint. Thus it is now shown that how the FCIBM system associated with a fuzzy interval mixed square bimatrix model works.

Suppose there are several experts working on the same problem but using different number of concepts in the FCIM model, then certainly the fuzzy interval mixed square bimatrix model will not serve anymore so we should try to build a general new model is built.

The construction of the most generalized FCIM model which we denoted by FCItM, $t \geq 3$ is described below.

Suppose we have a set of experts working on the same problem P using the FCIBM model .

We have p_1 experts working with n_1 concepts on the problem P and p_2 experts working with n_2 concepts on the same problem P. ($n_1 \neq n_2$) and so on and p_t experts working with n_t concepts on the same problem P, using the FCIBM model with $p_i \neq p_j$ (if $i \neq j$). Then a single dynamical system associated with the FCIM will not serve the purpose, so we have to construct a special dynamical system to work with the problem simultaneously. To this end we do the following construction which we term as Fuzzy Cognitive Interval n-matrix model (FCInM).

Using the p_1 experts and n1 concepts on the problem first we form the fuzzy interval $n_1 \times n_1$ square matrix associated with the FCIM. Let us denote this fuzzy interval $n_1 \times n_1$ square matrix associated with the FCIM on the problem P by $[A_1, B_1]$ i.e., we form the FCIM described earlier.

Likewise we form the $[A_i, B_i]$ fuzzy interval $n_i \times n_i$ square matrices associated with FCIM using the pi experts. We do this for the collection of all the set of $p_1, p_2, \ldots, p_t$ experts i.e., $i = 1, 2, \ldots, t$.

Let $[A, B] = [A_1, B_1] \cup [A_2, B_2] \cup \ldots \cup [A_t, B_t]$ where [A,B] according to the definition, is a fuzzy interval mixed square matrices and each of the fuzzy interval square matrices $[A_i, B_i]$ contains almost $p_i + 4$ number of $n_i \times n_i$ square fuzzy matrices and $[A_i, B_i]$ is the associated fuzzy interval $n_i \times n_i$ square matrix of the FCIM for the problem P i.e., the FCIM model associated with P. This is true for $i = 1, 2, \ldots, t$. Thus we see [A, B] is the collection of all fuzzy mixed square t matrices.

Any element M in the set [A, B] will be of the form
$M = M_1 \cup M_2 \cup M_3 \cup \ldots \cup M_t$ where $M_i \cup [A_i, B_i]$; $i = 1, 2, \ldots, t$. We call [A, B] the fuzzy interval mixed square t matrices associated with the FCItMs of the problem P. When $t = 1$ we get the fuzzy interval matrix associated with the FCIM of the problem P. When $t = 2$ we get the fuzzy interval bimatrix associated with the FCIBM of the problem P and so on.

Now we shall sketch the working when $t = 5$.

Let $[A, B] = [A_1B_1] \cup [A_2B_2] \cup \ldots \cup [A_5B_5]$ be a fuzzy interval mixed square 5 matrix of the FCI5M model on some problem P.

Let $M = M \cup M_2 \cup \ldots \cup M_5$, if X is a state 5 vector whose resultant we are interested in finding out, we set $X = X_1 \cup X_2 \cup \ldots \cup X_5$.

$$\begin{aligned} XM &= (X_1 \cup \ldots \cup X_5)(M_1 \cup M_2 \cup \ldots \cup M_5) \\ &= X_1 M_1 \cup X_2 M_2 \cup \ldots \cup X_5 M_5 \\ &= Y_1 \cup Y_2 \cup \ldots \cup Y_5 \\ &= Y \text{ (say)}. \\ YM &= Y_1M_1 \cup \ldots \cup Y_5M_5 \\ &= Z_1 \cup Z_2 \cup \ldots \cup Z_5 \quad = Z \text{ (say)}. \end{aligned}$$

Then we find ZM; we proceed with the same process until we get a fixed point a limit cycle of the FCI5M.