Introduction

Lotfi Asker Zadeh (1965) developed fuzzy set theory, which embraced classical mathematical areas such as algebra, graph theory, topology, etc and it enables one to work in uncertain and ambiguous situations and problems with incomplete information. A classical set is characterized by the characteristic function, assigning values 1 and 0 for membership and non membership functions respectively whereas a fuzzy set is characterized by its membership function assigning real numbers in the interval [0, 1]. The idea of fuzzy set is welcomed because it handles uncertainty and vagueness which crisp set could not address and it provides a meaningful and a powerful representation of measurement of uncertainties, as well as vague concepts expressed in natural languages.

The new set is an extension of the ordinary fuzzy set and the first research work followed up by step by step of the existing results of fuzzy sets. The new class of sets as an extension of standard fuzzy sets have many specific properties, which are not available in case of fuzzy sets. Atanassov (1986) introduced intuitionistic fuzzy set by adding a non-membership function (β) to the membership function (α) of fuzzy set with the condition $0 \leq \alpha + \beta \leq 1$. Due to the flexibility of Intuitionistic Fuzzy Set (IFS) in handling uncertainty, they are tool for a more human consistent reasoning under imperfectly defined facts and imprecise knowledge. There are situations where evaluation of membership values is not always possible due to the insufficiency in the information system and consequently, there remains a part on which indeterminacy prevails.

Yager (2013) proposed a brand-new extension of fuzzy set called Pythagorean fuzzy set (PFS), which has been successfully applied in many fields for decision making procedures. PFS is characterized by a membership and non-membership function satisfies the condition that the square sum of membership and non-membership is less than or equal to one. It is noted that not all Pythagorean fuzzy set are intuitionistic fuzzy set but an intuitionistic fuzzy set must be a Pythagorean fuzzy set.

Spherical fuzzy set is a generalization of picture fuzzy set and Pythagorean fuzzy set. There is a need of spherical fuzzy set to tackle an interesting scenario emerge when picture fuzzy sets and Pythagorean fuzzy sets both failed to handle. One can study the neutral degree in spherical fuzzy set where as in Pythagorean fuzzy sets and picture fuzzy sets it doesn't. In spherical fuzzy set, membership degrees are gratifying the condition $0 \leq P^2(x) + I^2(x) + N^2(x) \leq 1$.

The idea of neutrosophic set is introduced by Smarandache (1999) which is a generalization of the fuzzy set, intuitionistic fuzzy set. The neutrosophic sets are characterized by a truth function (T), an indeterminate function (I) and a false function (F) independently. Smarandache (2017) introduced the new concepts of neutrosophic perspectives: Triplets, Duplets, Multisets, Hybrid Operators, Modal Logic, Hedge Algebras and Applications. The neutrosophic set and its extensions play a vital role to deal with incomplete and inconsistent information that exist in real world. Also introduced neutrosophic quadruple sets and neutrosophic quadruple numbers.

A score function is used to find the values of measures, which is more convenient than the similarity measures. Chen et.al (1994) proposed two functions to compare two different IFNs and applied in Multi–Attributes Group Decision Making (MCDM) i.e., score function and accuracy function. The concept of cubic set is characterized by fuzzy set and interval valued fuzzy set, which is an important tool to deal with uncertainty and vagueness.

The hybrid platform of cubic set contains more information than a fuzzy set. Neutrosophic set combined with cubic sets gave the new concept of neutrosophic cubic set introduced by Jun et.al (2012). Further Smarandache et.al (2015) introduced the new idea of neutrosophic cubic graphs and their fundamental operations such as cartesian product, union and join of neutrosophic cubic graphs, composition, degree and order of neutrosophic cubic graphs and some results.

A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and the relations by edges. When there is vagueness in the description of the objects or in its relationship or in both, it is necessary to assign a fuzzy graph model. Crisp graph and fuzzy graph both are structurally similar. For example, a social network may be represented as a graph where the vertices represent people and the edges represent the relation between the people. If the relations among people are to be measured as good or bad according to the frequency of contacts among the people, then the network is represented by a fuzzy graph. This motivated to define fuzzy graphs. The first definition of fuzzy graph was given by Kauffman (1973) based on Zadeh's fuzzy relations (1971b). Rosenfeld (1975) who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs. A fuzzy graph is a class of vertices and edges with a degree of membership which assigns a real number to each vertex and each edge in the interval [0, 1]. Fuzzy graph theory is now finding numerous applications in modern science and technology especially in the fields of neural networks, expert systems, cluster analysis, medical diagnosis, etc.

Intuitionistic fuzzy graphs (IFGs) defined by Atanassov (1986) give us a way to incorporate uncertainty with an additional degree. IFGs are graphs that extend theory of fuzzy graphs by adding two new components, degree of non-membership and degree of indeterminacy in the definition of a fuzzy graph. The theory of IFGs extend by allowing the assessment of the elements by three functions such as membership (μ), non-membership (ν) and indeterminacy degree (π), which belong to the real unit interval [0, 1] and whose sum belongs to the same interval as well. Karunambigai et al (2006) presented a model, based on dynamic programming, to find the shortest paths in intuitionistic fuzzy graphs.

Naz et al (2018) introduced the idea of Pythagorean fuzzy graphs, an extension of intuitionistic fuzzy graphs. Akram et al (2020a) examined decision making methods based on spherical fuzzy graphs and extended the graph-theoretic concepts under a spherical fuzzy environment and discussed some operations, properties of irregular and edge-irregular spherical fuzzy graph with examples(2020b).

A wide variety of human decision making is based on double sided or bipolar judgmental thinking on a positive side and a negative side. For instance, cooperation and competition, friendship and hostility, common interest and conflict intersects, effect and side effect, etc are often the two sides in decision and coordination. The co-existence, equilibrium and harmony of the two sides are considered a key for the mental and physical health of a person as well as for the stability and prosperity of a social system. Fuzzy logic provides a way for dealing with vagueness and uncertainties but lacks the representational and reasoning capabilities for direct modelling of the coexistence and interaction of bipolar relationships. This is because the logical values in the classical logical model lie in the positive interval [0,1] and they are uni-polar models in nature.

Bipolar fuzzy sets is an extension of fuzzy sets, in which positive information represents the possible and negative information represents the impossible or surely false. In bipolar fuzzy sets, the elements which are irrelevant are indicated by membership degree zero, the elements which satisfies the corresponding property by (0,1] and the elements which satisfies implicit counter property by [-1,0). Akram (2011) introduced the concept of bipolar fuzzy graph and defined different operations on bipolar fuzzy graphs. Ye (2013) presented a method to obtain minimum spanning tree of a graph where nodes (samples) are represented in the form of single valued neutrosophic set and distance between two nodes which represents the dissimilarity between the corresponding samples has been derived.

The thesis is organized into six chapters, as given below:

Chapter I deals with the basic concepts of fuzzy set, intuitionistic fuzzy set, Pythagorean fuzzy set, spherical fuzzy set, fuzzy graph, intuitionistic fuzzy graph, Pythagorean fuzzy graph, spherical fuzzy graph, coloring which are used to constructing the properties relating to the study.

Chapter II is divided into four sections, in the first section, a short introduction to bipolar spherical fuzzy graph is provided. In the second section, bipolar spherical fuzzy set and bipolar spherical fuzzy graph are defined and studied. And also discussed it on a graph. In the third section, the operation such as symmetric difference with a description on degree and total degree of bipolar spherical fuzzy graphs are derived and explained with numerical example. In section four, the rejection operation of bipolar spherical fuzzy graph with a description on degree and total degree is introduced and explained with suitable example.

In Chapter III, the first section presents a short introduction to bipolar spherical fuzzy minimum spanning tree algorithm. In the second section, score function of bipolar spherical fuzzy set are defined and presented bipolar spherical fuzzy minimum spanning tree algorithm. In the third section, a minimum spanning tree problem is described and discussed it on a graph.

In Chapter IV, the first section begins with the introduction of bipolar spherical neutrosophic cubic graph. In the second section, bipolar spherical neutrosophic cubic graph and its algebraic operations such as degree, order, union, join, and composition are introduced and discussed some other results related with bipolar spherical neutrosophic cubic graph with examples. In third section, the real life applications of bipolar spherical neutrosophic cubic graph are provided.

In Chapter V, the first section gives an introduction to bipolar spherical neutrosophic cubic minimum spanning tree algorithm. In the second section, score function of bipolar spherical neutrosophic cubic set are defined and presented bipolar spherical neutrosophic cubic minimum spanning tree algorithm. In the third section, a minimum spanning tree problem is provided and discussed it on a graph.

Chapter VI is divided into four section, first section describes an introduction to vertex coloring of spherical neutrosophic graph. In the second section, vertex coloring of spherical neutrosophic graph, complement of spherical neutrosophic graph and the union of spherical neutrosophic graph are defined. Vertex coloring of complete spherical neutrosophic graph, vertex coloring of strong spherical neutrosophic graph and complement of strong spherical neutrosophic graph are introduced and discussed with an example. In section three, bipolar spherical neutrosophic graph coloring, complete bipolar spherical neutrosophic graph coloring and strong bipolar spherical neutrosophic graph coloring are proposed and discussed with an numerical example.

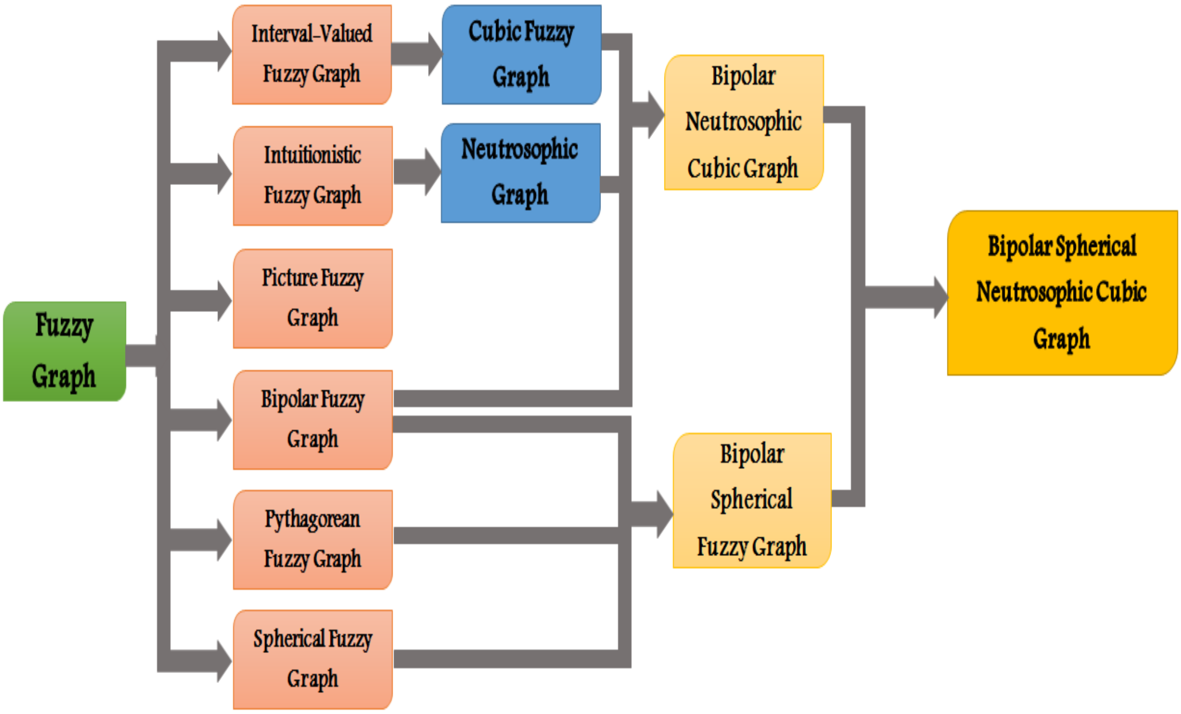

Fig. 1 Schematic representation of Bipolar Spherical Neutrosophic Cubic Graph