Manimegk

Avinashilingam Institute for Home Science and Higher Education for Women
(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)
Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B
Coimbatore - 641 043, Tamil Nadu, India

Bachelor's Degree Examination – March 2021
I Semester

Class : I UG **Time : 3 Hours**
Major : Mathematics **Max. Marks: 100**

18BMAC02 Classical Algebra and Theory of Numbers

Part A **10 x 1 = 10**
Choose the Correct Answer

1. A reciprocal equation of the standard form can always be depressed to another of _____ the dimensions.
   a. full b. half c. one d. zero

2. Find the remainder when $3x^3 + 8x^2 + 8x + 12 = 0$ is divided by $x - 4$ is
   a. 364 b. 246 c. 346 d. 463

3. A real matrix is unitary if and only if it is
   a. Symmetric b. Orthogonal c. Hermitian d. Orthonormal

4. Similar matrices have the same _____.
   a. determinant b. rank c. entries d. None of the above

5. What is value of Euler's function $\varphi(7) =$____.
   a. 5 b. 3 c. 4 d. 6

6. The integral part of $\left[\frac{3}{4}\right] =$ _____.
   a. 1 b. 0 c. 2 d. 3

7. The value of $26 \equiv 1\ mod(5) is$ _____.
   a. 4 b. 5 c. 0 d.1

8. Which one of the following is one of the properties of congruence?
   a. $ax \equiv b\ mod(0)$ b. $ax = bx\ mod(m)$ c. $a \equiv b\ mod(m)$ d. None of the above

9. If n is a prime number, then $\varphi(n) =$ ___ .
   a. n b. $n - 1$ c. $n + 1$ d. $n - 2$

10. The twelfth power of any number is of the form
    a. $12m$ b. $13m$ or $13m + 1$ c. $13m$ d. $13m$ or $13m - 1$

Part B 5 x 6 = 30

Answer ALL questions

Each answer should not exceed 400 words or two pages

11.a. Increase by 7 the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$.

(or)

11.b. Find the equation whose roots are the squares of the roots of $x^4 + x^3 + 2x^2 + x + 1 = 0$.

12.a. Show that the matrix $\frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ is orthogonal.

(or)

12.b. Obtain the characteristic roots and the associated characteristic vectors of the matrix $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$.

13.a. Determine the number of integers less than 600 and prime to it.

(or)

13.b. Find the highest power of 2 and 5 in $75!$.

14.a. Find the number of divisors and the sum of divisors of 480.

(or)

14.b. Verify that 220 an 284 are amicable numbers.

15.a. Show that $n^3 - n$ is divisible by 2730 if n is prime to 2730.

(or)

15.b. Show that $18! + 1$ is divisible by 437.

Part C 5 x 12 = 60

Answer ALL questions

Each answer should not exceed 800 words or four pages

16.a. Solve the equation $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$.

(or)

16.b. Show that the equation $x^4 - 3x^3 + 4x^2 - 2x + 1 = 0$ can be transformed into a reciprocal equation by diminishing the roots by unity. Hence solve the equation.

17.a. Show that the matrix $\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ satisfies the Cayley's-Hamilton theorem. Also find A^{-1}.

(or)

17.b. Diagonalise the matrix $\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$.

18.a. Prove that the highest power of a prime $p$containing $n!$ is zero or $= \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \cdots + \left[\frac{n}{p^{k-1}}\right]$ where $\left[\frac{n}{p^k}\right] = 0$ according as $n < p \quad or \quad n \geq p$.

(or)

18.b. Show that the product of 3 consecutive even integers(> 2) is divisible by 48. Deduce that $n(n^2 + 20)$is divisible by 48 where n is an even integer.

19.a. Show that$4^{2n+1} + 3^{n+2} = 0\ mod(13)$. Also find the smallest number with 24 divisors.

(or)

19.b. Find the remainder when $129^{8941}$is divisible by 29.

20.a. State and prove the Fermat's theorem.

(or)

20.b. State and prove the Wilson's theorem.

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