

# Intuitionist Fuzzy almost Generalized Semi-pre Closed Mappings

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## ABSTRACT

In this paper we introduce intuitionistic fuzzy almost generalized semi-pre closed mappings and intuitionistic fuzzy almost generalized semi-pre open mappings. We investigate some of their properties. Also we provide the relation between intuitionistic fuzzy almost generalized semi-pre closed mappings and other intuitionistic fuzzy closed mappings.

## Keywords

Intuitionistic fuzzy topology, intuitionistic fuzzy generalized semi-pre  $T_{1/2}$  space, intuitionistic fuzzy almost generalized semi-pre closed mappings.

## 1. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [13], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [1] is one among them. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological spaces. R. Santhi and D. Jayanthi [6] introduced the notion of intuitionistic fuzzy generalized semi-pre closed mappings and intuitionistic fuzzy generalized semi-pre open mappings. In this paper we introduce intuitionistic fuzzy almost generalized semi-pre closed mappings. We investigate some of its properties. Also we provide the relation between an intuitionistic fuzzy almost generalized semi-pre closed mapping and other intuitionistic fuzzy closed mappings.

## 2. PRELIMINARIES

**Definition 2.1:**[1] An intuitionistic fuzzy set (IFS in short)  $A$  in  $X$  is an object having the form

$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$  where the functions  $\mu_A : X \rightarrow [0,1]$  and  $\nu_A : X \rightarrow [0,1]$  denote the degree of membership (namely  $\mu_A(x)$ ) and the degree of non-membership (namely  $\nu_A(x)$ ) of each element  $x \in X$  to the set  $A$ , respectively, and  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$  for each  $x \in X$ .

Denote by  $IFS(X)$ , the set of all intuitionistic fuzzy sets in  $X$ .

**Definition 2.2:** [1] Let  $A$  and  $B$  be IFSs of the form  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$  and  $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$ . Then

(a)  $A \subseteq B$  if and only if  $\mu_A(x) \leq \mu_B(x)$  and  $\nu_A(x) \geq \nu_B(x)$  for all  $x \in X$

(b)  $A = B$  if and only if  $A \subseteq B$  and  $B \subseteq A$

(c)  $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$

(d)  $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$

(e)  $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation  $A = \langle x, \mu_A, \nu_A \rangle$  instead of  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ .

The intuitionistic fuzzy sets  $0 \sim = \{ \langle x, 0, 1 \rangle / x \in X \}$  and  $1 \sim = \{ \langle x, 1, 0 \rangle / x \in X \}$  are respectively the empty set and the whole set of  $X$ .

**Definition 2.3:** [11] The IFS  $c(\alpha, \beta) = \langle x, c_\alpha, c_{1-\beta} \rangle$  where  $\alpha \in (0, 1]$ ,  $\beta \in [0, 1)$  and  $\alpha + \beta \leq 1$  is called an intuitionistic fuzzy point (IFP for short) in  $X$ .

Note that an IFP  $c(\alpha, \beta)$  is said to belong to an IFS  $A = \langle x, \mu_A, \nu_A \rangle$  of  $X$  denoted by  $c(\alpha, \beta) \in A$  if  $\alpha \leq \mu_A$  and  $\beta \geq \nu_A$ .

**Definition 2.4:**[3] An intuitionistic fuzzy topology (IFT for short) on  $X$  is a family  $\tau$  of IFSs in  $X$  satisfying the following axioms.

(i)  $0 \sim, 1 \sim \in \tau$

(ii)  $G_1 \cap G_2 \in \tau$  for any  $G_1, G_2 \in \tau$

(iii)  $\cup G_i \in \tau$  for any family  $\{ G_i / i \in J \} \subseteq \tau$ .

In this case the pair  $(X, \tau)$  is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in  $\tau$  is known as an intuitionistic fuzzy open set (IFOS in short) in  $X$ . The complement  $A^c$  of an IFOS  $A$  in IFTS  $(X, \tau)$  is called an intuitionistic fuzzy closed set (IFCS in short) in  $X$ .

**Definition 2.5:**[3] Let  $(X, \tau)$  be an IFTS and  $A = \langle x, \mu_A, \nu_A \rangle$  be an IFS in  $X$ . Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$\text{int}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \}$

$\text{cl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}$

Note that for any IFS  $A$  in  $(X, \tau)$ , we have  $\text{cl}(A^c) = [\text{int}(A)]^c$  and  $\text{int}(A^c) = [\text{cl}(A)]^c$  [11].

**Definition 2.6:**[5] An IFS  $A = \langle x, \mu_A, \nu_A \rangle$  in an IFTS  $(X, \tau)$  is said to be an

(i) intuitionistic fuzzy semi closed set (IFSCS in short) if  $\text{int}(\text{cl}(A)) \subseteq A$

(ii) intuitionistic fuzzy pre closed set (IFPCS in short) if  $\text{cl}(\text{int}(A)) \subseteq A$

(iii) intuitionistic fuzzy  $\alpha$ closed set (IF $\alpha$ CS in short) if  $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$ .

The respective complements of the above IFCSs are called their respective IFOSs. The family of all IFSCSs, IFPCSs, and IF $\alpha$ CSs ( respectively IFSOSs, IFPOSS and IF $\alpha$ OSs ) of an IFTS  $(X, \tau)$  are respectively denoted by IFSC(X), IFPC(X) and IF $\alpha$ C(X) (respectively IFSO(X), IFPO(X) and IF $\alpha$ O(X)).

**Definition 2.7:**[11] An IFS  $A = \langle x, \mu_A, \nu_A \rangle$  in an IFTS  $(X, \tau)$  is said to be an

(i) intuitionistic fuzzy semi-pre closed set (IFSPCS for short) if there exists an IFPCS  $B$  such that  $\text{int}(B) \subseteq A \subseteq B$ .

(ii) intuitionistic fuzzy semi-pre open set (IFSPOS for short) if there exists an intuitionistic fuzzy pre open set (IFPOS for short)  $B$  such that  $B \subseteq A \subseteq \text{cl}(B)$ .

The family of all IFSPCSs (respectively IFSPOSs) of an IFTS  $(X, \tau)$  is denoted by IFSPC(X) (respectively IFSPO(X)).

Every IFSCS (respectively IFSOS) and every IFPCS (respectively IFPOS) is an IFSPCS (respectively IFSPOS). But the separate converses need not be true in general [11].

Note that an IFS  $A$  is an IFSPCS if and only if  $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$  and an IFSPOS if and only if  $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$  [7].

**Definition 2.8:**[7] Let  $A$  be an IFS in an IFTS  $(X, \tau)$ . Then the semi-pre interior and the semi-pre closure of  $A$  are defined by

$$\text{spint}(A) = \cup \{ G / G \text{ is an IFSPOS in } X \text{ and } G \subseteq A \},$$

$$\text{spcl}(A) = \cap \{ K / K \text{ is an IFSPCS in } X \text{ and } A \subseteq K \}.$$

Note that for any IFS  $A$  in  $(X, \tau)$ , we have  $\text{spcl}(A^c) = [\text{spint}(A)]^c$  and  $\text{spint}(A^c) = [\text{spcl}(A)]^c$  [7].

**Definition 2.9:**[10] An IFS  $A$  is an

(i) intuitionistic fuzzy regular closed set (IFRCS for short) if  $A = \text{cl}(\text{int}(A))$ .

(ii) intuitionistic fuzzy generalized closed set (IFGCS for short) if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IFOS.

The family of all IFRCSs and IFGCSs (respectively IFROSS and IFGOSs) of an IFTS  $(X, \tau)$  are respectively denoted by IFRC(X) and IFGC(X) (respectively IFRO(X) and IFGO(X)).

**Definition 2.10:**[7] An IFS  $A$  in an IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy generalized semi-pre closed set (IFGSPCS for short) if  $\text{spcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IFOS in  $(X, \tau)$ .

Every IFCS, IFGCS, IFSCS, IFPCS, IFRCS, IF $\alpha$ CS and IFSPCS is an IFGSPCS but the separate converses may not be true in general [7]. The family of all IFGSPCSs of an IFTS  $(X, \tau)$  is denoted by IFGSPC(X).

**Definition 2.11:**[6] Let  $A$  be an IFS in an IFTS  $(X, \tau)$ . Then the generalized semi-pre interior and the generalized semi-pre closure of  $A$  are defined by

$$\text{gspint}(A) = \cup \{ G / G \text{ is an IFGSPCS in } X \text{ and } G \subseteq A \}.$$

$$\text{gspcl}(A) = \cap \{ K / K \text{ is an IFGSPCS in } X \text{ and } A \subseteq K \}.$$

Note that for any IFS  $A$  in  $(X, \tau)$ , we have  $\text{gspcl}(A^c) = [\text{gspint}(A)]^c$  and  $\text{gspint}(A^c) = [\text{gspcl}(A)]^c$

**Remark 2.12:** If an IFS  $A$  in an IFTS  $(X, \tau)$  is an IFGSPCS in  $X$ , then  $\text{gspcl}(A) = A$ . But the converse may not be true in general, since intersection does not exist in IFGSPCSs [7]

**Remark 2.13:** If an IFS  $A$  in an IFTS  $(X, \tau)$  is an IFGSPCS in  $X$ , then  $\text{gspint}(A) = A$ . But the converse may not be true in general, since union does not exist in IFGSPCSs [7].

**Definition 2.14:**[7] The complement  $A^c$  of an IFGSPCS  $A$  in an IFTS  $(X, \tau)$  is called an intuitionistic fuzzy generalized semi-pre open set (IFGSPOS for short) in  $X$ .

Every IFOS, IFGOS, IFSOS, IFPOS, IFROS, IF $\alpha$ OS and IFSPOS is an IFGSPOS but the separate converses may not be true in general [7]. The family of all IFGSPOSs of an IFTS  $(X, \tau)$  is denoted by IFGSPO(X).

**Definition 2.15:**[9] Let  $c(\alpha, \beta)$  be an IFP of an IFTS  $(X, \tau)$ . An IFS  $A$  of  $X$  is called an intuitionistic fuzzy neighborhood (IFN for short) of  $c(\alpha, \beta)$  if there exists an IFOS  $B$  in  $X$  such that  $c(\alpha, \beta) \in B \subseteq A$ .

**Definition 2.16:**[8] If every IFGSPCS in  $(X, \tau)$  is an IFSPCS in  $(X, \tau)$ , then the space can be called as an intuitionistic fuzzy semi-pre  $T_{1/2}$  space (IFSP $T_{1/2}$  space for short).

**Definition 2.17:**[5] A map  $f: X \rightarrow Y$  is called an intuitionistic fuzzy closed mapping (IFCM for short) if  $f(A)$  is an IFCS in  $Y$  for each IFCS  $A$  in  $X$ .

**Definition 2.18:**[5] A map  $f: X \rightarrow Y$  is called an

(i) intuitionistic fuzzy semi open mapping (IFSOM for short) if  $f(A)$  is an IFOS in  $Y$  for each IFOS  $A$  in  $X$ .

(ii) intuitionistic fuzzy  $\alpha$  open mapping (IF $\alpha$ OM for short) if  $f(A)$  is an IF $\alpha$ OS in  $Y$  for each IFOS  $A$  in  $X$ .

(iii) intuitionistic fuzzy preopen mapping (IFPOM for short) if  $f(A)$  is an IFPOS in  $Y$  for each IFOS  $A$  in  $X$ .

**Definition 2.19:** The intuitionistic fuzzy semi closure and the intuitionistic fuzzy  $\alpha$  closure of an IFS  $A$  in an IFTS  $(X, \tau)$  are defined by

$$\text{scl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}.$$

$$\alpha \text{cl}(A) = \cap \{ K / K \text{ is an IF}\alpha\text{CS in } X \text{ and } A \subseteq K \}.$$

**Definition 2.20:**[6] Let  $c(\alpha, \beta)$  be an IFP in  $(X, \tau)$ . An IFS  $A$  of  $X$  is called an intuitionistic fuzzy semi neighborhood (IFSN for short) of  $c(\alpha, \beta)$  if there is an IFSPOS  $B$  in  $X$  such that  $c(\alpha, \beta) \in B \subseteq A$ .

**Definition 2.21:**[6] A mapping  $f: X \rightarrow Y$  is said to be an intuitionistic fuzzy generalized semi-pre closed mapping (IFGSPCM, for short) if  $f(A)$  is an IFGSPCS in  $Y$  for every IFCS  $A$  in  $X$ .

**Definition 2.22:**[6] A mapping  $f: X \rightarrow Y$  is said to be an intuitionistic fuzzy  $M$ -generalized semi-pre closed mapping (IFMGSPCM, for short) if  $f(A)$  is an IFGSPCS in  $Y$  for every IFGSPCS  $A$  in  $X$ .

**Definition 2.23:**[6] A mapping  $f: X \rightarrow Y$  is said to be an intuitionistic fuzzy almost generalized semi-pre continuous mapping (IFaGSP continuous mapping, for short) if  $f^{-1}(A)$  is an IFGSPCS in  $X$  for every IFRCS  $A$  in  $Y$ .

**Definition 2.24:** An IFS  $A$  is said to be an intuitionistic fuzzy dense (IFD for short) in another IFS  $B$  in an IFTS  $(X, \tau)$ , if  $\text{cl}(A) = B$ .

### 3. INTUITIONISTIC FUZZY ALMOST GENERALIZED SEMI-PRE CLOSED MAPPINGS AND INTUITIONISTIC FUZZY ALMOST GENERALIZED SEMI-PRE OPEN MAPPINGS

In this section we introduce intuitionistic fuzzy almost generalized semi-pre closed mappings and intuitionistic fuzzy almost generalized semi-pre open mappings. We study some of their properties.

**Definition 3.1:** A map  $f: X \rightarrow Y$  is called an intuitionistic fuzzy almost generalized semi-pre closed mapping (IFaGSPCM for short) if  $f(A)$  is an IFGSPCS in  $Y$  for each IFRCS  $A$  in  $X$ .

**Example 3.2:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$ ,  $G_2 = \langle y, (0.2_u, 0.3_v), (0.8_u, 0.7_v) \rangle$ . Then  $\tau = \{0_-, G_1, 1_-\}$  and  $\sigma = \{0_-, G_2, 1_-\}$  are IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ . Then  $f$  is an IFaGSPCM.

**Theorem 3.3:** Every IFCM is an IFaGSPCM but not conversely.

**Proof:** Let  $f: X \rightarrow Y$  be an IFCM. Let  $A$  be an IFRCS in  $X$ . Since every IFRCS is an IFCS,  $A$  is an IFCS in  $X$ . Then  $f(A)$  is an IFCS in  $Y$ . Since every IFCS is an IFGSPCS,  $f(A)$  is an IFGSPCS in  $Y$ . Hence  $f$  is an IFaGSPCM.

**Example 3.4:** In Example 3.2  $f$  is an IFGSPCM but not an IFCM, since  $G_1^c = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$  is an IFCS in  $X$  but  $f(G_1^c) = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v) \rangle$  is not an IFCS in  $Y$ , since  $\text{cl}(f(G_1^c)) = G_2 \neq f(G_1^c)$ .

**Theorem 3.5:** Every IFSCM is an IFaGSPCM but not conversely.

**Proof:** Let  $f: X \rightarrow Y$  be an IFSCM. Let  $A$  be an IFRCS in  $X$ . Since every IFRCS is an IFCS,  $A$  is an IFCS in  $X$ . Then  $f(A)$  is an IFSCS in  $Y$ . Since every IFSCS is an IFGSPCS,  $f(A)$  is an IFGSPCS in  $Y$ . Hence  $f$  is an IFaGSPCM.

**Example 3.6:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$ ,  $G_2 = \langle y, (0.5_u, 0.4_v), (0.2_u, 0.3_v) \rangle$ . Then  $\tau = \{0_-, G_1, 1_-\}$  and  $\sigma = \{0_-, G_2, 1_-\}$  are IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ . Then  $f$  is an IFaGSPCM but not an IFSCM, since  $G_1^c = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$  is an IFCS in  $X$  but  $f(G_1^c) = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v) \rangle$  is not an IFSCS in  $Y$ , since  $\text{int}(\text{cl}(f(G_1^c))) = 1_- \not\subseteq f(G_1^c)$ .

**Theorem 3.7:** Every IF $\alpha$ CM is an IFaGSPCM but not conversely.

**Proof:** Let  $f: X \rightarrow Y$  be an IF $\alpha$ CM. Let  $A$  be an IFRCS in  $X$ . Since every IFRCS is an IFCS,  $A$  is an IFCS in  $X$ . Then  $f(A)$  is an IF $\alpha$ CS in  $Y$ . Since every IF $\alpha$ CS is an IFGSPCS,  $f(A)$  is an IFGSPCS in  $Y$ . Hence  $f$  is an IFaGSPCM.

**Example 3.8:** In Example 3.6  $f$  is an IFaGSPCM but not an IF $\alpha$ CM, since  $G_1^c = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$  is an IFCS in  $X$  but  $f(G_1^c) = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v) \rangle$  is not an IF $\alpha$ CS in  $Y$ , since  $\text{cl}(\text{int}(\text{cl}(f(G_1^c)))) = 1_- \not\subseteq f(G_1^c)$ .

**Theorem 3.9:** Every IFPCM is an IFaGSPCM but not conversely.

**Proof:** Let  $f: X \rightarrow Y$  be an IFPCM. Let  $A$  be an IFRCS in  $X$ . Since every IFRCS is an IFCS,  $A$  is an IFCS in  $X$ . Then  $f(A)$

is an IFPCS in  $Y$ . Since every IFPCS is an IFGSPCS,  $f(A)$  is an IFGSPCS in  $Y$ . Hence  $f$  is an IFaGSPCM.

**Example 3.10:** In Example 3.2  $f$  is an IFaGSPCM but not an IFPCM, since  $G_1^c = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$  is an IFCS in  $X$  but  $f(G_1^c) = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v) \rangle$  is not an IFPCS in  $Y$ , since  $\text{cl}(\text{int}(f(G_1^c))) = G_2^c \not\subseteq f(G_1^c)$ .

**Theorem 3.11:** Every IFGSPCM is an IFaGSPCM but not conversely.

**Proof:** Let  $f: X \rightarrow Y$  be an IFGSPCM. Let  $A$  be an IFRCS in  $X$ . Since every IFRCS is an IFCS,  $A$  is an IFCS in  $X$ . Then  $f(A)$  is an IFGSPCS in  $Y$ . Hence  $f$  is an IFaGSPCM.

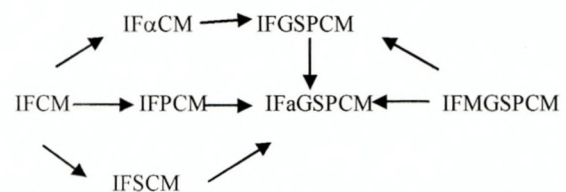
**Example 3.12:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $G_1 = \langle x, (0.2_a, 0.2_b), (0.4_a, 0.4_b) \rangle$ ,  $G_2 = \langle x, (0.2_a, 0_b), (0.5_a, 0.4_b) \rangle$ ,  $G_3 = \langle y, (0.5_u, 0.6_v), (0.2_u, 0_v) \rangle$  and  $G_4 = \langle y, (0.4_u, 0.1_v), (0.2_u, 0.1_v) \rangle$ . Then  $\tau = \{0_-, G_1, G_2, 1_-\}$  and  $\sigma = \{0_-, G_3, G_4, 1_-\}$  are IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ . Then  $f$  is an IFaGSPM but not an IFGSPCM, since  $G_2^c = \langle x, (0.5_a, 0.4_b), (0.2_a, 0_b) \rangle$  is an IFCS in  $X$  but  $f(G_2^c) = \langle y, (0.5_u, 0.4_v), (0.2_u, 0_v) \rangle$  is not an IFGSPCS in  $Y$ , since  $f(G_2^c) \subseteq G_3$  but  $\text{spl}(f(G_2^c)) = 1_- \not\subseteq G_3$ .

**Theorem 3.13:** Every IFMGSPCM is an IFaGSPCM but not conversely.

**Proof:** Let  $f: X \rightarrow Y$  be an IFMGSPCM. Let  $A$  be an IFRCS in  $X$ . Then  $A$  is an IFGSPCS in  $X$ . By hypothesis  $f(A)$  is an IFGSPCS in  $Y$ . Therefore  $f$  is an IFaGSPCM.

**Example 3.14:** In Example 3.12  $f$  is an IFaGSPCM but not an IFMGSPCM, since  $A = \langle x, (0.4_a, 0.2_b), (0.2_a, 0_b) \rangle$  is an IFGSPCS in  $X$  but  $f(A) = \langle y, (0.4_u, 0.2_v), (0.2_u, 0_v) \rangle$  is not an IFGSPCS in  $Y$ , since  $f(A) \subseteq G_3$  but  $\text{spl}(f(A)) = 1_- \not\subseteq G_3$ .

The relation between various types of intuitionistic fuzzy closedness is given in the following diagram.



The reverse implications are not true in general in the above diagram.

**Definition 3.15:** A map  $f: X \rightarrow Y$  is called an intuitionistic fuzzy almost generalized semi-pre open mapping (IFaGSPOM for short) if  $f(A)$  is an IFGSPOS in  $Y$  for each IFROS  $A$  in  $X$ .

**Theorem 3.16:** Let  $f: X \rightarrow Y$  be a mapping. Then the following are equivalent.

- (i)  $f$  is an IFaGSPOM
- (ii)  $f$  is an IFaGSPCM

**Proof:** Straightforward.

**Theorem 3.17:** Let  $c(\alpha, \beta)$  be an IFP in  $X$ . A mapping  $f: X \rightarrow Y$  is an IFaGSPCM if for every IFOS  $A$  in  $X$  with  $f^1(c(\alpha, \beta)) \in A$ , there exists an IFOS  $B$  in  $Y$  with  $c(\alpha, \beta) \in B$  such that  $f(A)$  is IFD in  $B$ .

**Proof:** Let  $A$  be an IFROS in  $X$ . Then  $A$  is an IFOS in  $X$ . Let  $f^1(c(\alpha, \beta)) \in A$ , then there exists an IFOS  $B$  in  $Y$  such that

$c(\alpha, \beta) \in B$  and  $cl(f(A)) = B$ . Since  $B$  is an IFOS,  $cl(f(A)) = B$  is also an IFOS in  $Y$ . Therefore  $int(cl(f(A))) = cl(f(A))$ . Now  $f(A) \subseteq cl(f(A)) = int(cl(f(A))) \subseteq cl(int(cl(f(A))))$ . This implies  $f(A)$  is an IFSPS in  $Y$  and hence an IFGSPS in  $Y$ . Thus  $f$  is an IFaGSPCM.

**Theorem 3.18:** Let  $f: X \rightarrow Y$  be a mapping where  $Y$  is an IFSP1/2space. Then the following are equivalent:

- (i)  $f$  is an IFaGSPCM.
- (ii)  $spcl(f(A)) \subseteq cl(A)$  for every IFSPS  $A$  in  $X$
- (iii)  $spcl(f(A)) \subseteq f(cl(A))$  for every IFSPS  $A$  in  $X$
- (iv)  $f(A) \subseteq spint(f(int(cl(A))))$  for every IFPOS  $A$  in  $X$ .

**Proof:** (i)  $\Rightarrow$  (ii) Let  $A$  be an IFSPS in  $X$ . Then  $cl(A)$  is an IFRCS in  $X$ . By hypothesis  $f(A)$  is an IFGSPS in  $Y$  and hence is an IFSPS in  $Y$ , since  $Y$  is an IFSP1/2 space. This implies  $spcl(f(A)) = f(cl(A))$ . Now  $spcl(f(A)) \subseteq spcl(f(cl(A))) = f(cl(A))$ . Thus  $spcl(f(A)) \subseteq f(cl(A))$ .

(ii)  $\Rightarrow$  (iii) Since every IFSPS is an IFSPS, the proof directly follows.

(iii)  $\Rightarrow$  (i) Let  $A$  be an IFRCS in  $X$ . Then  $A = cl(int(A))$ . Therefore  $A$  is an IFSPS in  $X$ . By hypothesis,  $spcl(f(A)) \subseteq f(cl(A)) = f(A) \subseteq spcl(f(A))$ . Hence  $f(A)$  is an IFSPS and hence is an IFGSPS in  $Y$ . Thus  $f$  is an IFaGSPCM.

(i)  $\Rightarrow$  (iv) Let  $A$  be an IFPOS in  $X$ . Then  $A \subseteq int(cl(A))$ . Since  $int(cl(A))$  is an IFROS in  $X$ , by hypothesis,  $f(int(cl(A)))$  is an IFGSPS in  $Y$ . Since  $Y$  is an IFSP1/2space,  $f(int(cl(A)))$  is an IFSPS in  $Y$ . Therefore  $f(A) \subseteq f(int(cl(A))) \subseteq spint(f(int(cl(A))))$ .

(iv)  $\Rightarrow$  (i) Let  $A$  be an IFROS in  $X$ . Then  $A$  is an IFPOS in  $X$ . By hypothesis,  $f(A) \subseteq spint(f(int(cl(A)))) = spint(f(A)) \subseteq f(A)$ . This implies  $f(A)$  is an IFSPS in  $Y$  and hence is an IFGSPS in  $Y$ . Therefore  $f$  is an IFaGSPCM.

**Theorem 3.19:** Let  $f: X \rightarrow Y$  be a mapping. Then  $f$  is an IFaGSPCM if for each IFP  $c(\alpha, \beta) \in Y$  and for each IFSPS  $B$  in  $X$  such that  $f^{-1}(c(\alpha, \beta)) \in B$ ,  $spcl(f(B))$  is an IFSN of  $c(\alpha, \beta) \in Y$ .

**Proof:** Let  $c(\alpha, \beta) \in Y$  and let  $A$  be an IFROS in  $X$ . Then  $A$  is an IFSPS in  $X$ . By hypothesis  $f^{-1}(c(\alpha, \beta)) \in A$ , that is  $c(\alpha, \beta) \in f(A)$  in  $Y$  and  $spcl(f(A))$  is an IFSN of  $c(\alpha, \beta)$  in  $Y$ . Therefore there exists an IFSPS  $B$  in  $X$  such that  $c(\alpha, \beta) \in B \subseteq spcl(f(A))$ . We have  $c(\alpha, \beta) \in f(A) \subseteq spcl(f(A))$ . Now  $B = \cup \{ c(\alpha, \beta) / c(\alpha, \beta) \in B \} = f(A)$ . Therefore  $f(A)$  is an IFSPS in  $Y$  and hence an IFGSPS in  $Y$ . Thus  $f$  is an IFaGSPOM. Hence by Theorem 3.16  $f$  is an IFaGSPCM.

**Theorem 3.20:** Let  $f: X \rightarrow Y$  be a mapping. If  $f$  is an IFaGSPCM, then  $gspcl(f(A)) \subseteq f(cl(A))$  for every IFSPS  $A$  in  $X$ .

**Proof:** Let  $A$  be an IFSPS in  $X$ . Then  $cl(A)$  is an IFRCS in  $X$ . By hypothesis  $f(A)$  is an IFGSPS in  $Y$ . Then  $gspcl(f(A)) = f(cl(A))$ . Now  $gspcl(f(A)) \subseteq gspcl(f(cl(A))) = f(cl(A))$ . That is  $gspcl(f(A)) \subseteq f(cl(A))$ .

**Corollary 3.21:** Let  $f: X \rightarrow Y$  be a mapping. If  $f$  is an IFaGSPCM, then  $gspcl(f(A)) \subseteq f(cl(A))$  for every IFSPS  $A$  in  $X$ .

**Proof:** Since every IFSPS is an IFSPS, the proof directly follows from the Theorem 3.20.

**Corollary 3.22:** Let  $f: X \rightarrow Y$  be a mapping. If  $f$  is an IFaGSPCM, then  $gspcl(f(A)) \subseteq f(cl(A))$  for every IFPOS  $A$  in  $X$ .

**Proof:** Since every IFPOS is an IFSPS, the proof directly follows from the Theorem 3.20.

**Theorem 3.23:** Let  $f: X \rightarrow Y$  be a mapping. If  $f$  is an IFaGSPCM, then  $gspcl(f(A)) \subseteq$

$f(cl(spint(A)))$  for every IFSPS  $A$  in  $X$ .

**Proof:** Let  $A$  be an IFSPS in  $X$ . Then  $cl(A)$  is an IFRCS in  $X$ . By hypothesis,  $f(A)$  is an IFGSPS in  $Y$ . Then  $gspcl(f(A)) \subseteq gspcl(f(cl(A))) = f(cl(A)) \subseteq f(cl(spint(A)))$ , since  $spint(A) = A$ .

**Corollary 3.24:** Let  $f: X \rightarrow Y$  be a mapping. If  $f$  is an IFaGSPCM, then  $gspcl(f(A)) \subseteq f(cl(spint(A)))$  for every IFSPS  $A$  in  $X$ .

**Proof:** Since every IFSPS is an IFSPS, the proof directly follows from the Theorem 3.23.

**Corollary 3.25:** Let  $f: X \rightarrow Y$  be a mapping. If  $f$  is an IFaGSPCM, then  $gspcl(f(A)) \subseteq f(cl(spint(A)))$  for every IFPOS  $A$  in  $X$ .

**Proof:** Since every IFPOS is an IFSPS, the proof directly follows from the Theorem 3.23.

**Theorem 3.26:** Let  $f: X \rightarrow Y$  be a mapping. If  $f(spint(B)) \subseteq spint(f(B))$  for every IFSPS  $B$

in  $X$ , then  $f$  is an IFaGSPOM.

**Proof:** Let  $B \subseteq X$  be an IFROS. By hypothesis,  $f(spint(B)) \subseteq spint(f(B))$ . Since  $B$  is an IFROS, it is an IFSPS in  $X$ . Therefore  $spint(B) = B$ . Hence  $f(B) = f(spint(B)) \subseteq spint(f(B)) \subseteq f(B)$ . This implies  $f(B)$  is an IFSPS and hence an IFGSPS in  $Y$ . Thus  $f$  is an IFaGSPOM.

**Theorem 3.27:** Let  $f: X \rightarrow Y$  be a mapping. If  $spcl(f(B)) \subseteq f(spcl(B))$  for every IFSPS  $B$  in  $X$ , then  $f$  is an IFaGSPCM.

**Proof:** Let  $B \subseteq X$  be an IFRCS. By hypothesis,  $spcl(f(B)) \subseteq f(spcl(B))$ . Since  $B$  is an IFRCS, it is an IFSPS in  $X$ . Therefore  $spcl(B) = B$ . Hence  $f(B) = f(spcl(B)) \subseteq spcl(f(B)) \subseteq f(B)$ . This implies  $f(B)$  is an IFSPS and hence an IFGSPS in  $Y$ . Thus  $f$  is an IFaGSPCM.

**Theorem 3.28:** Let  $f: X \rightarrow Y$  be a mapping where  $Y$  is an IFSP1/2space. Then the following are equivalent.

- (i)  $f$  is an IFaGSPOM
- (ii) for each IFP  $c(\alpha, \beta)$  in  $Y$  and each IFROS  $B$  in  $X$  such that  $f^{-1}(c(\alpha, \beta)) \in B$ ,  $cl(f(B))$  is an IFSN of  $c(\alpha, \beta)$  in  $Y$ .

**Proof:** (i)  $\Rightarrow$  (ii) Let  $c(\alpha, \beta) \in Y$  and let  $B$  be an IFROS in  $X$  such that  $f^{-1}(c(\alpha, \beta)) \in B$ . That is  $c(\alpha, \beta) \in f(B)$ . By hypothesis,  $f(B)$  is an IFGSPS in  $Y$ . Since  $Y$  is an IFSP1/2 space,  $f(B)$  is an IFSPS in  $Y$ . Now  $c(\alpha, \beta) \in f(B) \subseteq cl(f(B)) \subseteq cl(f(cl(B)))$ . Hence  $cl(f(B))$  is an IFSN of  $c(\alpha, \beta)$  in  $Y$ .

(ii)  $\Rightarrow$  (i) Let  $B$  be an IFROS in  $X$ . Then  $f^{-1}(c(\alpha, \beta)) \in B$ . This implies  $c(\alpha, \beta) \in f(B)$ . By hypothesis,  $cl(f(B))$  is an IFSN of  $c(\alpha, \beta)$ . Therefore there exists an IFSPS  $A$  in  $Y$  such that  $c(\alpha, \beta) \in A \subseteq cl(f(B))$ . Now  $A = \cup \{ c(\alpha, \beta) / c(\alpha, \beta) \in A \} = f(B)$ . Therefore  $f(B)$  is an IFSPS and hence an IFGSPS in  $Y$ . Thus  $f$  is an IFaGSPOM.

**Theorem 3.29:**The following are equivalent for a mapping  $f: X \rightarrow Y$  where  $Y$  is an  $IFSPT_{1/2}$ space.

- (i)  $f$  is an IFaGSPCM
- (ii)  $\text{spcl}(f(A)) \subseteq f(\alpha \text{cl}(A))$  for every IFSPOS  $A$  in  $X$
- (iii)  $\text{spcl}(f(A)) \subseteq f(\alpha \text{cl}(A))$  for every IFSOS  $A$  in  $X$
- (iv)  $f(A) \subseteq \text{spint}(f(\text{scl}(A)))$  for every IFPOS  $A$  in  $X$ .

**Proof:**(i)  $\Rightarrow$ (ii) Let  $A$  be an IFSPOS in  $X$ . Then  $\text{cl}(A)$  is an IFRCS in  $X$ . By hypothesis  $f(A)$  is an IFGSPCS in  $Y$  and hence is an IFSPCS in  $Y$ , since  $Y$  is an  $IFSPT_{1/2}$ space. This implies  $\text{spcl}(f(\text{cl}(A))) = f(\text{cl}(A))$ . Now  $\text{spcl}(f(A)) \subseteq \text{spcl}(f(\text{cl}(A))) = f(\text{cl}(A))$ . Since  $\text{cl}(A)$  is an IFRCS,  $\text{cl}(\text{int}(\text{cl}(A))) = \text{cl}(A)$ . Therefore  $\text{spcl}(f(A)) \subseteq f(\text{cl}(A)) = (\text{cl}(\text{int}(\text{cl}(A)))) \subseteq f(A \cup \text{cl}(\text{int}(\text{cl}(A)))) \subseteq f(\alpha \text{cl}(A))$ . Hence  $\text{spcl}(f(A)) \subseteq f(\alpha \text{cl}(A))$ .

(ii)  $\Rightarrow$ (iii) Let  $A$  be an IFSOS in  $X$ . Since every IFSOS is an IFSPOS, the proof is obvious.

(iii)  $\Rightarrow$ (i) Let  $A$  be an IFRCS in  $X$ . Then  $A = \text{cl}(\text{int}(A))$ . Therefore  $A$  is an IFSOS in  $X$ . By hypothesis,  $\text{spcl}(f(A)) \subseteq f(\alpha \text{cl}(A)) \subseteq f(\text{cl}(A)) = f(A) \subseteq \text{spcl}(f(A))$ . That is  $\text{spcl}(f(A)) = f(A)$ . Hence  $f(A)$  is an IFSPCS and hence is an IFGSPCS in  $Y$ . Thus  $f$  is an IFaGSPCM.

(i)  $\Rightarrow$ (iv) Let  $A$  be an IFPOS in  $X$ . Then  $A \subseteq \text{int}(\text{cl}(A))$ . Since  $\text{int}(\text{cl}(A))$  is an IFROS in  $X$ , by hypothesis,  $f(\text{int}(\text{cl}(A)))$  is an IFGSPOS in  $Y$ . Since  $Y$  is an  $IFSPT_{1/2}$ space,

$f(\text{int}(\text{cl}(A)))$  is an IFSPOS in  $Y$ . Therefore  $f(A) \subseteq f(\text{int}(\text{cl}(A))) \subseteq \text{spint}(f(\text{int}(\text{cl}(A)))) = \text{spint}(f(A \cup \text{int}(\text{cl}(A)))) = \text{spint}(f(\text{scl}(A)))$ . That is  $f(A) \subseteq \text{spint}(f(\text{scl}(A)))$ .

(iv)  $\Rightarrow$ (i) Let  $A$  be an IFROS in  $X$ . Then  $A$  is an IFPOS in  $X$ . By hypothesis,  $f(A) \subseteq \text{spint}(f(\text{scl}(A)))$ . This implies  $f(A) \subseteq \text{spint}(f(A \cup \text{int}(\text{cl}(A)))) \subseteq \text{spint}(f(A \cup A)) = \text{spint}(f(A)) \subseteq f(A)$ . Therefore  $f(A)$  is an IFSPOS in  $Y$  and hence an IFGSPOS in  $Y$ . Thus  $f$  is an IFaGSPCM, by Theorem 3.16.

**Theorem 3.30:**Let  $f: X \rightarrow Y$  be a mapping where  $Y$  is an  $IFSPT_{1/2}$ space. If  $f$  is an IFaGSPCM, then  $\text{int}(\text{cl}(\text{int}(f(B)))) \subseteq f(\text{spcl}(B))$  for every IFRCS  $B$  in  $X$ .

**Proof:**Let  $B \subseteq X$  be an IFRCS. By hypothesis,  $f(B)$  is an IFGSPCS in  $Y$ . Since  $Y$  is an  $IFSPT_{1/2}$ space,  $f(B)$  is an IFSPCS in  $Y$ . Therefore  $\text{spcl}(f(B)) = f(B)$ . Now  $\text{int}(\text{cl}(\text{int}(f(B)))) \subseteq f(B) \cup \text{int}(\text{cl}(\text{int}(f(B)))) = \text{spcl}(f(B)) = f(B) = f(\text{spcl}(B))$ . Hence  $\text{int}(\text{cl}(\text{int}(f(B)))) \subseteq f(\text{spcl}(B))$ .

**Theorem 3.31:**Let  $f: X \rightarrow Y$  be a mapping where  $Y$  is an  $IFSPT_{1/2}$ space. If  $f$  is an IFaGSPCM, then  $f(\text{spint}(B)) \subseteq \text{cl}(\text{int}(\text{cl}(f(B))))$  for every IFROS  $B$  in  $X$ .

**Proof:**This theorem can be easily proved by taking complement in Theorem 3.30.

Next we provide the characterization theorem for an IFaGSPOM.

**Theorem 3.32:**Let  $f: X \rightarrow Y$  be a bijective mapping. Then the following are equivalent.

- (i)  $f$  is an IFaGSPOM
- (ii)  $f$  is an IFaGSPCM
- (iii)  $f^{-1}$  is an IFaGSP continuous mapping.

**Proof:**(i)  $\Leftrightarrow$ (ii) is obvious from the Theorem 3.16.

(ii)  $\Rightarrow$ (iii) Let  $A \subseteq X$  be an IFRCS. Then by hypothesis,  $f(A)$  is an IFGSPCS in  $Y$ . That is  $(f^{-1})^{-1}(A)$  is an IFGSPCS in  $Y$ . This implies  $f^{-1}$  is an IFaGSP continuous mapping.

(iii)  $\Rightarrow$ (ii) Let  $A \subseteq X$  be an IFRCS. Then by hypothesis  $(f^{-1})^{-1}(A)$  is an IFGSPCS in  $Y$ . That is  $f(A)$  is an IFGSPCS in  $Y$ . Hence  $f$  is an IFaGSPCM.

#### 4. REFERENCES

- [1] Atanassov, K., Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 1986, 87- 96.
- [2] Chang, C., Fuzzy topological spaces, J. Math. Anal. Appl., 1968, 182-190.
- [3] Coker, D., An introduction to intuitionistic fuzzy topological space, Fuzzy sets and systems, 1997, 81-89.
- [4] Gurcay, H., Coker, D., and Haydar, Es. A., On fuzzy continuity in intuitionistic fuzzy topological spaces, The J. fuzzy mathematics, 1997, 365-378.
- [5] Joung Kon Jeon, Young Bae Jun, and Jin Han Park, Intuitionistic fuzzy alpha-continuity and intuitionistic fuzzy pre continuity, International Journal of Mathematics and Mathematical Sciences, 2005, 3091-3101.
- [6] Santhi, R. and Jayanthi, D., Intuitionistic fuzzy generalized semi-pre closed mappings (Submitted)
- [7] Santhi, R. and Jayanthi, D., Intuitionistic fuzzy generalized semi-pre closed set (Accepted by Tripura Mathematical Society, Tripura)
- [8] Santhi, R. and Jayanthi, D., Intuitionistic fuzzy generalized semi-pre continuous mappings (Accepted by The International journal of contemporary Mathematical Sciences, Bulgaria)
- [9] Santhi, R. and Jayanthi, D., Intuitionistic fuzzy almost generalized semi-pre continuous mappings (Submitted)
- [10] Seok Jong Lee and Eun Pyo Lee, The category of intuitionistic fuzzy topological spaces, Bull. Korean Math. Soc. 2000, 63-76.
- [11] Thakur, S. S and Rekha Chaturvedi, Regular generalized closed sets in intuitionistic fuzzy topological spaces, Universitatea Din Bacau, Studii Si Cercetari Stiintifice, Seria: Matematica, 2006, 257-272.
- [12] Young Bae Jun and Seok- Zun Song, Intuitionistic fuzzy semi-pre opensets and Intuitionistic semi-pre continuous mappings, jour. of Appl. Math & computing, 2005, 467-474.
- [13] Zadeh, L. A., Fuzzy sets, Information and control, 1965, 338-35