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ψgs- Closed Sets in Topological Spaces

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ABSTRACT

In this paper we introduce a new class of sets called ψgs-closed sets in topological spaces. We prove that this class lies between ψ-closed sets and ψg-closed sets. We study the properties of ψgs-closed sets.

Keywords: ψ-closed sets, ψg-closed sets and ψgs-closed sets

I. INTRODUCTION

In 1970, Levine [12] introduced the concept of generalized closed sets (briefly, g-closed set) in topological spaces. Levine [11] introduced the concepts semi-open sets in topological spaces. Veerakumar [18] introduced and studied ψ- closed sets in topological spaces. Ramya and Parvathi [15] introduced a new concept of ψ- generalized closed (briefly, ψg-closed) sets in topological spaces. In this paper we introduce a new class of sets namely ψgs-closed sets and study their basic properties.

II. PRELIMINARIES

Throughout this paper (X, τ) represents non-empty topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A), and int(A) denote the closure of A and the interior of A respectively.

Definition 2.1

A subset A of a topological space (X, τ) is called

(i) Semi-open set [11] if $A \subseteq cl(int(A))$
(ii) Regular open set [16] if $A=int(cl(A))$
(iii) α -open set if [10] $A \subseteq int(cl(int(A)))$
(iv) Pre-open set [9] if $A \subseteq int(cl(A))$
(v) Semi- pre-open set [2] if $A \subseteq cl(int(cl(A)))$

The complements of the above mentioned sets are called semi-closed, regular closed, α-closed, pre-closed and semi pre-closed sets respectively.

The intersection of all semi-closed (resp. regular -closed, α-closed, pre-closed and semi pre-closed) subsets of (X,τ) containing A is called the semi-closure (resp. regular-closure, α-closure, pre-closure and semi pre-closure) of A and is denoted by scl(A) (resp. rcl(A), αcl(A), pcl(A) and spcl(A)).

Definition 2.2

A subset A of a topological space (X,τ) is called

(i)generalized closed set (briefly g-closed)[12] if cl(A) ⊆U whenever A⊆U and U is open in (X, τ).

(ii)□-generalized closed set (briefly □g-closed)[13] if αcl(A) ⊆U whenever A⊆U and U is open set in (X, τ).

(iii)semi-generalized closed set (briefly sg-closed)[4] if scl(A)⊆U whenever A⊆U and U is semi-open in (X, τ).

(iv)generalized semi-pre -closed set [5] (briefly gsp-closed) if spcl(A) ⊆U whenever A⊆U and U is open in (X, τ).

(v) $\hat{g}$ - closed set [20] if cl(A)⊆U whenever A⊆U and U is semi-open in (X, τ).

(vi) g^*-closed set [19] if cl(A) ⊆U whenever A⊆U and U is g-open in (X, τ).

(vii) $^*g$- closed set [23] if cl(A) ⊆U whenever A⊆U and U is $\hat{g}$ -open in (X, τ).

(viii) $^{\#}$gs-closed set [22] if scl(A)⊆U whenever A⊆U and U is $^*g$ -open in (X, τ).

(ix) $\tilde{g}$-closed set [7] if cl(A)⊆U whenever A⊆U and U is $^{\#}$ gs-open in (X, τ).

(x) $\tilde{g}$-semi-closed set [17] if scl(A)⊆U whenever A⊆U and U is $^{\#}$ gs-open in (X, τ).

(xi) $\tilde{g}_{\alpha}$ -closed set [8] if αcl(A)⊆U whenever A⊆U and U is $^{\#}$ gs-open in (X, τ).

(xii) $\tilde{g}$-pre closed set [6] if pcl(A)⊆U whenever A⊆U and U is $^{\#}$ gs-open in (X, τ).

(xiii) αgs -closed set [14] if αcl(A) ⊆U whenever A⊆U and U is semi-open in (X, τ).

(xiv) α$\hat{g}$ - closed set [1] if αcl(A)⊆U whenever A⊆U and U is $\hat{g}$ -open in (X, τ).

(xv) ψ-closed set [18] if scl(A)⊆U whenever A⊆U and U is sg-open in (X, τ).

(xvi)) $g^*\psi$ -closed set [21] if ψcl(A)⊆U whenever A⊆U and U is g-open in (X, τ).

(xvii) ψg-closed set [15] if ψcl(A)⊆U whenever A⊆U and U is open in (X, τ).

(xviii) ψ$\hat{g}$ - closed set [15] if ψcl(A)⊆U whenever A⊆U and U is $\hat{g}$ -open in (X, τ).

(xix) ψg^{**} -closed set [3] if ψcl(A)⊆U whenever A⊆U and U is g^{**}- open in (X, τ).

The complements of the above mentioned sets are called their respective open-sets.

ψgs- closed sets

Definition 2.2.1 A subset A of a topological space (X, τ) is called **ψ generalized semi-closed** (briefly ψgs-closed) if ψcl(A)⊆U whenever A⊆U and U is semi-open in (X, τ).
The class of all ψgs-closed sets in (X, τ) is denoted by ψgsC(X, τ).

Example 2.2.2 Let X = {a, b, c} with topology τ={X, φ, {a}}. Then the subsets X, φ, {b}, {c} and {b, c} are ψgs- closed.

Proposition 2.2.3 Every closed set in (X, τ) is ψgs-closed in (X, τ) but not conversely.
Proof: Let A be a closed set in (X, τ) and U be any semi-open set containing A. Since A is closed, cl(A)=A. For every subset A of X ψcl(A)⊆cl(A)=A⊆U and so we have ψcl(A)⊆U. Hence A is ψgs- closed.

Example 2.2.4 Let X= {a, b, c} with topology τ={X, φ, {a}, {b}, {a, b}}. Then the subset {b} is ψgs-closed but not closed.

Proposition 2.2.5 Every regular closed set in (X, τ) is ψgs-closed in (X, τ) but not conversely.

Proof: The proof follows from the result that every regular closed set is closed and by **proposition 2.2.3.**

Example 2.2.6 Let X = {a, b, c} with topology τ={X, φ, {a}}.Then the subset {b} is ψgs –closed but not regular closed set.

Proposition 2.2.7 Every semi-closed set in (X, τ) is ψgs-closed in (X, τ) but not conversely.

Proof: Let A be a semi-closed set and U be any semi-open set containing A. Since A is semi-closed scl(A)=A. For every subset A of X ψcl(A)⊆scl(A)=A⊆U and so we have ψcl(A)⊆U. Hence A is ψgs closed.

Example 2.2.8 Let X= {a, b, c} with topology τ={X, φ, {a, b}}. Then the subset {a, c} is ψgs-closed but not semi-closed.

Proposition 2.2.9 Every α- closed set in (X, τ) is ψgs-closed in (X, τ) but not conversely.

Proof: Let A be an α- closed set and U be any semi-open set containing A. Since A is α- closed αcl(A)=A. For every subset A of X ψcl(A)⊆αcl(A)=A⊆U and so we have ψcl(A)⊆U. Hence A is ψgs closed.

Example 2.2.10 Let X= {a, b, c} with topology τ={X, φ, {a, b}}. Then the subset {b, c} is ψgs-closed but not α-closed.

Proposition 2.2.11 Every ψ- closed set in (X, τ) is ψgs-closed in (X, τ) but not conversely.

Proof: Let A be a ψ- closed set and U be any semi-open set containing A. Since every semi-open set is sg-open and A is ψ-closed, scl(A)⊆U. For every subset A of X, ψcl(A)⊆scl(A) and so we have ψcl(A)⊆U. Hence A is ψgs- closed.

Example 2.2.12 Let X= {a, b, c} with topology τ={X, φ, {a}, {b, c}} Then the subset {a, b} is ψgs-closed but not ψ-closed.

Proposition 2.2.13 Every αgs- closed set in (X, τ) is ψgs-closed in (X, τ) but not conversely.

Proof: Let A be an αgs-closed set and U be any semi-open set containing A. Since A is αgs –closed, αcl(A)⊆U. For every subset A of X, ψcl(A)⊆αcl(A) so we have ψcl(A)⊆U. Hence A is ψgs closed.

Example 2.2.14 Let X= {a, b, c} with topology τ={X, φ, {a}, {b}, {a, b}}. Then the subset {b} is ψgs-closed but not αgs-closed.

Proposition 2.2.15 Every g̃-closed set in (X, τ) is ψgs -closed in (X, τ) but not conversely.

Proof: Let A be a g̃ -closed set and U be any semi-open containing A. Since every semi-open set is #gs-open and A is g̃-closed cl(A)⊆U. For every subset A of X, ψcl(A)⊆cl(A) and so we have ψcl(A)⊆U. Hence A is ψgs- closed.

Example 2.2.16 Let X= {a, b, c} with topology τ={X, φ, {a}, {a, b}}. Then the subset {b} is ψgs-closed but not g̃ -closed.

Proposition 2.2.17 Every $\widetilde{g}_s$ -closed set in (X, τ) is ψgs-closed in (X, τ) but not conversely.

Proof: Let A be a $\widetilde{g}_s$ -closed set and U be any semi-open containing A. Since every semi-open set is #gs-open and A is $\widetilde{g}_s$ -closed. scl(A)⊆U. For every subset A of X, ψcl(A)⊆scl(A) and so we have ψcl(A)⊆U. Hence A is ψgs closed.

Example 2.2.18 Let X= {a, b, c} with topology τ={X, φ, {a}, {b, c}}.Then the subset {c} is ψgs-closed but not $\widetilde{g}_s$ -closed.

Proposition 2.2.19 Every $\widetilde{g}_{\alpha}$-closed set in (X, τ) is ψgs-closed in (X, τ) but not conversely.

Proof: Let A be a $\widetilde{g}_{\alpha}$- closed set and U be any semi-open containing A. Since every semi-open set is #gs-open and A is $\widetilde{g}_{\alpha}$-closed, αcl(A)⊆U. For every subset A of X, ψcl(A)⊆αcl(A)and so we have ψcl(A)⊆U. Hence A is ψgs closed.

Example 2.2.20 Let X= {a, b, c} with topology τ={X, φ,{a},{b},{a, b}}. Then the subset {a} is ψgs-closed but not $\widetilde{g}_{\alpha}$ -closed.

Proposition 2.2.21 Every ψgs -closed set in (X, τ) is ψg-closed in (X,τ) but not conversely.

Proof: Let A be a ψgs- closed set and U be any open set containing A. Since every open set is semi-open, and A is ψgs-closed, ψcl(A)⊆U and hence A is ψg- closed.

Example 2.2.22 Let X= {a, b, c} with topology τ={X, φ, {a}, {a, b}}. Then the subset {a, c} is ψg-closed but not ψgs-closed.

Proposition 2.2.23 Every ψgs-closed set in (X, τ) is gsp-closed in (X, τ) but not conversely.

Proof: Let A be a ψgs- closed set and U be any open set containing A. Since every open set is semi-open, and A is ψgs-closed, ψcl(A)⊆U. For every subset A of X. spcl(A)⊆ψcl(A) and so spcl(A)⊆U. Hence A is gsp- closed.

Example 2.2.24 Let X= {a, b, c} with topology τ={X, φ, {a}}.Then the subset {a, c} is gsp-closed but not ψgs -closed.

Remark 2.2.25 The following examples show that ψgs- closedness is independent from αg-closedness and g-closedness.

Example 2.2.26 Let X= {a, b, c} with τ={X, φ, {a}}.In this topology the subset {a, b} is αg-closed and g-closed but not ψgs-closed.

Example2.2.27 Let X= {a, b, c} with τ={X, φ, {a}, {b},{a, b}}.In this topology the subset {b} is ψgs-closed but not αg-closed and g-closed .

Remark 2.2.28 The following examples show that ψgs- closedness is independent from $g^*\psi$-closedness, g^*-closedness and ψg**-closedness.

Example 2.2.29 Let X= {a, b, c} with τ={X, φ, {a}, {a, b}}.In this topology the subset {a, c} is $g^*\psi$-closed, g^*-closed and ψg**-closed but not ψgs-closed.

Example 2.2.30 Let X= {a, b, c} with τ={X, φ, {a}, {b, c}}.In this topology the subset {b} is ψgs-closed but not $g^*\psi$ –closed, g^*-closed and ψg**-closed.

Remark 2.2.31 The following examples show that ψgs- closedness is independent from $\tilde{g}_p$-closedness and pre -closedness.

Example 2.2.32 Let X= {a, b, c} with τ={X, φ, {a, b}}.In this topology the subset {a} is $\tilde{g}_p$-closed and pre-closed but not ψgs-closed.

Example 2.2.33 Let X= {a, b, c} with τ={X, φ, {a}, {b}, {a, b}}.In this topology the subset {b} is ψgs-closed but not $\tilde{g}_p$-closed and pre-closed.

Remark 2.2.34 The following example show that ψgs- closedness is independent from $\psi\hat{g}$ -closedness, $^*g$-closedness and $\alpha\hat{g}$ -closedness.

Example 2.2.35 Let X= {a, b, c} with τ={X, φ, {a}}.In this topology the subset {a, b} is $\psi\hat{g}$-closed, $^*g$-closed and $\alpha\hat{g}$ -closed but not ψgs-closed.

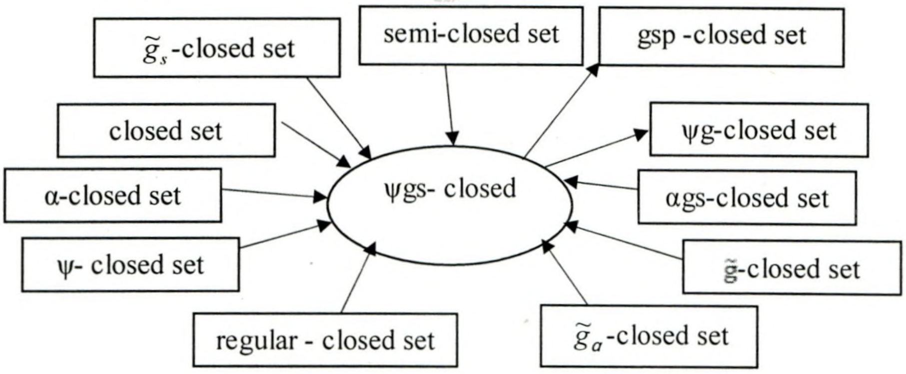
Example 2.2.36 Let X= {a, b, c} with τ={X, φ, {a}, {b, c}}.In this topology the subset {b} is ψgs-closed but not $\psi\hat{g}$ –closed, $^*g$-closed and $\alpha\hat{g}$ -closed.

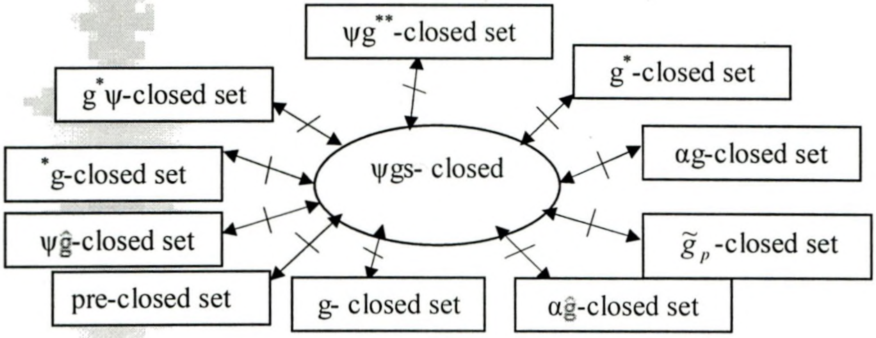
Remark 2.2.37 Union of two ψgs-closed sets need not be ψgs-closed sets as seen from the following example.

Example2.2.38 Let X={a, b, c} with τ={X, φ,{a},{b},{a, b}}.Then the subsets X, φ, {a}, {b}, {c}, {a, c} and {b, c} are ψgs-closed but {a}∪{b}={a, b} is not ψgs-closed.

Remark 2.2.39

The following diagrams show the relationship between ψgs –closed sets with already existing closed sets.





where A $\longrightarrow$ B represents A implies B and A $\not\longleftrightarrow$ B represents A and B are independent.

Theorem 2.2.40 Let A be a ψgs-closed set in (X, τ). Then ψcl(A)-A contains no non-empty closed set.

Proof: Suppose that A is ψgs-closed. Let F be a closed subset of ψcl(A)-A. Then F^c is open and hence semi-open such that A⊆ F^c. Since A is a ψgs-closed set, ψcl(A) ⊆ F^c. Thus F⊆$(ψcl(A))^c$. Also F⊆ψcl(A)-A. Therefore F⊆ψcl(A) ∩ $(ψcl(A))^c$ =φ. Hence F=φ.

Theorem 2.2.41 A set A is ψgs -closed in (X,τ) if and only if ψcl(A)-A contains no non empty semi-closed set.

Proof :(Necessity) Let A be a ψgs-closed subset of (X,τ). Let F be a semi-closed set contained in ψcl(A)-A. Since F^c is semi-closed with A⊆F^c and A is ψgs-closed ψcl(A)⊆F^c. Then F⊆$(ψcl(A))^c$. Also F⊆ ψcl(A)-A. Therefore F⊆$(ψcl(A))^c$∩(ψcl(A))=φ. Hence F=φ.

Sufficiency: Let ψcl(A)-A contains no non empty semi-closed set. Let A⊆G and G be semi-open. If ψcl(A) is not a subset of G then ψcl(A)∩G^c is a non-empty semi-closed subset of ψcl(A)-A, which is a contradiction. Therefore ψcl(A)⊆G and hence A is ψgs-closed.

Theorem 2.2.42 If a set A is semi-open and ψgs -closed set of (X, τ).Then A is ψ-closed set of X.

Proof: Since A is semi-open and ψgs -closed, ψcl(A) ⊆ A. Hence A is ψ-closed.

Theorem 2.2.43 If A is a ψgs -closed set in (X,τ) and A⊆B⊆ψcl(A) . Then B is also a ψgs-closed set.

Proof: Let U be a semi-open set of (X,τ) such that B⊆U. Then A⊆U. Since A is a ψgs -closed set, ψcl(A) ⊆ U. Also since B⊆ψcl(A), ψcl(B)⊆ ψcl(ψcl(A))= ψcl(A). Hence ψcl(B)⊆U. Therefore B is also a ψgs -closed set in (X, τ)

Theorem 2.2.44 For each x∈X either {x} is semi-closed or X-{x} is a ψgs –closed set in (X,τ).

Proof: Let x∈X and suppose that {x} is not semi-closed in X. Then X-{x} is not semi-open in X. Hence X is the only semi-open set containing X-{x}. That is (X-{x}) ⊆ X. Therefore ψcl(X-{x}) ⊆ X which implies that X-{x} is ψgs- closed set in (X, τ).

Theorem 2.2.45 Let A be a ψgs-closed set in (X,τ). Then A is ψ-closed if and only if ψcl(A)-A is semi-closed.

Proof :(Necessity) Let A be a ψ-closed subset of (X,τ).Then ψcl(A)=A and therefore ψcl(A)-A=φ which is semi-closed.

Sufficiency: Let ψcl(A)-A be a semi-closed set. Since A is ψgs-closed by **theorem 2.2.41** ψcl(A)-A contains no non- empty semi-closed set which implies ψcl(A)-A=φ. That is ψcl(A)=A. Hence A is ψ-closed.

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