

SUMMARY AND CONCLUSION

This thesis systematically explored the evolution and application of neutrosophic set variants, focusing on their mathematical properties, topological structures, approximation spaces, decision-making methodologies, and distance measures.

In PNS, the truth, indeterminacy, and falsity membership values are interdependent, and these membership functions can take values greater than 1, up to $\sqrt{2}$. Pythagorean Neutrosophic Sets (PNS) are capable of modelling situations in which both the degrees of acceptance and rejection are relatively high, by accommodating indeterminacy, thus providing greater flexibility. When introducing PNS, challenges arise in defining the universal set, null set, and set complement. These are addressed by incorporating a maximum–minimum condition into the definitions of the universal and null sets.

NSS is an enhanced form of traditional neutrosophic set theory, developed to manage uncertainty, vagueness, and indeterminacy in data more effectively than PNS. Its spherical design enables the modelling of situations where both acceptance and rejection can be relatively high and effectively captures indeterminacy that reflects meaningful hesitation in decision-making. In NSS, the membership values also remain interdependent but are restricted to the interval $[0,1]$, making it straightforward to define the whole set, null set, and complement. When compared PNS, and NSS with FNS, it is shown that FNS represents a generalised case of these sets, as demonstrated through graphs and comparison tables.

The study then extends to cubic and temporal concepts for FNS. In Fermatean neutrosophic cubic set, the dual representation of cubic set, combining precise and

interval-based membership, enables one to effectively model complex real-world problems involving mixed uncertainties. In Fermatean neutrosophic cubic sets, internal and external FNCS are defined, along with P-order, R-order, their unions, intersections, and related properties.

Fermatean Temporal neutrosophic sets, assigns time-dependent membership degrees over time, capturing more nuanced uncertainty dynamics. In Fermatean temporal neutrosophic sets, the fundamental set-theoretic operations are extended to include a temporal component.

Topological concepts are introduced to neutrosophic variants in the senses of Chang, Lowen, and Sostak. Since these interpretations differ, the topology of neutrosophic variants is formulated separately. The study also defined Fermatean neutrosophic gradation of openness, subspaces, gradation-preserving maps, bases and subbases, product Fermatean neutrosophic topological spaces, Fermatean neutrosophic compactness, and explored Tychonoff's theorem. Fermatean Temporal neutrosophic topology was also defined in a similar manner and several propositions are proved.

The rough set concept is then integrated with FNS, leading to the definition of Fermatean neutrosophic rough sets. Level cuts are defined along with some of their properties. The temporal concept is also incorporated to define Fermatean temporal neutrosophic rough sets, and their topological properties are developed.

By integrating FNS, Fermatean neutrosophic topology, Fermatean neutrosophic rough sets, and the temporal component, an attribute reduction rule is developed, supported by an illustrative example.

Furthermore, aggregation operators — both arithmetic and geometric — are defined for NSS and FNS, and a CODAS method is developed. An example is provided, and the method is compared with that of neutrosophic sets, with results presented.

Finally, distance measures for NSS and FNS are defined and compared with existing distance measures. These proposed measures are applied to enhance the TOPSIS method.

From this research, it is concluded that PNS and NSS can be applied in specific situations where their characteristics are most suitable. In the dependence case, the Fermatean neutrosophic set (FNS) is a generalised form, encompassing all types of fuzzy sets, intuitionistic fuzzy sets, neutrosophic sets, and their variants. Under independence, the FNS space satisfies $FNS \subseteq NS$, whereas under dependence, it becomes generalised and $NS \subseteq FNS$. Therefore, FNS is more specific than NS in certain applications under independence, yet broader in scope under dependence. Therefore, the theory for FNS paves as a framework that is capable of addressing all lower cases within a single model.

The future research directions based on this study are outlined as follows:

- Exploring lower and higher separation axioms, convergence properties, and compactness.
- Implementing the validated theoretical concepts, initially demonstrated through assumed numerical examples, in a practical, real-time application specifically in pattern recognition and machine learning.