

CHAPTER - III

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MODIFIED FUZZY TOPSIS METHOD

This method includes modifications in Fuzzy Multiple Criteria Decision-Making (MCDM) theory to strengthen the comprehensiveness and reasonableness of the decision making process using Fuzzy TOPSIS. Considering the fuzziness in the decision data and group decision making process, linguistic variables are used to assess the weights of all criteria and the ratings of each alternative with respect to each criterion. It is possible to convert the decision matrix into a fuzzy decision one and construct a weighted normalized fuzzy decision matrix once the decision makers' fuzzy ratings have been pooled. According to the concept of TOPSIS, the Fuzzy Positive Ideal Solution (FPIS) and the Fuzzy Negative Ideal Solution (FNIS) are defined. Then, a new method to calculate the distance between two triangular fuzzy ratings is used. Using the idea of comparison between two fuzzy numbers, the distance of each alternative from FPIS and FNIS are calculated respectively. In other words a new distance measure for Fuzzy TOPSIS is proposed in this method. Finally, a closeness coefficient of each alternative is used to determine the ranking order of all alternatives. The higher value of closeness coefficient indicates that an alternative is closer to FPIS and farther from FNIS simultaneously.

Definition 3.1 [58]

The *fuzzy comparison function* of two given discrete fuzzy numbers (\tilde{m} and \tilde{n}) is denoted as $\text{fuzzycmp}(\tilde{m}, \tilde{n})$ be the truth of that the fuzzy number \tilde{m} be greater than or equal to the fuzzy number \tilde{n} and is defined as

$$\text{fuzzycmp}(\tilde{m}, \tilde{n}) = \max[\min\{\mu_m(\tilde{m}_i), \mu_n(\tilde{n}_j)\}], \forall i, j : \tilde{m}_i \geq \tilde{n}_j \quad (1)$$

where \tilde{m}_i and \tilde{n}_j are the universe elements of discrete fuzzy numbers \tilde{m} and \tilde{n} , respectively.

Definition 3.2 [58]

The distance of two fuzzy numbers \tilde{m} and \tilde{n} using the fuzzy comparison function is $d(\tilde{m}, \tilde{n}) = | \text{fuzzycmp}(\tilde{m}, \tilde{n}) - \text{fuzzycmp}(\tilde{n}, \tilde{m}) |$ (2)

This describes the idea more closer two fuzzy numbers be, more nearer are the truths of one being greater than or equal to the other.

In this method the importance weight of each criterion can be obtained by either directly assign or indirectly using pairwise comparisons [27]. Here, the decision makers use the linguistic variables to evaluate the importance of the criteria and the ratings of alternatives with respect to various criteria. Assume that a decision group has K persons, and then the importance of the criteria and the rating of alternatives with respect to each criterion can be calculated as:

$$\tilde{x}_{ij} = \frac{1}{K} [\tilde{x}_{ij}^1 (+) \tilde{x}_{ij}^2 (+) \dots (+) \tilde{x}_{ij}^K] \quad (3)$$

$$\tilde{w}_j = \frac{1}{K} [\tilde{w}_j^1 (+) \tilde{w}_j^2 (+) \dots (+) \tilde{w}_j^K] \quad (4)$$

where \tilde{x}_{ij}^k and \tilde{w}_j^k are the rating and the importance weight of the k^{th} decision maker and (+) indicates the fuzzy arithmetic summation function.

A fuzzy multi-criteria group decision making problem can be concisely expressed in matrix format as

$$D = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & \tilde{x}_{mn} \end{bmatrix} \quad (5)$$

$$W = [\tilde{w}_j^1, \tilde{w}_j^2, \dots, \tilde{w}_j^K] \quad (6)$$

where \tilde{x}_{ij}^k and \tilde{w}_j^k are linguistic variables that can be shown by triangular fuzzy numbers $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ and $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$

The linear scale transformation is used to transform the various criteria scales into a comparable scale. Therefore, it is possible to obtain the normalized fuzzy decision matrix denoted by R

$$R = [\tilde{r}_{ij}]_{m \times n} \quad (7)$$

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+} \right), j \in B \quad (8)$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}} \right), j \in C \quad (9)$$

where B and C are the set of benefit criteria and cost criteria, respectively, and

$$c_j^+ = \max_i c_{ij}, \text{ if } j \in B \quad (10)$$

$$a_j^- = \min_i a_{ij}, \text{ if } j \in C \quad (11)$$

The normalization method mentioned above is to preserve the property that the ranges of normalized triangular fuzzy numbers belong to [0, 1].

Considering the different importance of each criterion, the weighted normalized fuzzy decision matrix is constructed as

$$\tilde{V} = [\tilde{v}_{ij}]_{m \times n}, i = 1, 2, \dots, m, j = 1, 2, \dots, n, \text{ where } \tilde{v}_{ij} = \tilde{r}_{ij}(\cdot) \tilde{w}_j \quad (12)$$

Then the Fuzzy Positive Ideal Solution (FPIS, A^+) and Fuzzy Negative Ideal Solution (FNIS, A^-) is defined as

$$A^+ = (\tilde{v}_1^+, \tilde{v}_2^+, \dots, \tilde{v}_n^+)$$

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^-) \quad (13)$$

where $\tilde{v}_i^+ = (1.0, 1.0, 1.0)$ and $\tilde{v}_i^- = (0.0, 0.0, 0.0)$

The distance of each alternative A_i ($i=1, 2, \dots, m$) from A^+ and A^- can be calculated as

$$d_i^+ = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_j^+), \forall i = 1, 2, \dots, m$$

$$d_i^- = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_j^-), \forall i = 1, 2, \dots, m \quad (14)$$

where $d(\bullet, \bullet)$ is the new distance measurements between two fuzzy numbers as is proposed by Eq. (2).

Moreover, a closeness coefficient is usually defined to determine the ranking order of all alternatives once the d_i^+ and d_i^- of each alternative A_i ($i=1, 2, \dots, m$) has been calculated. The closeness coefficient of each alternative is calculated as

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-}, \forall i = 1, 2, \dots, m \quad (15)$$

ALGORITHM:

The discussed modified calculations for multi-criteria group decision making fuzzy TOPSIS procedure can now be implemented in following general implementing steps proposed by Chen [11]

Step 1: Form a committee of decision-makers, and then identify the evaluation criteria.

Step 2: Choose the appropriate linguistic variables for the importance weight of the criteria and the linguistic ratings for alternatives with respect to criteria.

Step 3: Aggregate the weight of criteria to get the aggregated fuzzy weight \tilde{w}_j of criterion C_j , and pool the decision makers' opinions to get the aggregated fuzzy rating \tilde{x}_{ij} of alternative A_i under criterion C_j .

Step 4: Construct the normalized fuzzy decision matrix.

Step 5: Construct the weighted normalized fuzzy decision matrix.

Step 6: Determine Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS).

Step 7: Calculate the distance of each alternative from FPIS and FNIS, respectively using modified distance measure defined by Eq(2).

Step 8: Calculate the closeness coefficient of each alternative.

Step 9: According to the closeness coefficient, determine the ranking order of all alternatives.