6

γ Generalized Connected Space in Intuitionistic Fuzzy Topological Spaces

6.1 Introduction

Balachandran, Sundaram and Maki (1991) introduced GO connectedness in topological spaces. Narmadadevi, Roja and Uma (2013) introduced γ basically disconnected spaces. Rajasethupathy and Lakshmivarman (1977) have introduced connectedness in fuzzy topological spaces. Hanafy (2003) introduced the concept of γ connectedness in fuzzy topological spaces. Intuitionistic fuzzy C_5-connectedness is introduced by Coker (1997). Ozeag and Coker (1998) have introduced connectedness in intuitionistic fuzzy topological spaces. In this chapter we have introduced intuitionistic fuzzy γ generalized connected space, intuitionistic fuzzy γ generalized super connected space and intuitionistic fuzzy γ generalized extremally disconnected space. We investigated some of their properties. Also we have characterized the intuitionistic fuzzy γ generalized super connected space.

6.2 Intuitionistic fuzzy γ generalized connected spaces

Definition 6.2.1: An IFTS (X, τ) is said to be an ***intuitionistic fuzzy γ generalized*** (IFγG) ***connected space*** if the only IFSs which are both an IFγGCS and an IFγGOS are $0_\sim$ and $1_\sim$.

Proposition 6.2.2: Every IFγG connected space is an IFC_5-connected space but not conversely in general.

Proof: Let (X, τ) be an IFγG connected space. Suppose (X, τ) is not an IFC_5-connected space, then there exists a proper IFS B which is both an IFOS and an IFCS in (X, τ). That is B is both an IFγGOS and an IFγGCS in (X, τ). This implies that (X, τ) is not an IFγG connected space. This is a contradiction. Therefore (X, τ) is an IFC_5-connected space.

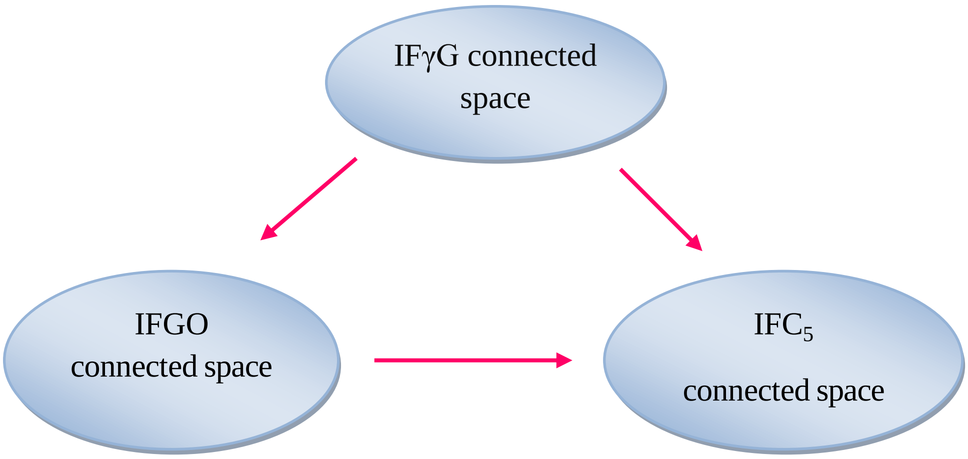
Example 6.2.3: Let X = {a, b} and τ = {$0_\sim$, A, $1_\sim$} be an IFT on X, where A = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩. Then (X, τ) is an IFC_5-connected space but not an IFγG connected space, since the IFS A in τ is both IFγGOS and IFγGCS in (X, τ).

Proposition 6.2.4: Every IFγG connected space is an IFGO-connected space but not conversely in general.

Proof: Let (X, τ) be an IFγG connected space. Suppose (X, τ) is not an IFGO-connected space, then there exists a proper IFS A which is both an IFGOS and an IFGCS in (X, τ). That is, A is both an IFγGOS and an IFγGCS in (X, τ). This implies that (X, τ) is not an IFγG connected space. This is a contradiction. Therefore (X, τ) is an IFGO-connected space.

Example 6.2.5: : Let X = {a, b} and τ = {$0_\sim$, A, $1_\sim$} be an IFT on X, where A = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩. Then (X, τ) is an IFGO-connected space but not an IFγG connected space, since the IFS A in τ is both IFγGOS and IFγGCS in (X, τ)..

The relation among various types of intuitionistic fuzzy connectedness is given in the following diagram.



The reverse implications are not true in general in the above diagram.

Proposition 6.2.6: An IFTS (X, τ) is an IFγG connected space if and only if there exist no non-zero IFγGOSs A and B in (X, τ) such that A = B^c.

Proof: Necessity: Let A and B be two IFγGOSs in (X, τ) such that $A \neq 0_\sim \neq B$ and $A = B^c$. Therefore B^c is an IFγGCS in X. Since $B \neq 0_\sim$, $A = B^c \neq 1_\sim$. This implies A is a proper IFS which is both an IFγGOS and an IFγGCS in (X, τ). Hence (X, τ) is not an IFγG connected space. But this is a contradiction to our hypothesis. Thus there exist no non-zero IFγGOSs A and B in (X, τ) such that $A = B^c$.

Sufficiency: Let A be both an IFγGOS and an IFγGCS in (X, τ) such that $1_\sim \neq A \neq 0_\sim$. Now let $B = A^c$. Then B is an IFγGOS and $B \neq 1_\sim$. This implies $B^c = A \neq 0_\sim$, which is a contradiction to our hypothesis. Therefore (X, τ) is an IFγG connected space.

Proposition 6.2.7: An IFTS (X, τ) is an IFγG connected space if and only if there exists no non-zero IFγGOSs A and B in (X, τ) such that $B = A^c$, $B = (\gamma cl(A))^c$ and $A = (\gamma cl(B))^c$.

Proof: Necessity: Assume that there exist IFSs A and B such that $A \neq 0_\sim \neq B$, $B = A^c$, $B = (\gamma cl(A))^c$ and $A = (\gamma cl(B))^c$. Since $(\gamma cl(A))^c$ and $(\gamma cl(B))^c$ are IFγGOSs in (X, τ), A and B are IFγGOSs in (X, τ). This implies (X, τ) is not an IFγG connected space, which is a contradiction. Therefore there exists no non-zero IFγGOSs A and B in (X, τ) such that $B = A^c$, $B = (\gamma cl(A))^c$ and $A = (\gamma cl(B))^c$.

Sufficiency: Let A be both an IFγGOS and an IFγGCS in (X, τ) such that $1_\sim \neq A \neq 0_\sim$. Now by taking $B = A^c$, we obtain a contradiction to our hypothesis. Hence (X, τ) is an IFγG connected space.

Proposition 6.2.8: Let (X, τ) be an IF$\gamma_c T_{1/2}$ space, then the following are equivalent:

(i) (X, τ) is an IFγG connected space,

(ii) (X, τ) is an IFGO-connected space,

(iii) (X, τ) is an IFC_5-connected space.

Proof:(i) ⇒(ii) is obvious from Proposition 6.2.4.

(ii) ⇒ (iii) is obvious.

(iii) ⇒ (i) Let (X, τ) be an IFC_5-connected space. Suppose (X, τ) is not an IFγG connected space, then there exists a proper IFS A in (X, τ) which is both an IFγGOS and an IFγGCS in (X, τ). But since (X, τ) is an $IF\gamma_cT_{1/2}$ space, A is both an IFOS and an IFCS in (X, τ). This implies that (X, τ) is not an IFC_5-connected space, which is a contradiction to our hypothesis. Therefore (X, τ) is an IFγG connected space.

Proposition 6.2.9: If f : (X, τ) → (Y, σ) is an IFγG continuous mapping and (X, τ) is an IFγG connected space, then (Y, σ) is an IFC_5-connected space.

Proof: Let (X, τ) be an IFγG connected space. Suppose (Y, σ) is not an IFC_5-connected space, then there exists a proper IFS A which is both an IFOS and an IFCS in (Y, σ). Since f is an IFγG continuous mapping, $f^{-1}(A)$ is both an IFγGOS and an IFγGCS in (X, τ). But this is a contradiction to hypothesis. Hence (Y, σ) is an IFC_5-connected space.

Proposition 6.2.10: If f : (X, τ) → (Y, σ) is an IFγG irresolute surjection mapping and (X, τ) is an IFγG connected space, then (Y, σ) is also an IFγG connected space.

Proof: Suppose (Y, σ) is not an IFγG connected space, then there exists a proper IFS B which is both an IFγGOS and an IFγGCS in (Y, σ). Since f is an IFγG irresolute mapping, $f^{-1}(B)$ is both IFγGOS and IFγGCS in (X, τ). But this is a contradiction to hypothesis. Hence (Y, σ) is an IFγG connected space.

Definition 6.2.11: An IFTS (X, τ) is called ***IFγG connected between*** two IFSs A and B if there is no IFγGOS E in (X, τ) such that A ⊆ E and $E\ {}_q^c\ B$.

Example 6.2.12: Let X = {a, b} and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$. Then,

The IFTS (X, τ) is an IFγG connected between the two IFSs $A = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)\rangle$ and $B = \langle x, (0.9_a, 0.8_b), (0.1_a, 0.2_b)\rangle$ as there exists no IFγGOS E such that A ⊆ E and $E\ {}_q^c\ B$.

Proposition 6.2.13: If an IFTS (X, τ) is IFγG connected between two IFSs A and B, then it is IFC_5-connected between two IFSs A and B.

Proof: Suppose (X, τ) is not IFC_5-connected between A and B, then there exists an IFOS E in (X, τ) such that $A \subseteq E$ and $E\ _q^c\ B$. Since every IFOS is an IFγGOS, there exists an IFγGOS E in (X, τ) such that $A \subseteq E$ and $E\ _q^c\ B$. This implies (X, τ) is not IFγG connected between A and B, a contradiction to our hypothesis. Therefore (X, τ) is IFC_5-connected between A and B.

Proposition 6.2.14: An IFTS (X, τ) is IFγG connected between two IFSs A and B if and only if there is no IFγGOS E in (X, τ) such that $A \subseteq E \subseteq B^c$.

Proof: Necessity: Let (X, τ) be IFγG connected between two IFSs A and B. Suppose that there exists an IFγGOS E in (X, τ) such that $A \subseteq E \subseteq B^c$, then $E\ _q^c\ B$ and $A \subseteq E$. This implies (X, τ) is not IFγG connected between A and B, by Definition 6.2.11. A contradiction to our hypothesis. Therefore there is no IFγGOS E in (X, τ) such that $A \subseteq E \subseteq B^c$.

Sufficiency: Suppose that (X, τ) is not IFγG connected between A and B. Then there exists an IFγGOS E in (X, τ) such that $A \subseteq E$ and $E\ _q^c\ B$. This implies that there is an IFγGOS E in (X, τ) such that $A \subseteq E \subseteq B^c$. But this is a contradiction to our hypothesis. Hence (X, τ) is IFγG connected between A and B.

Proposition 6.2.15: If an IFTS (X, τ) is IFγG connected between two IFSs A and B, $A \subseteq A_1$ and $B \subseteq B_1$, then (X, τ) is IFγG connected between A_1 and B_1.

Proof: Suppose that (X, τ) is not IFγG connected between A_1 and B_1, then by Definition 6.2.11, there exists an IFγGOS E in (X, τ) such that $A_1 \subseteq E$ and $E\ _q^c\ B_1$. This implies $E \subseteq B_1^c$ and $A_1 \subseteq E$ implies $A \subseteq A_1 \subseteq E$. Hence $A \subseteq E$. Since $E \subseteq B_1^c$, $B_1 \subseteq E^c$, $B \subseteq B_1 \subseteq E^c$. Hence $E \subseteq B^c$. Therefore (X, τ) is not IFγG connected between A and B, which is a contradiction to our hypothesis. Thus (X, τ) is IFγG connected between A_1 and B_1.

Proposition 6.2.16: Let (X, τ) be an IFTS and A and B be IFSs in (X, τ). If $A\ _q\ B$, then (X, τ) is IFγG connected between A and B.

Proof: Suppose (X, τ) is not IFγG connected between A and B. Then there exists an IFγGOS E in (X, τ) such that $A \subseteq E$ and $E \subseteq B^c$. This implies that $A \subseteq B^c$. That is

A $_q{}^c$ B. But this is a contradiction to our hypothesis. Therefore (X, τ) is IFγG connected between A and B.

Definition 6.2.17: An IFγGOS A in an IFTS (X, τ) is called an ***intuitionistic fuzzy regular γ generalized open set*** (IFRγGOS) if A = γgint(γgcl(A)). The complement of an IFRγGOS is called an ***intuitionistic fuzzy regular γ generalized closed set*** (IFRγGCS) in (X, τ).

Definition 6.2.18: An IFTS (X, τ) is called an ***intuitionistic fuzzy γ generalized*** (IFγG) ***super connected space*** if there exists no proper intuitionistic fuzzy regular γ generalized open set in (X, τ).

Theorem 6.2.19: Let (X, τ) be an IFTS, then the following are equivalent:

(i) (X, τ) is an IFγG super connected space,

(ii) For every non-zero IFRγGOS A, γgcl(A) = $1_\sim$,

(iii) For every IFRγGCS A with A ≠ $1_\sim$, γgint(A) = $0_\sim$,

(iv) There exists no IFRγGOSs A and B in (X, τ) such that A ≠ $0_\sim$ ≠ B, A ⊆ B^c,

(v) There exists no IFRγGOSs A and B in (X, τ) such that A ≠ $0_\sim$ ≠ B,

B = $(\gamma gcl(A))^c$, A = $(\gamma gcl(B))^c$,

(vi) There exists no IFRγGCSs A and B in (X, τ) such that A ≠ $1_\sim$ ≠ B,
B = $(\gamma gint(A))^c$, A = $(\gamma gint(B))^c$.

Proof: (i) ⇒(ii) Assume that there exists an IFRγGOS A in (X, τ) such that A ≠ $0_\sim$ and γgcl(A) ≠ $1_\sim$. Now let B = γgint(γgcl(A))^c. Then B is a proper IFRγGOS in (X, τ). But this is a contradiction to the fact that (X, τ) is an IFγG super connected space. Therefore γgcl(A) = $1_\sim$.

(ii) ⇒ (iii) Let A ≠ $1_\sim$ be an IFRγGCS in (X, τ). If B = A^c, then B is an IFRγGOS in (X, τ) with B ≠ $0_\sim$. Hence γgcl(B) = $1_\sim$, by hypothesis. This implies $(\gamma gcl(B))^c$ = $0_\sim$. That is γgint(B^c) = $0_\sim$. Hence γgint(A) = $0_\sim$.

(iii) ⇒ (iv) Suppose A and B be two IFRγGOSs in (X, τ) such that $A \neq 0_{\sim} \neq B$, $A \subseteq B^c$. Since B^c is an IFRγGCS in (X, τ) and $B \neq 0_{\sim}$ implies $B^c \neq 1_{\sim}$, $B^c = \gamma gcl(\gamma gint(B^c))$ and we have $\gamma gint(B^c) = 0_{\sim}$. But $A \subseteq B^c$. Therefore $0_{\sim} \neq A = \gamma gint(\gamma gcl(A)) \subseteq \gamma gint(\gamma gcl(B^c)) = \gamma gint(\gamma gcl(\gamma gcl(\gamma gint(B^c)))) = \gamma gint(\gamma gcl(\gamma gint(B^c))) = \gamma gint(B^c) = 0_{\sim}$. A contradiction arises. Therefore (iv) is true.

(iv) ⇒ (i) Suppose $0_{\sim} \neq A \neq 1_{\sim}$ be an IFRγGOS in (X, τ). If we take $B = (\gamma gcl(A))^c$, then B is an IFRγGOS, since $\gamma gint(\gamma gcl(B)) = \gamma gint(\gamma gcl(\gamma gcl(A))^c) = \gamma gint(\gamma gint(\gamma gcl(A)))^c = \gamma gint(A^c) = (\gamma gcl(A))^c = B$. Also we get $B \neq 0_{\sim}$, since otherwise, if $B = 0_{\sim}$, this implies $(\gamma gcl(A))^c = 0_{\sim}$. That is $\gamma gcl(A) = 1_{\sim}$. Hence $A = \gamma gint(\gamma gcl(A)) = \gamma gint(1_{\sim}) = 1_{\sim}$, which is a contradiction. Therefore $B \neq 0_{\sim}$ and $A \subseteq B^c$. But this is a contradiction to (iv). Therefore (X, τ) is an IFγG super connected space.

(i) ⇒ (v) Suppose A and B are any two IFRγGOSs in (X, τ) such that $A \neq 0_{\sim} \neq B$, $B = (\gamma gcl(A))^c$ and $A = (\gamma gcl(B))^c$. Now we have $\gamma gint(\gamma gcl(A)) = \gamma gint(B^c) = (\gamma gcl(B))^c = A$, $A \neq 0_{\sim}$ and $A \neq 1_{\sim}$, since if $A = 1_{\sim}$, then $1_{\sim} = (\gamma gcl(B))^c \Rightarrow \gamma gcl(B) = 0_{\sim} \Rightarrow B = 0_{\sim}$. But $B \neq 0_{\sim}$. Therefore $A \neq 1_{\sim} \Rightarrow$ A is a proper IFRγGOS in (X, τ), which is a contradiction to (i). Hence (v) is true.

(v) ⇒ (i) Suppose A is an IFRγGOS in (X, τ) such that $0_{\sim} \neq A \neq 1_{\sim}$. Now take $B = (\gamma gcl(A))^c$. In this case we get $B \neq 0_{\sim}$ and B is an IFRγGOS in (X, τ), $B = (\gamma gcl(A))^c$ and $(\gamma gcl(B))^c = (\gamma gcl(\gamma gcl(A))^c)^c = \gamma gint(\gamma gcl(A)^c)^c = \gamma gint(\gamma gcl(A)) = A$. But this is a contradiction to (v). Therefore (X, τ) is an IFγG super connected space.

(v) ⇒ (vi) Suppose A and B be two IFRγGCSs in (X, τ) such that $A \neq 1_{\sim} \neq B$, $B = (\gamma gint(A))^c$ and $A = (\gamma gint(B))^c$. Taking $C = A^c$ and $D = B^c$, C and D become IFRγGOSs in (X, τ) with $C \neq 0_{\sim} \neq D$, $D = (\gamma gcl(C))^c = (\gamma gcl(D))^c$, which is a contradiction to (v). Hence (vi) is true.

(vi) ⇒ (v) can be proved easily by the similar way as in (v) ⇒ (vi).