

REVIEW OF LITERATURE

## REVIEW OF LITERATURE

Topology is an important branch of mathematics. It has become a powerful topic in mathematical research. The concept of closedness is fundamental with respect to the investigation of topological spaces. In the study of topological spaces many concepts of topology have been generalized by considering the concepts of semi open sets and generalized open sets due to Levine (1963,1970) instead of open sets. Njastad (1965), Velicko (1968), Mashour et al (1982), Abd El- Monsef et al (1983), Bhattacharyya and Lahiri (1987), Tong (1989), Maki et al (1991), Palaniappan and Rao (1993), and Dontchev (1995) introduced respectively  $\alpha$ -open sets,  $\theta$ -open and  $\delta$ -open, preopen sets,  $\beta$ -open sets, semi-generalized open sets, strong regular open sets, generalized preopen sets, regular generalized open sets and generalized semi-preopen sets which are some weaker forms of open sets. Various types of sets have been studied by Dontchev (1998), Dunham (1982), Maki (1991, 1993, 1994, 1996). Ganster and Reily (1989) and Balachandran et al (1996) have introduced locally closed sets and generalized locally closed sets which are weaker than open and closed sets. Also in recent years, Ekici(2004,2005,2006,2007,2008, 2009) has published some papers related to various classes of closed sets in topological spaces. Apart from the above said concepts the following important and relevant results are reviewed.

### 1. Semi-open sets and semi-continuity in topological spaces

#### Levine (1963)

The notions of semi open sets and semi continuous functions are introduced and their properties are explored.

### 2. Generalized closed sets in topology

#### Levine (1970)

The author has introduced the notion of generalized closed sets in topological spaces and shown that compactness, locally compactness,

countably compactness, paracompactness and normality etc are g-closed hereditary. Further he introduced a separation axiom called  $T_{1/2}$  between  $T_1$  and  $T_0$ .

### **3. Somewhat continuous functions**

**Gentry and Hoyee (1971)**

The authors have introduced and studied the properties of somewhat continuous functions and somewhat homeomorphisms.

### **4. A generalization of closed mappings**

**Noiri (1973)**

In this article, the author has introduced semi-closed mappings which contains the class of closed mappings and discussed some characterizations of such mappings.

### **5. On semi homeomorphisms**

**Zbigniew Piotrowski (1979)**

The author has shown that if  $f: X \rightarrow Y$  is a closed function from a regular space  $X$  to a regular and dense in itself space  $Y$ , then  $f$  is open and continuous if and only if  $f$  is pre semi open and irresolute.

### **6. Generalized closed maps**

**Malghan (1982)**

The author has investigated generalized closed maps and discussed some of their properties.

## **7. On almost continuous function**

**Noiri (1988)**

The author has obtained some characterizations of almost continuous functions. It is shown that every nearly almost open and almost weakly continuous function is almost continuous.

## **8. Locally closed sets and LC-continuous functions**

**Ganster and Reilly (1989)**

The authors have introduced and investigated some properties of locally closed sets which are weaker than both open and closed sets.

## **9. Almost irresolute functions**

**Cammamroto and Noiri (1989)**

The authors have introduced and studied the notion of almost irresolute functions in topological spaces.

## **10. Operations on topological spaces and associated topology**

**Ogata (1991)**

In this article the author has introduced and studied the properties of  $(\alpha, \beta)$ -homeomorphic, irresolute and continuous functions.

## **11. On Generalized homomorphism in topological spaces**

**Maki et al (1991)**

In this article, the authors have defined the notion of generalized homeomorphism and gc-homeomorphism which are generalizations of homeomorphism and investigated some properties of generalized homeomorphism and gc-homeomorphism from the quotient space to other spaces.

## **12. Semi-generalized homeomorphisms and generalized semi-homeomorphism in topological spaces**

**Devi et al (1995)**

The authors have introduced two classes of mappings namely generalized semi homeomorphism and semi generalized homeomorphism and Investigated some properties of the mappings from the quotient space to other spaces.

## **13. Generalized $\alpha$ -closed maps and $\alpha$ -generalized closed maps**

**Devi et al (1997)**

The authors have introduced and investigated the concept of generalized  $\alpha$ -closed maps,  $\alpha$ -generalized closed maps and  $\alpha$ -regular spaces as a generalizations of closed maps, generalized closed maps and regular spaces respectively.

## **14. On generalizing semi open sets and preopen sets**

**Maki et al (1999)**

In this article the authors have generalized the notions of semi open sets and pre-open sets in an universal set and obtained a formula for the generalized closure of a subset. The new characterizations of semi-open sets, pre open sets,  $\delta$  preopen sets and semi preopen sets are analyzed.

## **15. Contra semi continuous functions**

**Dontchev and Noiri (1999)**

The authors have introduced and studied contra semi continuous function and further investigated strongly S closed spaces. Some new decompositions of generalized continuous functions are obtained.

## 16. More on generalized homeomorphisms in topological spaces

**Miguel Caldas(2001)**

The author has introduced and developed new classes of maps, namely generalized  $\Lambda_s$ -open, generalized  $\Lambda_s^c$ -homeomorphisms and generalized  $\Lambda_s^f$ -s-homeomorphisms, by using  $g\Lambda_s$ -sets, which are generalizations of semi-open maps and generalizations of homeomorphisms. Properties, characterizations and applications are discussed.

## 17. On $\alpha$ -quasi-irresolute functions

**Saeid Jafari and Takashi Noiri (2001)**

The authors have introduced  $\alpha$ -quasi-irresolute functions which is weaker than  $(\theta, s)$ -continuous functions and obtained some interesting results.

## 18. Properties of $(\theta, s)$ -continuous functions

**Takashi Noiri and Saeid Jafari (2002)**

In this article, the authors have studied the properties of  $(\theta, s)$  continuous functions and obtained relationships between  $(\theta, s)$ -continuity, contra-continuity and regular set-connectedness.

## 19. On $\pi$ gp-continuous functions in topological spaces

**Park and Park (2004)**

In this article the authors have characterized  $\pi$ gp-irresolute and  $\pi$ gp-continuous functions via the concept of  $\pi$ gp-closed sets and related these concepts to the classes of  $\pi$ GPO-compact spaces and  $\pi$ GP-connected spaces.

## 20. Slightly $\gamma$ -Continuous Functions

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**Erdal Ekici and Miguel Caldas (2004)**

The authors have introduced and studied strongly  $\gamma$ -irresoluteness,  $\gamma$ -irresoluteness,  $\gamma$ -continuity, precontinuity, semi-continuity,  $\gamma$ -continuity and slightly continuity. In this paper, slightly  $\gamma$ -continuity. Basic properties and preservation theorems of slightly  $\gamma$ -continuous functions are investigated and relationships between slightly  $\gamma$ -continuous functions and graphs are discussed.

## 21. Some properties of contra- $\gamma$ -continuous functions

**Arafa A. Nasef (2005)**

The author has introduced and discussed a new class of functions called contra- $\gamma$ -continuous functions which lies between classes of contra semi-continuous functions and contra-  $\beta$  continuous functions.

## 22. On $\pi$ gs-closed sets in topological spaces

**Aslim et al (2006)**

The authors have introduced  $\pi$ gs -closed sets and its properties are analyzed. Moreover the notions of  $\pi$ gs- $T_{1/2}$  spaces and  $\pi$ gs -continuity are investigated.

## 23. On almost $\pi$ gp-continuous functions

**Erdal Ekici (2007)**

The author has investigated the notion of almost  $\pi$ gp-continuous functions. Characterizations of almost  $\pi$ gp-continuous functions and preservation theorems are obtained. The implications between all related forms of continuities and almost  $\pi$ gp-continuity are discussed.

## **24. On contra $\pi g$ -continuous functions**

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**Ekici (2008)**

The author has introduced the notion of contra  $\pi g$ -continuous functions and obtained fundamental properties of contra  $\pi g$ -continuous functions. The relationships between contra  $\pi g$ -continuity and other related functions are obtained.

## **25. Quasi S normal spaces and pre $\pi g s$ -closed functions**

**Caksu Guler et al. (2009)**

In this article, the authors have introduced and studied quasi S-normal spaces and pre  $\pi g s$ -closed functions. Some preservation theorems of quasi S normal spaces are discussed.

## **26. On quasi $\alpha\psi$ -open functions in topological spaces**

**Devi and Parimala (2009)**

In this article, the authors have introduced quasi  $\alpha\psi$ -open function and investigated some of its fundamental properties and its characterizations.

## **27. On $\Pi g\beta$ -closed sets in topological spaces**

**Sanjay Tahiliani (2010)**

In this article, the author has introduced a new class of sets called  $\Pi g\beta$ -closed sets and its properties are studied. Moreover the notions of  $\Pi g\beta - T_{1/2}$  spaces and  $\Pi g\beta$ -continuity are introduced and its properties are analysed.

## **28. $\pi\Omega$ Closed and $\pi\Omega$ s- closed set in supra topological spaces**

**Arockiarani and Trinita Pricilla (2011)**

The authors have defined and investigated a weaker class of supra  $\pi\Omega$ - closed set in supra topological spaces. Characterizations of supra  $\pi\Omega$ s- closed set are established. The concept of supra  $\pi\Omega$  and supra  $\pi\Omega$ s- continuity are introduced and obtained their applications.

## **29. GRW-open and GRW-homeomorphism in a topological space**

**Pushpalatha and Rajarubi (2011)**

In this article, the authors have introduced GRW-open, GRW-homeomorphism and GRWC-homeomorphism in a topological spaces. Some of its properties are investigated.

## **30. On $\pi$ -Normality, Weak Regularity and the product of topological spaces**

**Sadeq Ali Saad Thabit and Hailiza Kamarulhaili (2011)**

The authors have investigated weakly (resp. almost) regular spaces and  $\pi$ -normal spaces. Properties, characterizations and preservation theorems of  $\pi$ -normal spaces are analyzed.

## **31. On g- $\alpha$ -irresolute functions**

**Bal and Zuo (2011)**

The authors have introduced the notion of g- $\alpha$ -irresolute functions in generalized topological spaces. Properties and several characterizations of this type of functions are studied.

## **32. Regular Pre-closed mappings**

**Krishanamurthy and Balasubramanian (2011)**

The authors have introduced the concept of  $rp$ -closed mappings and obtained the interrelationships between  $rp$ -closed mappings with other closed mappings.

### **33. Almost Contra $g\delta$ s-continuous functions in topological spaces**

**Benchalli et al (2012)**

In this article, the authors have introduced the notion of  $g\delta$ s-open sets in topological spaces and studied almost contra  $g\delta$ s-continuous functions. Some of its characterizations and properties are analyzed.

### **34. On Generalized $\alpha b$ - continuous maps, on generalized $\alpha b$ - open maps and on generalized $\alpha b$ - closed maps**

**Vinayagamoorthis and Nagaveni (2012)**

The authors have introduced generalized  $\alpha b$  - continuous maps, generalized- $\alpha b$  open maps and generalized- $\alpha b$  closed maps and studied some of its properties.

### **35. Some new concepts of continuity in generalized topological space**

**Powar and Rajak (2012)**

The authors have introduced and studied the concept of generalized completely Irresolute functions in generalized topological space, completely fine-irresolute maps and  $G$ -continuous functions in fine space. Properties and applications are discussed.

### **36. On $\pi gb$ - closed sets and related topics**

**Sinem Çağlar Akgün and Gülhan Aslim (2012)**

In this article, the authors have introduced  $\pi gb$ -closed sets, almost  $\pi gb$  continuous functions which are generalizations of both  $\pi gb$  continuity and

almost  $b$ -continuity,  $\pi gb$  compactness and quasi  $b$  normality in topological spaces. Some of their properties are studied.

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### **37. $\pi p$ -Normal topological spaces**

**Sadeq Ali Saad Thabi and Hailiza Kamarulhaili (2012)**

The authors have studied weaker version of  $p$  normality called  $\pi p$  normality, which lies between  $p$ -normality and almost  $p$ -normality. They have proved that  $p$ -normality is a topological property and it is a hereditary property with respect to closed domain subspaces. Some basic properties, examples, characterizations and preservation theorems of this property are obtained.

#### **Bi $\check{C}$ ech closure spaces**

Cech closure spaces were introduced by  $\check{C}$ ech (1968) and studied by many authors. Bi $\check{C}$ ech closure spaces were introduced by Chandrasekhara Rao, Gowri and Swaminathan (2008). Devi et al.(2010) introduced the concept of  $\alpha\psi$ - closed sets in Bi $\check{C}$ ech closure spaces. Boonpok (2009) studied the concept of closed Maps in Bi $\check{C}$ ech Closure Spaces. A few studies on Bi $\check{C}$ ech closure spaces are listed below.

#### **1. On Closed Maps in Bi $\check{C}$ ech Closure Spaces**

**Boonpok (2009)**

The author has studied and investigated some properties of closed maps in bi $\check{C}$ ech closure spaces.

#### **2. $C_0$ -Bi $\check{C}$ ech Spaces And $C_1$ -Bi $\check{C}$ ech Spaces**

**Boonpok (2009)**

The author has introduced the concepts of  $C_0$ -bi $\check{C}$ ech spaces and  $C_1$ -bi $\check{C}$ ech spaces and studied its basic properties.

### 3. Bicontinuous Maps in Biclosure Spaces

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**Boonpok (2010)**

The author has introduced and studied the concept of bicontinuous maps in biclosure spaces and investigated its behaviour.

### 4. Pairwise Closed Sets in Biclosure Spaces

**Boonpok (2010)**

In this article, the author has introduced the concept of pairwise closed sets in biclosure spaces and studied their fundamental properties. Also, introduced preserve pairwise closed maps by using pairwise closed sets and investigated some of their characterizations.

### 5. $\pi\alpha$ -Separation Axioms in Bi-Cech Spaces

**Arockiarani et al (2011)**

The authors have introduced the concept of  $\pi\alpha$  generalized closed sets in bi-Cech closure space. Properties and characterizations are obtained.

### **BIMINIMAL STRUCTURE SPACES**

Maki (1996) introduced minimal structures and minimal spaces. Some results about minimal spaces can be found in Lugojan (1982), Maki (1996), Popa (2000) Casazar (2002) and Noiri (2003). Boonpok (2010) enhanced the concept of biminimal structure spaces and studied the fundamental properties of  $m_X^1$   $m_X^2$ -closed sets,  $m_X^1$   $m_X^2$ -open sets and M-continuous functions. The following are some of the references on minimal and Biminimal structure spaces.

## 1. On m-continuous functions

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### **Popa and Noiri (2000)**

The authors have introduced M-continuous functions in minimal spaces. They obtained some characterizations and properties of such functions. Further, the concepts m-compactness and m-connectness are introduced and their properties are obtained.

## 2. Biminimal Structure Spaces

### **Boonpok (2010)**

The author has introduced the concept of biminimal structure spaces and studied some fundamental properties of  $m_X^1$   $m_X^2$ -closed sets and  $m_X^1$   $m_X^2$  open sets in biminimal structure spaces.

## 3. Generalized m-Closed Sets in Biminimal Structure Spaces

### **Boonpok et al (2011)**

The authors have introduced the concept of generalized gm-closed sets in biminimal structure spaces and obtained some properties of generalized m-closed sets. Further he investigated the notions of  $m^{(i,j)}T_{1/2}$  spaces and  $gM^{(i,j)}$ -continuous functions. Some of their characterizations are discussed.

## 4. Weakly M-Precontinuous Functions on Biminimal Structure Spaces

### **Boonpok et al (2011)**

In this article, the authors have introduced weakly M-precontinuous functions on biminimal structure spaces and obtained several characterizations and properties of these functions.

## 5. Neighborhood and Accumulation Points in Biminimal Structure Spaces

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### Supunnee Sompong and Sa-at Muangchan (2012)

The authors have introduced the concepts of neighborhoods and accumulation points in biminimal structure spaces and some fundamental properties of neighborhoods and accumulation points in biminimal structure spaces are obtained.

### Grill

Choquet (1947) initiated the idea of grills on topological spaces. This concept is a powerful supporting tool like nets and filters for getting a deeper insight into further study of topological spaces. Roy and Mukherjee (2007) and Roy et al.(2008) defined and studied a topology by associating the existing topology and a grill. Dhananjoy Mandal and Mukherjee (2012) have defined and studied  $\mathcal{G}$ -g-closed sets in grill topological spaces. Arockiarani and Karthika (2012) introduced the notion of generalized B-closed sets in Grill Topology. A few studies on Grill spaces are listed below.

#### 1. On H-closed spaces and grills

##### Mukherjee and Atasi Debray (1998)

The authors have studied and investigated (quasi) H-closed spaces in terms of grills, and obtained some interesting characterizations.

#### 2. On $\theta$ -Principal Extension and H-Closedness

##### Mukherjee and Atasi Debray (2000)

In this article, a type of extensions called  $\theta$ -extension of topological spaces and their  $\theta$ -equivalence and  $\theta$ -trace systems are introduced, which ultimately characterize H-closed  $\theta$ -extensions of a Hausdorff topological space. Also, the notion of  $\theta$ -principal extensions is defined. A typical  $\theta$ -principal extension consisting of certain grills on a Hausdorff space is

constructed, and finally, some characterizations of H-closedness of a Hausdorff space are obtained.

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### 3. p-closed topological spaces in terms of grills

#### **Mukherjee and Roy (2006)**

The concept of p-closedness, a kind of covering property for topological spaces, has already been studied with meticulous care from different angles and via different approaches. The authors have continued the investigation in terms of different concepts viz. grills. The deliberations in the article include certain characterizations and a few necessary conditions for the p-closedness of a space, the latter conditions are also shown to be equivalent to p-closedness in a pre-almost regular space. All these and the associated discussions and results are studied with respect to grills as the prime supporting tool.

### 4. Concerning topologies induced by principal Grills

#### **Roy and Mukherjee (2009)**

Given a grill  $\mathcal{G}$  on a topological space  $(X, \tau)$ , a new topology  $\tau_{\mathcal{G}}$  on  $X$ , induced by the operator  $\Psi : \mathcal{P}(X) \rightarrow \mathcal{P}(X)$ , given by  $\psi(A) = A \cup \Phi(A)$  (for  $A \in \mathcal{P}(X)$ ), where  $\Phi(A) = \{x \in X : U \cap A \in \mathcal{G}, \text{ for all open neighbourhoods } U \text{ of } x\}$ . In this article, the authors have studied some interesting properties and behaviors of the topology  $\tau_{\mathcal{G}}$  where  $\mathcal{G}$  is a principal grill. Certain separation axioms and some well known covering properties are investigated. The simultaneous sharing of these properties by this topology  $\tau_{\mathcal{G}}$  (with  $\mathcal{G}$  a principal grill) and the preassigned topology on a set are discussed.

## 5. $P_\omega$ -closedness and its generalization with respect to a grill

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**Basu and Uzzal afsan (2011)**

In this article, the authors have introduced a new kind of covering axiom pre- $\omega$ -closedness ( $P_\omega$ -closedness, for short), stronger than p-closedness in terms of pre- $\omega$ -open sets. Several characterizations via filter bases and grills along with various properties of this concept are obtained. Grill generalizations of  $P_\omega$ -closedness and associated concepts have also been investigated.

## 6. On a type of generalized closed sets

**Dhananjay Mandal and Mukherjee (2012)**

The authors have introduced and studied a new class of generalized closed sets in a topological space  $X$ , defined in terms of a grill  $\mathcal{G}$  on  $X$ . Explicit characterization of such sets along with certain other properties of them are obtained. As applications, some characterizations of regular and normal spaces are achieved by use of the introduced classes of sets.

## 7. On a Type Of Generalized B-Closed Sets In Grill Topology

**Arockiarani and Karthika (2012)**

The authors have introduced and studied a new class of generalized closed sets in a Grill topological space  $(X, \tau, \mathcal{G})$ . The characterizations of such sets along with certain other properties of them are analyzed. The notions of this new class of sets with b-regular and b-normal spaces are also studied.