



# Avinashilingam Institute for Home Science and Higher Education for Women

Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD (now MoE)

Re-accredited with A++ Grade by NAAC. CGPA 3.65/4, Category I by UGC

Coimbatore - 641 043, Tamil Nadu, India

Continuous Internal Assessment Test I – August 2025

Semester V

Class : III UG  
Major : Mathematics

Time : 2 Hours  
Max. Marks : 60

## 23BMAC09 – Set Theory and Metric Spaces

### Course Outcomes:

CO1: learn basic facts about the cardinality of a set.

CO2: understand several standard concepts of metric spaces and their properties like openness and closedness.

CO3: identify the continuity of a function defined on metric spaces and homeomorphisms.

CO4: know the concepts of compactness, Bolzano-Weierstrass property and Totally Bounded sets.

CO5: recognize the difference between connected and disconnected sets and their properties.

### PART A

6 x 1 = 10

#### Choose the Correct Answer

- The cardinal number of the uncountable set of real number is denoted as \_\_\_\_\_ CO1K1  
a)  $c$       b)  $\infty$       c)  $\aleph_0$       d)  $\phi$
- The set inclusion is a \_\_\_\_\_ relation. CO1K2  
a) Partial order      b) total order  
c) linear order      d) either b or c
- The supremum of the set  $(0,1) \cup \{2\}$  is \_\_\_\_\_ CO1K2  
a) 1    b) 2      c) 0      d) does not exist
- Which one of the following is a neighbourhood of each of its points? CO2K2  
a)  $\mathbb{R}$       b)  $\mathbb{Z}$       c)  $\mathbb{N}$       d)  $\mathbb{Q}$
- If  $A$  is a Cantor set, then  $A^\circ =$ ----- CO2K2  
a)  $\mathbb{N}$       b)  $\mathbb{R}$       c)  $\phi$       d)  $\mathbb{Z}$
- The limit point of the sequence  $\{1/n\}$  is CO2K1  
a) 0      b)  $n$       c)  $\infty$       d) 1

### Part B

3 x 6 = 18

#### Answer ALL questions

- a. Prove that the set of all rational number is countable. CO1K3  
(or)
- b. Prove that any countable union of countable set is countable. CO1K3
- a. State and prove Cantors theorem. CO1K3  
(or)
- b. Prove that  $(X, d)$  is a metric space where  $X = \mathbb{R}$  and  $d(x, y) = |x-y|$ . CO2K4
- a. Prove that in a metric space  $(X, d)$ , finite intersection of open sets is open. CO2K4  
(or)
- b. State and prove Hausdorff property. CO2K4

**Part C**

**3 x 12 = 36**

**Answer ALL questions**

10. a. State and prove Schroeder-Bernstein theorem.

CO1K4

(or)

10. b. Prove that the set of all real numbers is uncountable.

CO1K3

11. a. Prove that the Cartesian product of natural numbers is countable.

CO1K3

(or)

11. b. Prove that every non-empty open set in  $\mathbb{R}_n$  is the union of a countable disjoint class of open interval..

CO2K4

12. a. Let  $(X, d)$  be a metric space and  $A \subset X$ . Then prove that  $A$  is closed if and only if  $A$  contains all its limit points.

CO2K4

(or)

12. b. Let  $(X, d)$  be a metric space, then prove that each closed sphere in  $X$  is a closed set.

CO2K4

**No. of copies : 22**