INTRODUCTION

Topology is a widely studied branch of Mathematics emerged through the study initiated by the great mathematician Henri Poincare in the 19th century. Topology has been developed as a field of study out of Geometry and Set Theory. The topological structures are suitable for not only the quantitative data but also for the qualitative data. So the notions of sets and functions in topological spaces are highly developed and used in many pure and applied Mathematics. The influence of topological spaces can be observed in the fields of computer graphics, pattern recognition, artificial intelligence, data mining, rough set theory, information systems and quantum physics and so on.

The notion of open sets is a powerful tool for defining a topological space. The idea of regular open sets was initiated by Stone (1937) which is a stronger form of open sets. In order to extend some important properties of open sets to a larger family of sets, the concept of semi open sets, which is a weaker form of open sets, was pioneered by Norman Levine (1963).

In 1968 Velicko introduced δ-open sets which are stronger than open sets in order to investigate the characterization of H-closed spaces in terms of arbitrary filter bases and showed that the collection of all δ-open sets, denoted by τ_δ, is a coarser topology on X, where a set is δ-open if it is the union of regular open sets. The collection of all regular open sets forms a base for a coarser topology $\tau_s \subseteq \tau$ but $\tau_s = \tau_\delta$. The family τ_s is called the semi-regularization of a given topology τ. In a semi-regular space, $\tau = \tau_s = \tau_\delta$. Further Norman Levine (1970) initiated the concept of generalized closed (denoted as g-closed) sets which are the weaker form of closed sets, the complement of open sets. Since then several mathematicians intensively contributed their work in the evolution of generalised closed sets.

Through the semi-regularization of a given topology and the associated δ-closure operator, Julian Dontchev, et.al., (1996) established a stronger form of g-closedness namely δg-closedness which is properly placed between δ-closedness and g-closedness.

δ-closedness → δg-closedness → g-closedness

He also proved that in semi regular spaces, the notions of δg-closed sets and g-closed sets coincide. Majority of closedness related concepts considered by Bhattacharya, et.al.,(1987), and Maki, et.al.,(1993, 1994) are weaker than generalized closedness. Julian Dontchev (2000) introduced and studied two new classes of generalized closed sets, namely, gδ-closed and $\delta g^{\ddagger}$-closed sets. Through these concepts new characterizations of almost weakly Hausdorff spaces were obtained and a new space $T_{3/4}$ emerged as an example of digital line.

The fruitfulness of the notion of generalized closed sets inspired the mathematicians to pioneer weaker and stronger forms of generalized closedness for the past four decades. Sudha (2012) initiated a stronger form of δg-closed sets namely, δg^*-closed sets.

δ-closedness → δg^*-closedness → δg-closedness → gδ-closedness

Norman Levine (1970) established $T_{1/2}$-spaces in which g-closed and closed sets coincide. Dunham (1977) showed that $T_{1/2}$-spaces are the spaces in which singletons are open or closed. The associated separation axioms are below $T_{1/2}$-separation axiom. A $T_{3/4}$ separation axiom was introduced by Julian Dontchev (1996) in which every δg-closed set is δ-closed and proved that it is properly placed between T_1 and $T_{1/2}$-spaces.

T_1 → $T_{3/4}$ → $T_{1/2}$

Norman Levine (1970) initiated the idea of continuous functions. Noiri, T (1980) introduced δ-continuous functions. The generalised continuity (briefly, g-continuity) was studied by Balachandran.K (1991).With the aid of g-open sets, many topologists introduced, and investigated various continuous functions which are the core concept of topology. In this regard, many topologists have enriched the field of generalized closed sets to a considerable extent through their detailed study.

Open maps and closed maps are very useful in topological spaces. The concept of homeomorphisms plays an important role in topological spaces. For researchers on various closed sets, the study is not complete without extending their definitions to open (closed) maps and homeomorphisms. Malghan (1982) introduced the concept of generalized closed maps in topological spaces. Maki et al. (1991) introduced generalized homeomorphisms and gc-homeomorphisms and studied their properties.

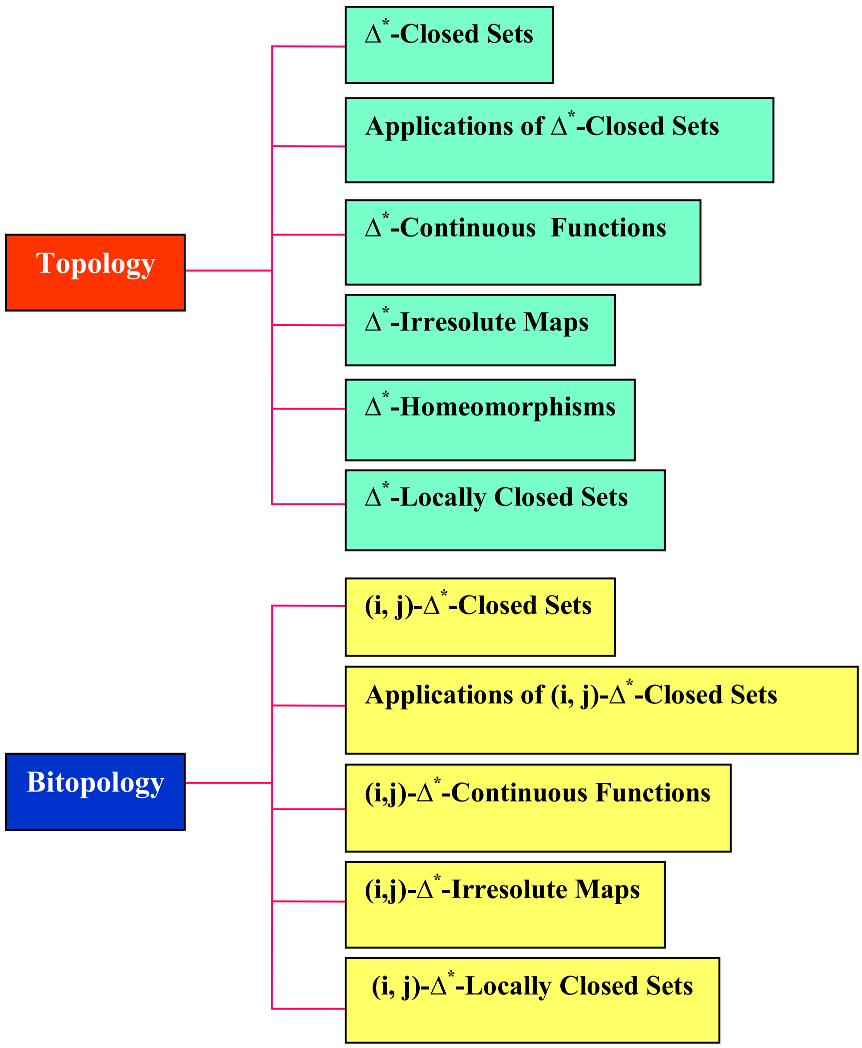
Ganster and Reilly (1989) described locally closed sets in topological spaces. Maki.H et al. (1996) developed the concept of generalized locally closed sets to obtain different notions of generalized continuities.

Kelly.J.C.(1963) introduced the idea of bitopological spaces and thereafter the theory has been developed by different mathematicians from different aspects. It is confined in considering the pairwise properties of the two topologies and their interrelations. Fukutake (1986) extended the concept of g-closed sets to bitopological spaces. Sheik John et al. (2004) proposed g*-closed sets in bitopological spaces. Mohammed et al. provided the concept of δ-open sets in bitopological spaces.

The endeavor of the present work is to introduce the concept of Δ^*-closed (Delta star closed) sets, to analyze their relations with various closed sets and to obtain their properties and some characterizations. It is proved that the Δ^*-closedness is stonger than gδ-closedness.

δ-closedness → δg^*-closedness → Δ^*-closedness → gδ-closedness

The deliberations in this research work include the following topics.



Notations : Throughout the thesis, the following notations are used.

- (X, τ), (Y, σ) and (Z, η) denote non empty topological spaces on which no separation axioms are mentioned unless it is stated specifically.
- The closure and interior of a subset A of a topological space is denoted by cl(A) and int(A) respectively.

- **In all the diagram: A ⟶ B represents A implies B but not conversely.**

  A ⟷ B (crossed) represents A and B are independent with each other.

Methodology : The analysis of Δ^*-closed sets has been done by the following methods.

- Analytical method of comparing Δ^*-closed sets with existing closed sets.
- Producing Counter examples wherever necessary.
- Analysis of preservation of topological properties by Δ^*-closed sets.
- Obtaining Characterization theorems.

Chapter 1 deals with preliminary definitions and results on various existing closed sets which are used to accomplish the present study.

In Chapter 2, a new class of closed sets, namely, Δ^*-closed (Delta star closed) sets is introduced in topological spaces. The class of all Δ^*-closed sets in a topological space (X, τ) is denoted by $\Delta^*\mathcal{C}$(X, τ). The class $\Delta^*\mathcal{C}$(X, τ) of all Δ^*-closed sets is properly lies between the classes of $\delta g^*\mathcal{C}$(X, τ) and $g\delta\mathcal{C}$(X, τ).

i.e., **$\delta g^*\mathcal{C}(X, \tau)$ □ $\Delta^*\mathcal{C}(X, \tau)$ □ $g\delta\mathcal{C}(X, \tau)$**

A comparative study between Δ^*-closed sets with other existing closed sets, properties of Δ^*-closed sets and characterizations of Δ^*-closed sets are analysed in this chapter. A similar study has also been developed for Δ^*-open sets in this chapter.

The following definitions are introduced in this chapter.

- A subset A of a topological space (X, τ) is called **Δ^*-closed set** if $\delta cl(A) \subseteq U$ whenever $A \subseteq U$, U is δg-open in (X, τ). The class of all Δ^*-closed sets of (X, τ) is denoted by $\Delta^*\mathcal{C}$(X, τ).

- The **Δ^*-closure operator** of a topological space (X, τ) is denoted by **$\Delta^*cl(A)$** and defined as follows.

$\Delta^*cl(A) = \cap$ { F □ X : A □ F and F is Δ^*-closed in (X, τ) }.

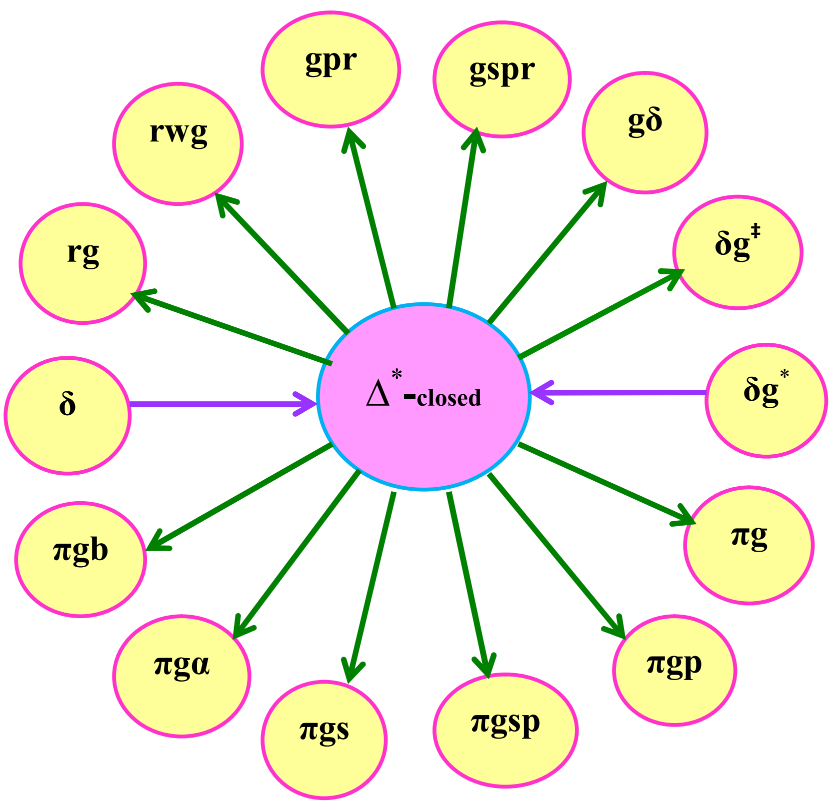
❖ Let U be any subset of (X,τ). Using Δ^*-closure operator, a new class of sets denoted by $\Delta^*\tau^\#$ is defined as follows.

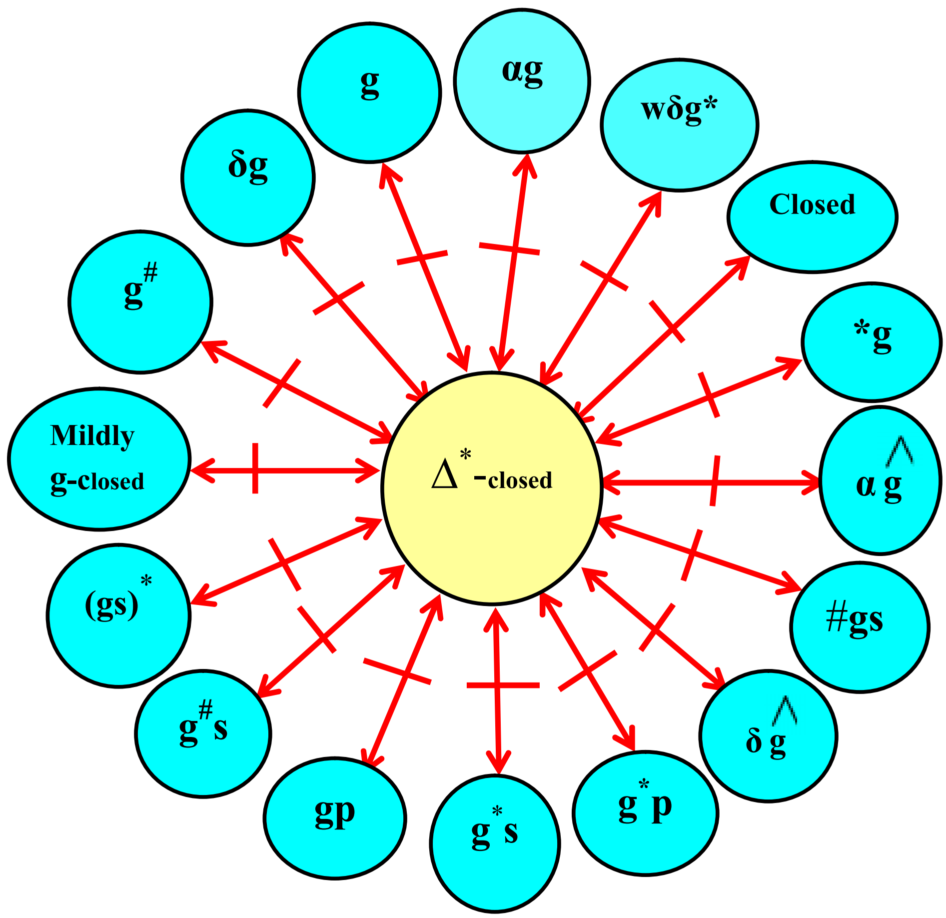
$\Delta^*\tau^\# = \{ U : \Delta^*cl(X - U) = X - U \}$.

❖ A subset A of a topological space(X, τ) is called Δ^***-open** if its complement A^c is Δ^*-closed in (X, τ). The collection of all Δ^*-open sets in (X, τ) is denoted by $\Delta^*O(X, \tau)$.

The following important results are analysed in this chapter.

❖ The comparative study of Δ^*-closed set with various existing closed sets is studied and depicted by the following diagrams.





The following significant preservation of properties of closed sets by Δ^*-closed sets are proved in this chapter.

- Finite union of Δ^*-closed sets is Δ^*-closed.
- For any two subsets A & B of (X, τ), if A □ B then Δ^* cl(A) □Δ^* cl(B).
- For any two subsets A & B of (X, τ), Δ^* cl(A□B) = Δ^*cl(A)□Δ^* cl(B).
- If A is Δ^*-closed then Δ^* cl(A)=A.
- If A is a Δ^*-closed set then δcl(A)−A contains no non empty δg-closed set.
- If A is Δ^*-closed and A □B □ δcl(A) then B is Δ^*-closed.
- If A is Δ^*-closed then δcl(A)−A is Δ^*-open.
- The intersection of a Δ^*-closed set and a δ-closed set is always Δ^*-closed.

The following characterizations of Δ^*-closed sets are derived in this chapter.

- If A is a Δ^*-closed set then it is δ-closed if and only if $\delta cl(A) - A$ is δg-closed.
- For each $x \in X$, $x \in \Delta^* cl(A)$ if and only if $(U \cap A) \neq \phi$, for all Δ^*-open sets U containing x.
- Every Δ^*-closed set is δ-closed in (X, τ) if and only if $\Delta^* \tau^{\#} = \tau_{\delta}$.

The following fascinating characterizations of Δ^*-closed sets are derived on different topological spaces.

1. In a **semi regular space** (X, τ), a subset A is Δ^*-closed if and only if A is δg^*-closed in (X, τ).
2. In an **almost weakly Hausdorff space** (X, τ), the g-closed subsets of (X, τ_s) are Δ^*-closed in (X, τ).
3. For a compact subset of a **R_1-topological space** (X, τ), A is Δ^*-closed if and only if A is g^*-closed in (X, τ).
4. In a **Hausdorff space** (X, τ), a finite set A is g^*-closed if and only if A is Δ^*-closed.
5. Every subset of (X, τ) is Δ^*-closed if and only if the semi regularization (X, τ_s) is a **$T_{1/2}$-Partition space** .

Chapter 3 is devoted to the study of separation axioms. Four new spaces are introduced and their properties are established as applications of Δ^*-closed sets.

The following definitions are introduced in this chapter.

A space (X, τ) is said to be a

- **$_{\Delta^*}T_{\delta}$ -space** if every Δ^*-closed set is δ-closed in (X, τ).
- **$_{\Delta^*}T_{\delta g^*}$ -space** if every Δ^*-closed set is δg^*-closed in (X, τ).
- **$_{g\delta}T_{\Delta^*}$ -space** if every gδ-closed set is Δ^*-closed in (X, τ).
- **$_{\delta g^{\ddagger}}T_{\Delta^*}$ -space** if every $\delta g^{\ddagger}$- set is Δ^*-closed in (X, τ).

The interesting properties and results proved in this chapter are listed below.

- All the four spaces are independent of T_b, T_c and T_d-spaces.
- If (X, τ) is a $_{\Delta^*}T_{\delta}$-space then $\Delta^*cl(B) = \delta cl(B)$ for each subset B of X.
- For a topological space (X, τ), the following statements are true.

  i) (X, τ) is a $_{\Delta^*}T_{\delta}$-space.

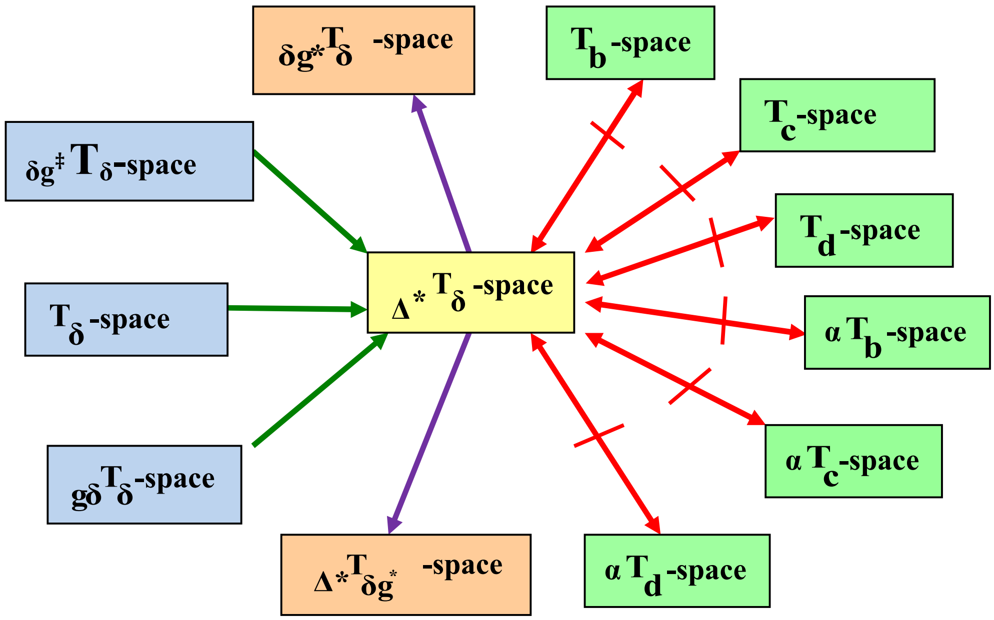
  ii) $\tau_{\delta} = \Delta^*\tau^{\#}$ holds.

  iii) Every singleton {x} is either δg-closed or δ-open.

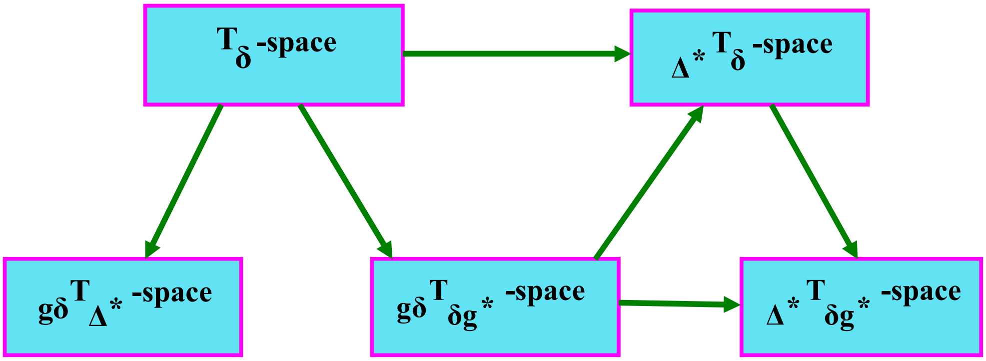
  iv) Every singleton {x} is either δg-closed or regular open.

- If (X, τ) is a $_{\Delta^*}T_{\delta}$-space then for every subset A of (X, τ), $\Delta^*cl(A)$ is δ-closed in (X, τ).
- If (X, τ) is a $_{\delta g}T_{\delta g^*}$ -space and a $_{\Delta^*}T_{\delta}$-space then it is a $T_{3/4}$-space.
- If (X, τ) is a $_{\Delta^*}T_{\delta g^*}$ -space then $\Delta^*cl(A) = \delta g^*cl(A)$ for each subset A of X.

- The association of ${}_{\Delta^*}T_\delta$-space with various existing spaces is portrayed by the following diagram.



- If (X, τ) is a ${}_{\Delta^*}T_{\delta g^*}$-space and a $T_{3/4}$-space then it is a ${}_{\Delta^*}T_\delta$-space.
- If (X, τ) is a ${}_{g\delta}T_{\Delta^*}$-space and a ${}_{\Delta^*}T_\delta$-space then it is a T_δ-space.
- If (X, τ) is a ${}_{g\delta}T_{\Delta^*}$-space and a ${}_{\Delta^*}T_{\delta g^*}$-space then it is a ${}_{g\delta}T_{\delta g^*}$-space.
- If (X, τ) is a ${}_{g\delta}T_{\delta g^*}$-space then it is a ${}_{\Delta^*}T_{\delta g^*}$-space.
- The association of ${}_{\Delta^*}T_\delta$-space, ${}_{\Delta^*}T_{\delta g^*}$-space, ${}_{g\delta}T_{\Delta^*}$-space and ${}_{g\delta}T_{\delta g^*}$-space with T_δ-space is depicted by the following diagram.



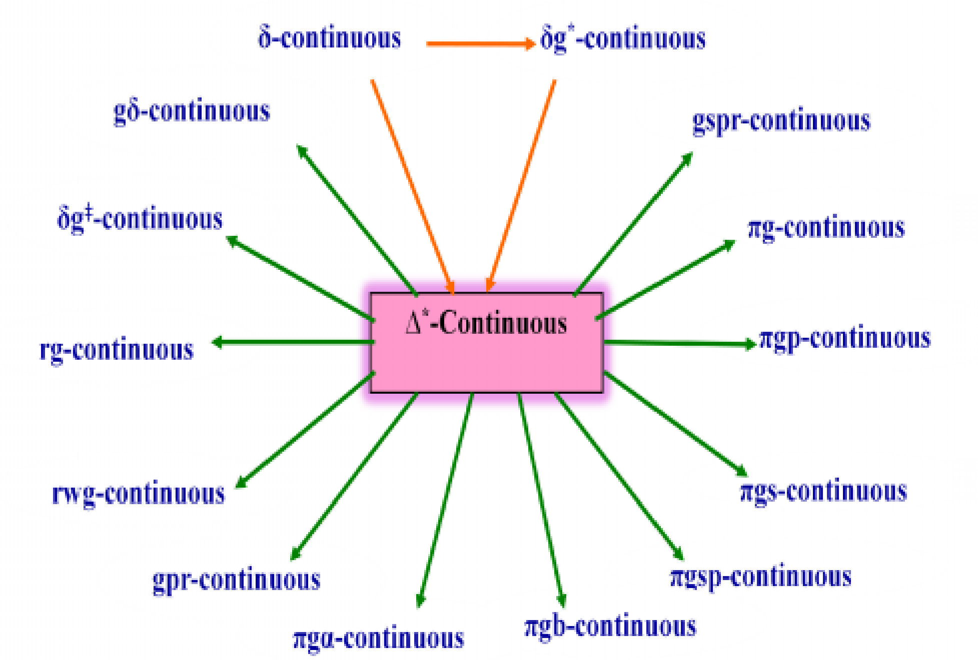
In chapter 4, the concept of continuity using Δ^*-closed sets is established and various types of continuous functions namely, quasi Δ^*-continuous functions, perfectly Δ^*-continuous maps, totally Δ^*-continuous maps, strongly totally Δ^*-continuous maps and contra Δ^*-continuous maps are defined.

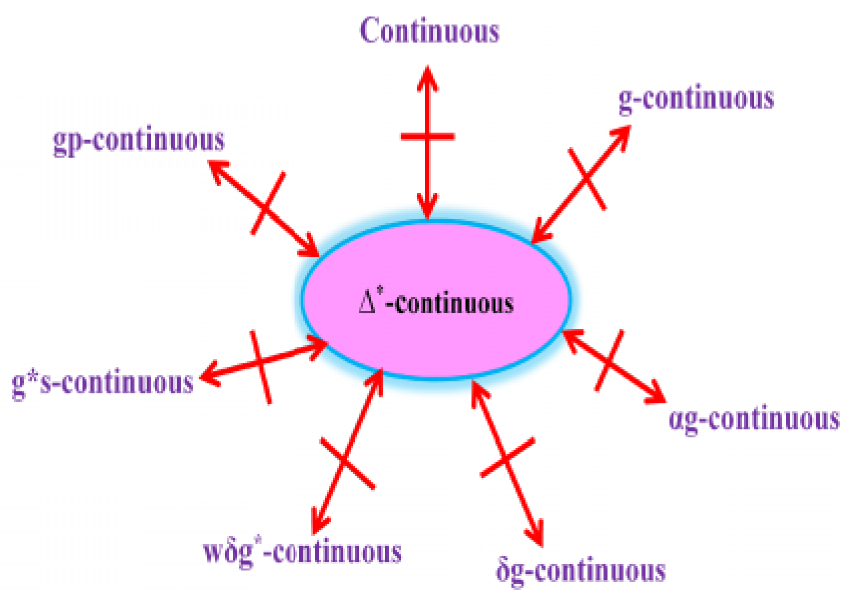
The following definitions are introduced in this chapter.

- A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be Δ^***-continuous** if the inverse image of every closed set in (Y, σ) is Δ^*-closed in (X, τ).
- A map $f : (X, \tau) \to (Y, \sigma)$ is called **quasi** Δ^***-continuous** if the inverse image of every Δ^*-open set in (Y, σ) is open in (X, τ).
- A map $f : (X, \tau) \to (Y, \sigma)$ is called **perfectly** Δ^***-continuous** if the inverse image of every Δ^*-open set in (Y, σ) is clopen set in (X, τ).
- A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be **totally** Δ^***-continuous** if for every open set V of (Y, σ), the inverse image $f^{-1}(V)$ is both Δ^*-open set and Δ^*-closed set in (X, τ). i.e., $f^{-1}(V)$ is a Δ^*-clopen set in (X, τ).
- A map $f : (X, \tau) \to (Y, \sigma)$ is said to be **strongly totally** Δ^***-continuous** if the inverse image of every subset of (Y, σ) is a Δ^*-clopen set in (X, τ).
- A map $f : (X, \tau) \to (Y, \sigma)$ is said to be **contra** Δ^***-continuous** if the inverse image of every closed set of (Y, σ) is Δ^*-open in (X, τ).

The following important properties are proved in this chapter.

- A mapping f : (X, τ) → (Y, σ) is Δ^*-continuous if and only if the inverse image of every open set in (Y, σ) is Δ^*-open in (X, τ).
- If f : (X, τ) → (Y, σ) be a Δ^*-continuous map then for every subset A of (X, τ), f(Δ^*cl(A)) □ cl(f(A)).
- If f : (X, τ) → (Y, σ) be a δ-continuous (or δg^*-continuous) function then for every subset A of (X, τ), f(Δ^*cl(A)) □ cl(f(A)).
- Let f : (X, τ) → (Y, σ) be a map. If for each point x □ X and each open set V in (Y, σ) containing f(x), there exists a Δ^*-open set U in (X, τ) containing x such that f(U) □ V then for every subset A of (X, τ), f(Δ^*cl(A)) □ cl(f(A)).
- The dependency of Δ^*-continuous maps with other existing continuous function is picturized as follows.





The following important properties using specific spaces are also analysed in this chapter.

- If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a Δ^*-continuous map and (X, τ) is a ${}_{\Delta^*}T_{\delta}$-space then f is a δ-continuous function.
- If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a Δ^*-continuous map and (X, τ) is a ${}_{\Delta^*}T_{\delta g^*}$-space then f is δg^*-continuous.
- If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a gδ-continuous function and (X, τ) is a $g\delta T_{\Delta^*}$-space then f is a Δ^*-continuous function.
- If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a $\delta g^{\ddagger}$-continuous function and (X, τ) is a $\delta g^{\ddagger}T\Delta^*$-space then f is a Δ^*-continuous function.
- If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a Δ^*-continuous function and (X, τ) is a ${}_{\Delta^*}T_{\delta}$-space then f is δg^*-continuous.

- If f : (X, τ) → (Y, σ) is a gδ-continuous function and (X, τ) is a T_δ-space then f is Δ^*-continuous.
- If f : (X, τ) → (Y, σ) is Δ^*-continuous where (X, τ) is a $_{\Delta^*}T_{\delta g^*}$ -space as well as $T_{3/4}$-space then f is δ-continuous.
- If f : (X, τ) → (Y, σ) is gδ-continuous function where (X, τ) is a $g\delta T_{\Delta^*}$ -space as well as $_{\Delta^*}T_{\delta g^*}$ -space then f is δg^*-continuous.
- If f : (X, τ) → (Y, σ) is totally continuous and (Y, σ) is a $_{\Delta^*}T_\delta$ - space then it is quasi Δ^*-continuous but not conversely. This result does not hold good if (Y, σ) is not a $_{\Delta^*}T_\delta$ -space.

In addition to the above results, the following imperative preservation of properties under the composition of mappings are established in this chapter.

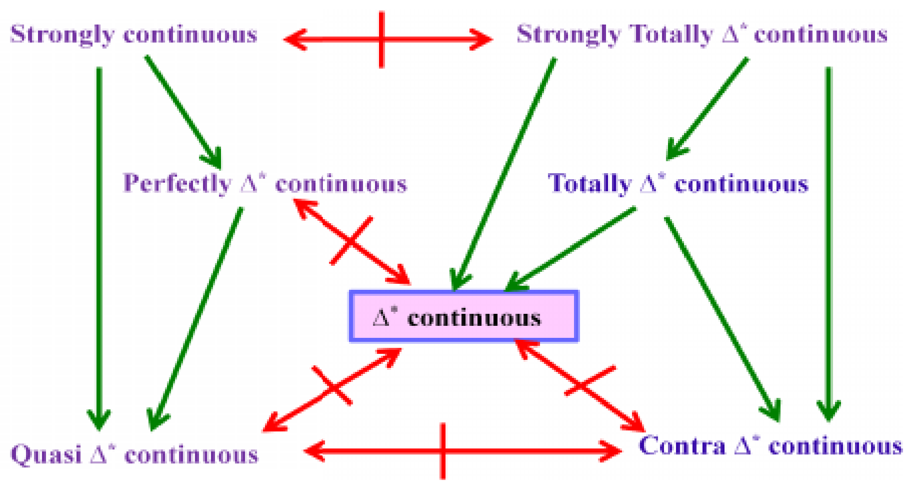
- The composition of two Δ^*-continuous functions need not be Δ^*-continuous which is proved by counter examples. But slightly changing the conditions for the function in four different ways the composition is preserved.
    - ♣ If f : (X, τ) → (Y, σ) is Δ^*-continuous and g : (Y, σ) → (Z, η) is continuous then (g ∘ f) : (X, τ) → (Z, η) is Δ^*-continuous.
    - ♣ If f : (X, τ) → (Y, σ) is Δ^*-continuous and g : (Y, σ) → (Z, η) is δ-continuous then (g ∘ f) : (X, τ) → (Z, η) is Δ^*-continuous.
    - ♣ If f : (X, τ) → (Y, σ) is Δ^*-continuous in which (Y, σ) is a T_δ -space and g : (Y, σ) → (Z, η) is gδ-continuous then (g ∘ f) : (X, τ) → (Z, η) is Δ^*-continuous.
    - ♣ If f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) are both Δ^*-continuous and (Y, σ) is a T_δ -space then (g ∘ f) : (X, τ) → (Z, η) is Δ^*-continuous.

♣ Let f : (X, τ) → (Y, σ) and g : (Y, σ) ⟶ (Z, η) be any two maps. Then their composition map (g ∘ f) : (X, τ) → (Z, η) is

a) Δ^*-continuous if g is strongly continuous and f is Δ^*-continuous.

b) Perfectly Δ^*-continuous if g is perfectly Δ^*-continuous and f is continuous.

➢ The association of various Δ^*-continuous continuous functions is depicted by the following diagram.



In Chapter 5, a new class of irresolute maps called Δ^*-irresolute and contra Δ^*-irresolute maps are introduced and their properties are obtained.

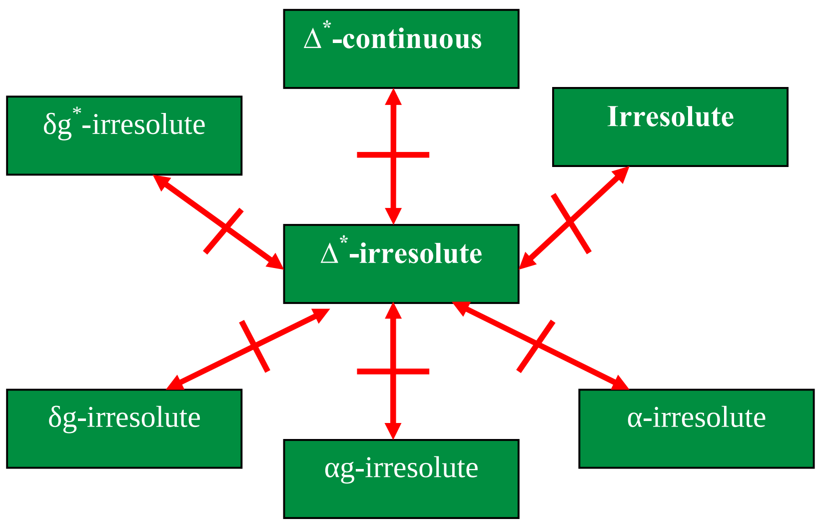
The following definitions are introduced in this chapter.

➢ A map f : (X, τ) → (Y, σ) is called **Δ^*-irresolute** if $f^{-1}(V)$ is Δ^*-open set in (X, τ) for every Δ^*-open set V in (Y, σ).

➢ A map f : (X, τ) → (Y, σ) is said to be **Contra Δ^*-irresolute** if $f^{-1}(V)$ is Δ^*-open in (X, τ) for every Δ^*-closed set V in (Y, σ).

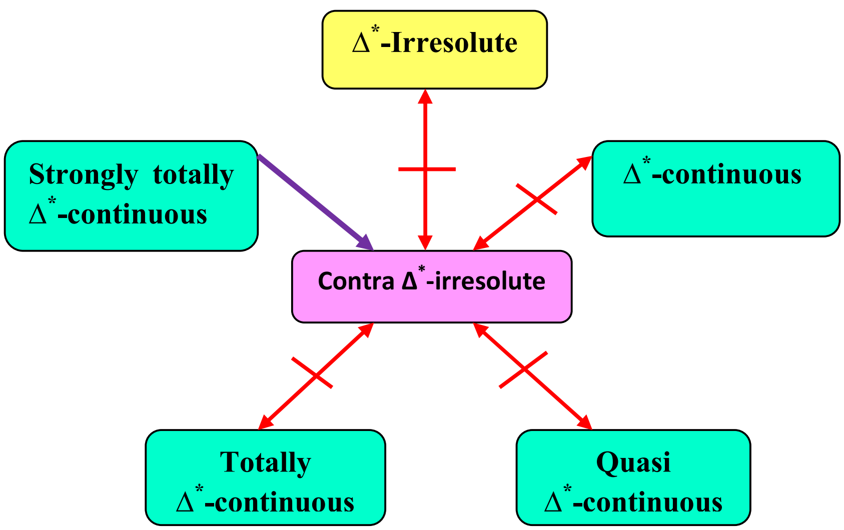
The following properties of Δ^*-irresolute maps and contra Δ^*-irresolute maps are proved in this chapter.

- The independency of Δ^*-irresolute maps with other existing irresolute maps and Δ^*-continuous maps is portrayed by the following diagram.



- A map f : (X, τ) → (Y, σ) is Δ^*-irresolute if and only if for every Δ^*-closed set V in (Y, σ), $f^{-1}(V)$ is a Δ^*- closed set in (X, τ).
- If a map f : (X, τ) → (Y, σ) is Δ^*-irresolute then for every subset A of (X, τ), f [Δ^*cl(A)] □ δcl[f(A)].
- If a map f : (X, τ) → (Y, σ) is Δ^*- irresolute then for any subset B □ Y, Δ^*cl[(f^{-1}(B)] □ f^{-1}[δcl(B)].
- If f : (X, τ) → (Y, σ) is a δ-open surjective and Δ^*-irresolute map where (X, τ) is a $_{\Delta^*}T_{\delta}$-space then (Y, σ) is a $_{\Delta^*}T_{\delta}$-space.
- If f : (X, τ) → (Y, σ) is a δg*-open surjective and Δ^*-irresolute map where (X, τ) is a $_{\Delta^*}T_{\delta g^*}$-space then (Y, σ) is a $_{\Delta^*}T_{\delta g^*}$-space.
- If f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) are two Δ^*-irresolute maps then their composition (g ∘ f) : (X, τ) → (Z, η) is a Δ^*-irresolute map.

- If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a Δ^*-irresolute map and $g : (Y, \sigma) \longrightarrow (Z, \eta)$ is a Δ^*-continuous map then their composition mapping $(g \circ f) : (X, \tau) \rightarrow (Z,\eta)$ is a Δ^*-continuous map.
- If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a Δ^*-irresolute map and $g :(Y, \sigma) \rightarrow (Z, \eta)$ is a contra Δ^*-continuous map then their composition mapping $(g \circ f) : (X, \tau) \rightarrow (Z, \eta)$ is a contra Δ^*-continuous function.
- If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a map then the following statements are equivalent.
  a) f is a contra Δ^*-irresolute map.
  b) The inverse image of every Δ^*-open set in (Y, σ) is Δ^*-closed in (X, τ).
- The association of contra Δ^*-irresolute maps with other maps is depicted by the following diagram.



In Chapter 6, the extensions of homeomorphisms via Δ^*-open sets namely, Δ^*-homeomorphisms and $\Delta^*\mathcal{C}$-homeomorphisms are introduced and their significance in topological spaces are studied.

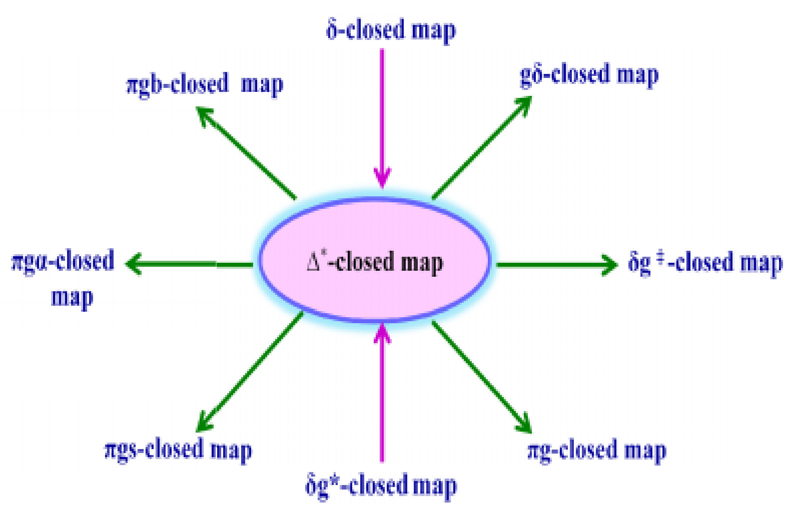
The following definitions are introduced in this chapter.

- A map f : (X, τ) → (Y, σ) is called a Δ^***-closed map** if the image of each closed set in (X, τ) is a Δ^*-closed set in (Y, σ).
- A map f : (X, τ) → (Y, σ) is called a Δ^***-open map** if the image of each open set in (X, τ) is a Δ^*-open set in (Y, σ).
- A bijection map f : (X, τ) → (Y, σ) is called a Δ^***-homeomorphism** if f is both Δ^*-continuous and Δ^*-open map.
- A bijection map f : (X, τ) → (Y, σ) is said to be a $\Delta^*\mathcal{C}$**-homeomorphism** if both f and f^{-1} are Δ^*-irresolute maps.

  The family of all $\Delta^*\mathcal{C}$-homeomorphism of a topological space (X, τ) onto itself is denoted by $\Delta^*\mathcal{C}\hbar$(X, τ).

The following imperative properties and results of Δ^*-closed maps are proved in this chapter.

- The dependency of Δ^*-closed maps with other closed maps is picturized below.



- The Δ^*-closed map is independent of Δ^*-continuous, g-closed map, δg-closed map, $\delta\hat{g}$-closed map, $\alpha\hat{g}$-closed map and αg-closed map.

- A map f : (X, τ) → (Y, σ) is Δ^*-closed if and only if for each subset U of (Y, σ) and for each open set V of (X, τ) containing f^{-1}(U) there exists a Δ^*-open set G of (Y, σ) such that U □ G and f^{-1}(G) □ V.
- A bijection mapping f : (X, τ) → (Y, σ) is a Δ^*-closed map if and only if f(U) is a Δ^*-open set in (Y, σ) for every open set U in (X, τ). In this result the bijection condition on f is a necessary condition.
- If f : (X, τ) → (Y, σ) is δg-irresolute and Δ^*-closed map then f(A) is a Δ^*-closed subset of (Y, σ) where A is a Δ^*-closed subset of (X, τ).
- If f : (X, τ) → (Y, σ) is a Δ^*-closed map and A is a closed subset of (X, τ) then $f/A : (A, \tau/A) \rightarrow (Y, \sigma)$ is Δ^*-closed.

The following significant results on composition of Δ^*-closed maps are discussed in this chapter .

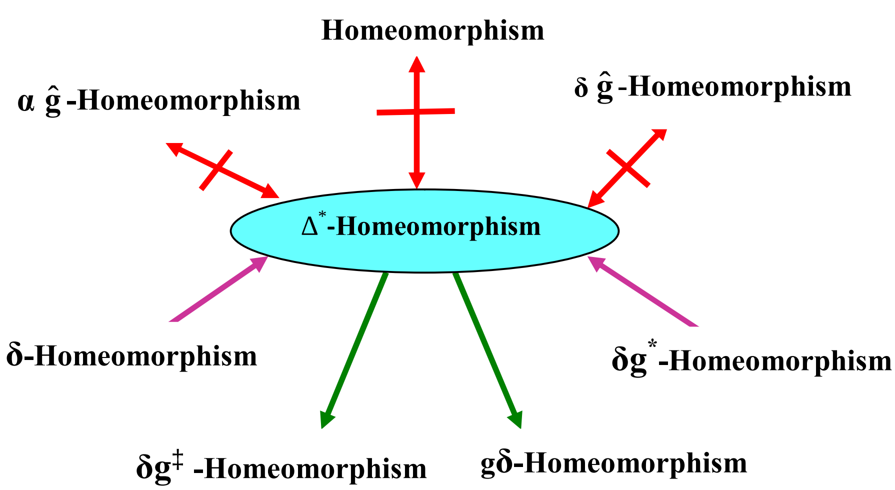
- The composition of two Δ^*-closed maps need not be a Δ^*-closed map.
- If f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) are Δ^*-closed maps where (Y, σ) is a ${}_{\Delta^*}T_{\delta}$-space then their composition (g ∘ f) : (X, τ) → (Z, η) is a Δ^*-closed map.
- If f : (X, τ) → (Y, σ) is a Δ^*-closed map and g : (Y, σ) → (Z, η) is a closed map then their composition need not be a Δ^*-closed map.
- If f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) are Δ^*-closed maps where (Y, σ) is a ${}_{\Delta^*}T_{\delta g^*}$-space as well as ${}_{\delta g^*}T_{\delta}$-space then their composition (g ∘ f) : (X, τ) → (Z, η) is a Δ^*-closed map.
- If f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) are any two maps then the following results are true.
  - If (g ∘ f) : (X, τ) → (Z, η) is a Δ^*-closed map and g is a Δ^*-irresolute injective map then f is a Δ^*-closed map.
  - If (g ∘ f) : (X, τ) → (Z, η) is a Δ^*-irresolute map and g is a Δ^*-closed injective map then f is a Δ^*-continuous map.

- ❖ If $(g \circ f) : (X, \tau) \to (Z, \eta)$ is a Δ^*-closed map and f is a continuous map then g is a Δ^*-closed map.

- ➢ If $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ are any two maps such that their composition map $(g \circ f) : (X, \tau) \to (Z, \eta)$ is a Δ^*-closed map then the following statements are true.
  - ❖ If f is a surjective continuous map then g is a Δ^*-closed map.
  - ❖ If f is a surjective g-continuous map and (X, τ) is a $T_{1/2}$-space then g is a Δ^*-closed map.
  - ❖ If f is a quasi Δ^*-continuous and injective map then f is a closed map.

The following significant results on Δ^*-homeomorphism are proved in this chapter.

- ➢ The association of Δ^*-homeomorphism with various existing homeomorphism are depicted by the following diagram.



- ➢ For the bijective Δ^*-continuous map $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent.
  - ❖ f is a Δ^*-open map.
  - ❖ f is a Δ^*-homeomorphism.
  - ❖ f is a Δ^*-closed map.

- Every Δ^*-homeomorphism from a $_{\Delta^*}T_\delta$-space into another $_{\Delta^*}T_\delta$-space is a homeomorphism.
- In general if f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) are Δ^*-homeomorphisms then the composition map (g ∘ f) : (X, τ) → (Z, η) is not a Δ^*-homeomorphism. But by imposing a condition on (Y, σ) that if it is a $_{\Delta^*}T_\delta$-space then it is proved the composition map (g ∘ f) : (X, τ) → (Z, η) is a Δ^*-homeomorphism.
- $\Delta^*\mathcal{C}$-homeomorphism and $\delta g^*\mathcal{C}$-homeomorphism are independent.

In addition to the above results the following group properties of $\Delta^*\mathcal{C}$-homeomorphisms are analyzed in this chapter.

- The composition of two $\Delta^*\mathcal{C}$-homeomorphisms is a $\Delta^*\mathcal{C}$-homeomorphism.
- The set $\Delta^*\mathcal{C}$-$\mathcal{h}$(X,τ) is a **group under the composition of maps**.
- If f : (X, τ) → (Y, σ) is a $\Delta^*\mathcal{C}$-homeomorphism then f induces an isomorphism from the group $\Delta^*\mathcal{C}$-$\mathcal{h}$(X, τ) onto the group $\Delta^*\mathcal{C}$-$\mathcal{h}$(Y, σ).
- The $\Delta^*\mathcal{C}$-homeomorphism is **an equivalence relation** in the collection of all topological spaces.

In Chapter 7, three types of locally closed sets namely, Δ^*lc-sets, Δ^*lc*-sets and Δ^*lc**-sets are introduced and their properties are studied. Also the nature of these sets in three different spaces called Δ^*-door space, Δ^*-submaximal space and Δ^{**}-submaximal space are discussed. Furthermore Δ^*-locally continuous and Δ^*-locally irresolute maps are defined and their properties are analysed.

The following important definitions are defined in this chapter.

- ❖ Let A be a subset of (X, τ). Then A is called a
  - Δ^*-**locally closed set**, i.e., Δ^***lc-set** if there exists a Δ^*-open set U and a Δ^*-closed set F of (X, τ) such that A = U ∩ F.

- Δ^***lc*-set** if there exists a Δ^*-open set U and a δ-closed set F of (X, τ) such that A = U ∩ F.
- Δ^***lc**-set** if there exist a δ-open set U and a Δ^*-closed set F of (X, τ) such that A = U ∩ F.

❖ The collection of all Δ^*lc-sets (respectively Δ^*lc*-sets, Δ^*lc**-sets) of (X, τ) is denoted by Δ^*LC(X, τ) (respectively Δ^*LC*(X, τ), Δ^*LC**(X, τ)).

❖ A subset A of (X, τ) is called a Δ^***-dense set** if Δ^*cl(A) = X.

❖ A topological space (X, τ) is called a Δ^***-submaximal** (resp.,Δ^{**}-submaximal) space if every Δ^*-dense (resp., δ-dense) subset is Δ^*-open in (X, τ).

❖ A map f : (X, τ) → (Y, σ) is called

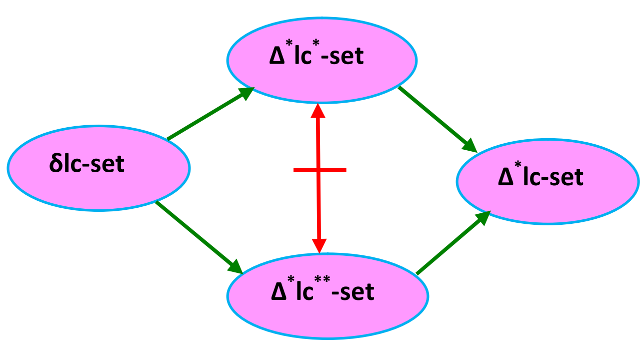
- Δ^***LC-continuous** if $f^{-1}(V) \in \Delta^*LC(X, \tau)$ for each $V \in \sigma$.
- Δ^***LC*-continuous** if $f^{-1}(V) \in \Delta^*LC^*(X, \tau)$ for each $V \in \sigma$.
- Δ^***LC**-continuous** if $f^{-1}(V) \in \Delta^*LC^{**}(X, \tau)$ for each $V \in \sigma$.

❖ Let f : (X, τ) → (Y, σ) be a map. Then f is called

- Δ^***LC-irresolute** if $f^{-1}(V) \in \Delta^*LC(X, \tau)$ for each $V \in \Delta^*LC(Y, \sigma)$.
- Δ^***LC*-irresolute** if $f^{-1}(V) \in \Delta^*LC^*(X, \tau)$ for each $V \in \Delta^*LC^*(Y, \sigma)$.
- Δ^***LC**-irresolute** if $f^{-1}(V) \in \Delta^*LC^{**}(X, \tau)$ for each $V \in \Delta^*LC^{**}(Y, \sigma)$.

The following significant properties are proved in this chapter.

❖ The relations between Δ^*lc-sets, Δ^*lc*- sets, Δ^*lc**- sets and δlc-sets are represented by the following diagram.



- If (X, τ) is a Δ^*-door space then $\Delta^*LC(X, \tau) = P(X)$.

- If (X,τ) is a $T_{\Delta^*\delta}$-space then the following results hold:
  - $\Delta^*LC(X, \tau) = \delta LC(X, \tau) = \Delta^*LC^*(X, \tau) = \Delta^*LC^{**}(X, \tau)$
  - $\Delta^*LC(X, \tau) \subseteq GLC(X, \tau)$
  - $\Delta^*LC(X, \tau) \subseteq GLSC(X, \tau)$

- For a subset A of (X, τ), $A \in \Delta^*LC(X, \tau) \Longrightarrow A = U \cap \Delta^*cl(A)$ for some Δ^*-open set U in (X, τ).

- For a subset A of (X, τ), $A \in \Delta^*LC^{**}(X, \tau) \Longrightarrow A = U \cap \Delta^*cl(A)$ for some δ-open set U in (X, τ).

- In a topological space (X, τ), every Δ^*-dense set is δ-dense set.

- In a topological space (X, τ), every πg-dense set is Δ^*-dense set.

- The relations between Δ^*-submaximal space, Δ^{**}-submaximal space and πg-submaximal space are derived as follows.

Δ^{**}-submaximal $\longrightarrow$ Δ^*-submaximal $\longrightarrow$ πg-submaximal

The following preservation of properties under Δ^*-locally closed sets are also proved in this chapter.

- The following results are true for any two subsets A and B of (X, τ).

- If A,B $\in \Delta^{*}LC^{*}(X, \tau)$, then $A \cap B \in \Delta^{*}LC^{*}(X, \tau)$.
- If A $\in \Delta^{*}LC(X, \tau)$ and B is Δ^{*}-open then $A \cap B \in \Delta^{*}LC(X, \tau)$.
- If A $\in \Delta^{*}LC^{*}(X, \tau)$ and B is Δ^{*}-open then $A \cap B \in \Delta^{*}LC^{*}(X, \tau)$.
- If A $\in \Delta^{*}LC^{**}(X, \tau)$ and B is Δ^{*}-open then $A \cap B \in \Delta^{*}LC(X, \tau)$.

The following various significant properties and outcomes on Δ^{*}-locally continuous as well as Δ^{*}-locally irresolute maps are proved in this chapter.

- If $f : (X, \tau) \rightarrow (Y, \sigma)$ is any map then the following results are true.
    - If f is δLC-continuous then it is Δ^{*}LC-continuous, $\Delta^{*}LC^{*}$-continuous and $\Delta^{*}LC^{**}$continuous. The reverse implication is true by imposing a condition on (X, τ) that it is a $_{\Delta^{*}}T_{\delta}$-space.
    - If f is $\Delta^{*}LC^{*}$-continuous or $\Delta^{*}LC^{**}$-continuous then it is Δ^{*}LC-continuous.
    - If f is a Δ^{*}-irresolute map then f is Δ^{*}LC-irresolute.
    - The Δ^{*}LC-irresolute maps and Δ^{*}LC-continuous maps are independent.
    - Any map defined on a δ-door space is Δ^{*}LC-irresolute.

The following results are analyzed in $_{\Delta^{*}}T_{\delta}$-space.

- Let f : (X, τ) → (Y, σ) be any map in which (Y, σ) is a $_{\Delta^*}T_{\delta}$-space then the following results are proved.
  - If f is Δ^*LC-continuous as well as contra δ-continuous then it is a Δ^*LC-irresolute map.
  - If f is Δ^*LC-continuous as well as contra Δ^*-irresolute then it is a Δ^*LC-irresolute map.
  - If f : (X, τ) → (Y, σ) is Δ^*LC^*-continuous as well as contra δ-continuous then it is a Δ^*LC^*-irresolute map.
- Let f : (X, τ) → (Y, σ) be any map in which (X, τ) is a $_{\Delta^*}T_{\delta}$-space as well as Δ^*-submaximal space then f is a Δ^*LC-irresolute map.

Results proved on composition of Δ^*-locally continuous and Δ^*-locally irresolute maps are stated below.

- Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) be any two maps. Then
  - (g ∘ f) : (X, τ) → (Z, η) is Δ^*LC-irresolute (resp. Δ^*LC^*-irresolute, Δ^*LC^{**}-irresolute) if f is Δ^*LC-irresolute (resp. Δ^*LC^*-irresolute, Δ^*LC^{**}-irresolute) and g is also Δ^*LC-irresolute.(resp. Δ^*LC^*-irresolute, Δ^*LC^{**}-irresolute).
  - (g ∘ f) : (X, τ) → (Z, η) is Δ^*LC- continuous if f is Δ^*LC-irresolute and g is Δ^*LC-continuous.
- The composition of two Δ^*LC- continuous maps need not be a Δ^*LC- continuous.
- For any two maps f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η), the following statements are true.
  - (g ∘ f) : (X, τ) → (Z, η) is Δ^*LC- continuous if f is Δ^*LC-irresolute and g is δLC-continuous (resp., Δ^*LC^*-continuous, Δ^*LC^{**}-continuous).
  - (g ∘ f) : (X, τ) → (Z, η) is Δ^*LC- continuous (resp. Δ^*LC^*-continuous) if f is Δ^*LC-irresolute (resp. Δ^*LC^*-irresolute) and g is Δ^*-continuous.

- ❖ $(g \circ f) : (X, \tau) \rightarrow (Z, \eta)$ is Δ^*LC- continuous (resp., Δ^*LC^*-continuous, Δ^*LC^{**}-continuous) if f is Δ^*LC- continuous and g is δ-continuous.
- ➢ Let $X = A \cup B$ where A and B are Δ^*-closed sets of (X, τ) and $f : (X, \tau_A) \rightarrow (Y, \sigma)$ and $h : (B, \tau_B) \rightarrow (Y, \sigma)$ are compatible functions. If f and h are Δ^*LC^{**}-continuous (resp., Δ^*LC^{**}-irresolute) then $(f \nabla h) : X \rightarrow Y$ is Δ^*LC^{**}-continuous (resp., Δ^*LC^{**}-irresolute).
- ➢ The **pasting lemma** holds good for Δ^*LC^{**}-continuous (resp., Δ^*LC^{**}-irresolute) functions.

In Chapter 8, a new class of closed sets called (i, j)-Δ^*-closed sets are established and their association between various existing generalized notions of (i, j)-closed sets are studied in bitopological spaces. As an application of (i, j)-Δ^*-closed sets, four new spaces namely, (i, j)-${}_{\Delta^*}T_{\delta}$-space, (i, j)-${}_{\Delta^*}T_{\delta g^*}$-space, (i, j)-${}_{g\delta}T_{\Delta^*}$-space and (i, j)-${}_{\delta g^{\ddagger}}T_{\Delta^*}$-spaces are introduced and their properties are also derived. Further (i, j)-Δ^*-σ_k-continuous functions, (i, j)-Δ^*-irresolute maps and (i, j)-Δ^*-locally closed sets are defined and their properties are analyzed in this chapter.

The following definitions are introduced in this chapter.

- ❖ A subset A of a bitopological space (X, τ_i, τ_j) is called a **(i, j)-Δ^*-closed set** if τ_j- δcl(A)   U whenever A □ U, U is τ_i-δg-open in (X, τ_i) where i = 1, 2 and $i \neq j$.

  The family of all (i, j)-Δ^*-closed sets in (X, τ_i, τ_j) is denoted by $D_{\Delta^*}(i, j)$.
- ❖ Let B □ Y □ X. A subset B of Y is said to be **(i, j)-Δ^*-closed relative to Y** if B is (i, j)-Δ^*-closed in the subspace Y.

- For each subset A of a topological space (X, τ_i, τ_j), the **(i, j)-Δ^*-closure of A** is denoted by (i, j)-Δ^*cl(A) and is defined as follows.

  $(i, j)\text{-}\Delta^*cl(A) = \bigcap\{F \subseteq X / A \subseteq F \text{ and } F \text{ is } (i, j)\text{-}\Delta^*\text{closed}\}$.

- Two subsets A and B are said to be **separated (i, j) -Δ^*-open sets** if the following conditions hold good.

  i) $[\tau_i\text{-}\delta cl(A)] \cap B = \phi$ ii) $A \cap [\tau_i\text{-}\delta cl(B)] = \phi$

  iii) $[\tau_j\text{-}\delta cl(A)] \cap B = \phi$ iv) $A \cap [\tau_j\text{-}\delta cl(B)] = \phi$

- A bitopological space (X, τ_i, τ_j) is said to be a
  - **(i, j)-${}_{\Delta^*}T_\delta$-space** if every (i, j)-Δ^*-closed set is a τ_j-δ-closed set.
  - **(i, j)-${}_{\Delta^*}T_{\delta g^*}$-space** if every (i, j)-Δ^*-closed set is a (i, j)-δg^*-closed set.
  - **(i, j)-${}_{g\delta}T_{\Delta^*}$-space** if every (i, j)-gδ-closed set is a (i, j)-Δ^*-closed set.
  - **(i, j)-$\delta g^\ddagger T_{\Delta^*}$-space** if every (i, j)-$\delta g^\ddagger$-closed set is a (i, j)-Δ^*-closed set.

- A map $f : (X, \tau_i, \tau_j) \longrightarrow (Y, \sigma_i, \sigma_j)$ is called a **(i, j)-Δ^*-σ_k-continuous** if the inverse image of every σ_k-closed set in Y is a (i, j)-Δ^*-closed set in X for i, j, k =1, 2 and $i \neq j$.

- A map $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$is called **(i, j)-$\Delta^*$-irresolute** if the inverse image of every (i, j)-Δ^*-closed set in (Y, σ_i, σ_j) is (i, j)-Δ^*-closed in (X, τ_i, τ_j).

- A subset A of a bitopological space (X, τ_i, τ_j) is said to be a
  - **(i, j)-Δ^*-locally closed set** if A = G∩F where G is τ_i-Δ^*-open and F is τ_j-Δ^*-closed in (X, τ_i, τ_j).
  - **(i, j)-Δ^*lc* set** if A = G∩F where G is τ_i-Δ^*-open and F is τ_j-δ-closed in (X, τ_i, τ_j).

- **(i, j)-Δ^{*}lc** set** if A = G∩F where G is τ_i-δ-open and F is τ_j-Δ^{*}-closed in (X, τ_i, τ_j).

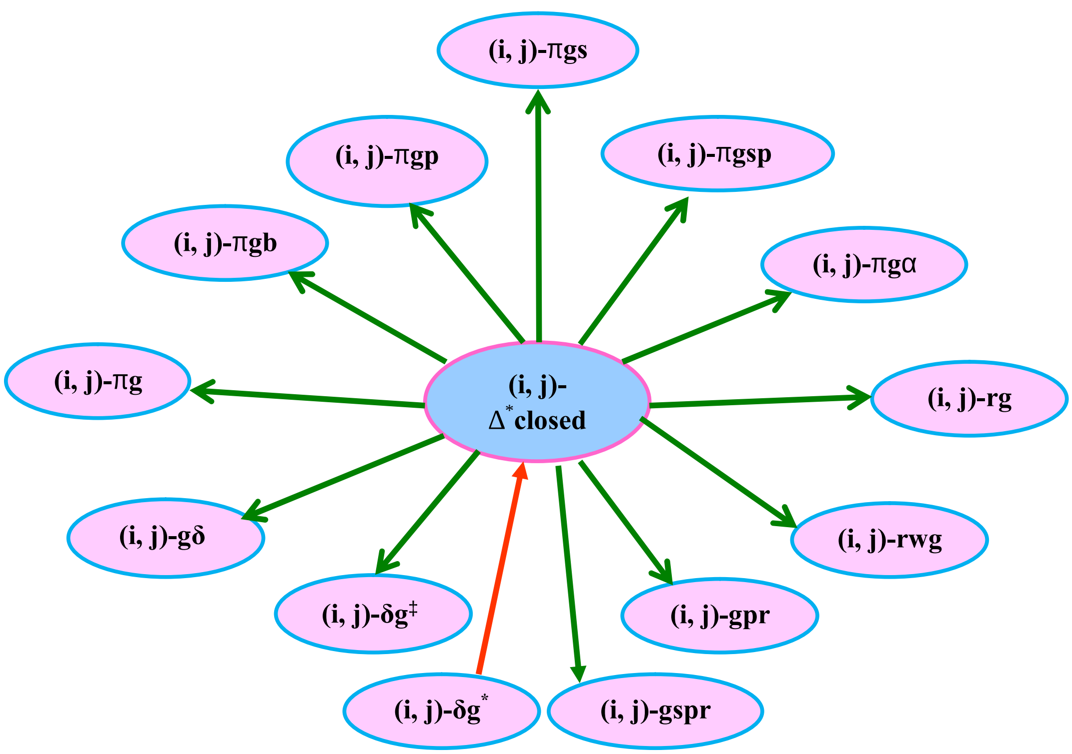
❖ The class of all (i, j)-Δ^{*}lc sets in (X, τ_i, τ_j) is denoted by $(i, j)\text{-}\Delta^{*}LC(X, \tau_i, \tau_j)$.

❖ The class of all (i, j)-Δ^{*}lc* sets in (X, τ_i, τ_j) is denoted by $(i, j)\text{-}\Delta^{*}LC^{*}(X, \tau_i, \tau_j)$.

❖ The class of all (i, j)-Δ^{*}lc** sets in (X, τ_i, τ_j) is denoted by $(i, j)\text{-}\Delta^{*}LC^{**}(X, \tau_i, \tau_j)$.

The following properties and results of (i, j)-Δ^{*}-closed sets are proved in this chapter.

- The dependency of (i, j)-Δ^{*}-closed set with various closed sets is analysed and it is picturized below.



- The (i, j)-Δ^*-closed set is independent of (i, j)-δg, (i, j)-gp, (i, j)-αg and (i, j)-mildly g-closed sets.
- If A is a τ_j-δ-closed subset of (X, τ_i, τ_j) then A is a (i, j)-Δ^*-closed set
- If A is both τ_i-δg-open and (i, j)-Δ^*-closed then A is τ_j-δ-closed.
- If A is both τ_i-δg-open and (i, j)-Δ^*-closed then A is τ_j-closed.
- Let A be τ_i-δg-open and (i, j)-Δ^*-closed in (X, τ_i, τ_j). Suppose that F is τ_j-δ-closed in (X, τ_i, τ_j). Then $(A \cap F)$ is (i, j)-Δ^*-closed in (X, τ_i, τ_j).
- If A is a (i, j)-Δ^*-closed set then $\tau_i \delta cl(X) \cap A \neq \phi$ holds for each x ∈ τ_j-δcl(A).
- An (i, j)-Δ^*-closed set need not be a τ_i-Δ^*-closed set or a τ_j-Δ^*-closed set.
- If A and B are (i, j)-Δ^*-closed sets then (A∪B) is also a (i, j)-Δ^*-closed set.
- The intersection of two (i, j)-Δ^*-closed sets need not be a (i, j)-Δ^*-closed set.
- For each point x of a space (X, τ_i, τ_j), {x} is τ_i-δg-closed or $\{x\}^c$ is (i, j)-Δ^*-closed.
- If A is a (i, j)-Δ^*-closed set in (X, τ_i, τ_j) then τ_j-δcl(A)−A contains no non empty τ_i-δg-closed set.
- If A is a (i, j)-Δ^*-closed set in (X, τ_i, τ_j) then A is τ_j-δ-closed if and only if τ_j-δcl(A)−A is τ_j-δg-closed.
- If A is a (i, j)-Δ^*-closed set in (X, τ_i, τ_j) and A ⊆ B ⊆ τ_j-δcl(A) then B is (i, j)-Δ^*-closed.
- If A is a (i, j)-Δ^*-closed set in (X, τ_i, τ_j) and A ⊆ B ⊆ τ_j-δcl(A) then τ_j-δcl(B)−B contains no non empty τ_i-δg-closed.

- Let B □ A □ X where A is τ_i -δg-open and (i, j)-Δ^*-closed in (X, τ_i, τ_j). Then B is (i, j)-Δ^*-closed relative to A if and only if B is (i, j)-Δ^*-closed relative to (X, τ_i, τ_j).
- In a bitopological space (X, τ_i, τ_j), δGO(X, τ_i)   δC(X, τ_j) if and only if every subset of (X, τ_i, τ_j) is (i, j)-Δ^*-closed.
- If A and B are any two subsets of (X, τ_i, τ_j) then the following results are proved.
  - (i, j)-Δ^*cl(ϕ) = ϕ and (i, j)-Δ^*-cl(X) = X
  - If A   B then (i, j)-Δ^*cl(A)   (i, j)-Δ^*cl(B)
  - A □ (i, j)-Δ^*cl(A) □ τ_j-δcl(A)
  - If A is (i, j)-Δ^*-closed then (i, j)-Δ^*cl(A) = A
  - (i, j)-Δ^*cl(A ∩ B)   (i, j)-Δ^*cl(A) ∩ (i, j)-Δ^*cl(B)
  - (i, j)-Δ^*cl(A ∪ B) = (i, j)-Δ^*cl(A) ∪ (i, j)-Δ^*cl(B)
- Let x and y be any two points of (X, τ_i, τ_j). If (i, j)-Δ^*cl({x}) = (i, j)-Δ^*cl({y}) and δGC(X, τ_j) = δC(X, τ_j) then x = y.

All the above properties and results are analysed under (i, j)-Δ^*-open sets which are the complement of (i, j)-Δ^*-closed sets.

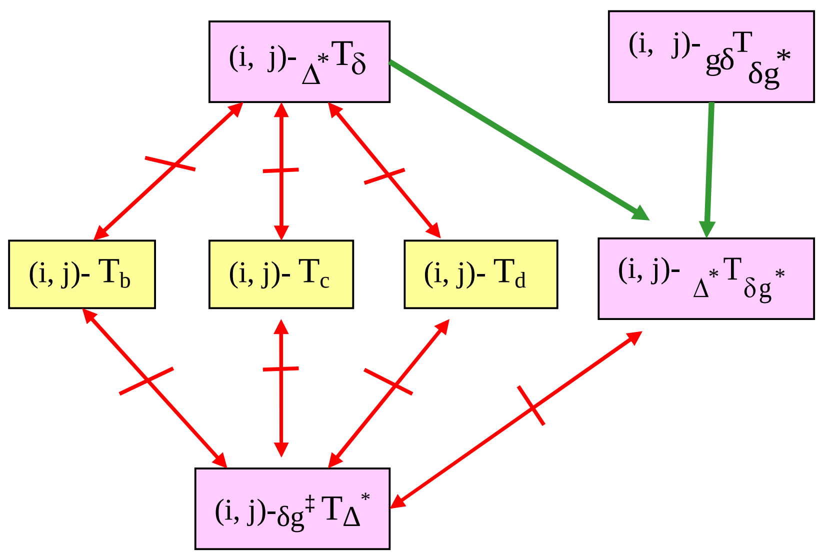
As an application of (i, j)-Δ^*-closed sets, the following properties and outcomes are derived in this chapter.

- Every (i, j)- ${}_{\Delta^*}T_{\delta}$ -space is a ${}_{\Delta^*}T_{\delta g^*}$ -space.
- If (X, τ_i, τ_j) is both (i, j)- ${}_{\Delta^*}T_{\delta g^*}$ -space and (i, j)- ${}_{\delta g^*}T_{\delta}$ -space then it is a (i, j)- ${}_{\Delta^*}T_{\delta}$ -space.
- For a space (X, τ_i , τ_j) the following are equivalent.

a) X is a (i, j)- $_{\Delta^*}T_{\delta}$ - space.

b) Every singleton is either τ_i -δg-closed or τ_j -δ-open for $i \neq j$.

- The interrelations between (i, j)- $_{\Delta^*}T_{\delta}$ -space , (i, j)- $_{\Delta^*}T_{\delta g^*}$ -space, (i, j)- $_{g\delta}T_{\delta g^*}$ -space , (i, j)-$_{\delta g^{\ddagger}}T_{\Delta^*}$-space, (i, j)- $T_{b,}$ T_c and T_d-spaces are portrayed below.



- If (X, τ_i, τ_j) is a (i, j)-$_{g\delta}T_{\Delta^*}$ -space and a (i, j)- $_{\Delta^*}T_{\delta}$ -space then it is a (i, j)- T_{δ}-space.

- If f : (X, τ_i, τ_j) $\longrightarrow$ (Y, σ_i, σ_j) is (i, j)-Δ^*-σ_k-continuous then f is (i, j)-gδ-σ_k-continuous, (i, j)-rg-σ_k-continuous, (i, j)-gpr-σ_k-continuous, (i, j)-rwg-σ_k-continuous, (i, j)-πgp-σ_k-continuous, (i,j)-πgs-σ_k-continuous, and (i, j)-πgsp-σ_k-continuous.

- The (i, j)-Δ^*-σ_k-continuous is independent with (i, j)-δg-σ_k-continuous, (i, j)-αg-σ_k-continuous and (i, j)-gp-σ_k-continuous.

- The composition of two (i, j)-Δ^*-σ_k-continuous maps need not be a (i, j)-Δ^*-σ_k-continuous map which is proved by counter example.

- If f : (X, τ_i, τ_j) $\longrightarrow$ (Y, σ_i, σ_j) is (i, j)-Δ^*-σ_k-continuous then f [(i, j)-Δ^*cl(A)] □ $\sigma_k$cl[f(A)].

- The composition of two (i, j)-Δ^*-irresolute maps need not be a (i, j)-Δ^*-σ_k-continuous map.

- If f : (X, τ_i, τ_j) $\longrightarrow$ (Y, σ_i, σ_j) is (i, j)-Δ^*-irresolute then for every subset V of (X, τ_1, τ_2) such that (i, j)-Δ^*-closed in (Y, σ_i, σ_j), f [(i, j) Δ^*cl(V)] (i, j) Δ^*cl [f(V)].

- If f : (X, τ_i, τ_j) $\longrightarrow$ (Y, σ_i, σ_j) is (i, j)-Δ^*-irresolute then for every (i, j)-Δ^*-closed set U in (Y, σ_i, σ_j), (i, j)Δ^*cl[f^{-1}(U)] f^{-1}[(i, j)Δ^*cl(U)]

- If f : (X, τ_i, τ_j) $\longrightarrow$ (Y, σ_i, σ_j) is bijective, τ_i -δg-open and (i, j)-Δ^*-σ_j continuous, then f is (i, j)-Δ^*-irresolute.

- If f : (X, τ_i, τ_j) $\longrightarrow$ (Y, σ_i, σ_j) is a (i, j)-Δ^*-irresolute and X is a (i, j)-$_{\Delta^*}T_\delta$ -space then f is τ_j -δ-irresolute.

- If f : (X, τ_i, τ_j) $\longrightarrow$ (Y, σ_i, σ_j) is a τ_i -δg-irresolute and τ_j -δ-closed, then for every (i, j)-Δ^*-closed set A of X, f(A) is a (i, j)-Δ^*-closed set of (Y, σ_i, σ_j).

- If f : (X, τ_i, τ_j) $\longrightarrow$ (Y, σ_i, σ_j) and g : (Y, σ_i, σ_j) $\rightarrow$ (Z, η_i, η_j) are two functions, then the following results are true.

- If f is (i, j)-Δ^*-irresolute and g is (i, j)-Δ^*-η_k-continuous, then (g ∘ f) is (i, j)-Δ^*-η_k-continuous.

- If f is (i, j)-лg-irresolute and g is (i, j)-Δ^*-η_k-continuous, then (g ∘ f) is $(i, j)-\pi g-\eta_k$ -continuous.

- If f is (i, j)-gδ-irresolute and g is (i, j)-Δ^*-η_k-continuous, then (g ∘ f) is $(i, j)-g\delta-\eta_k$ -continuous.

The following important properties and results of (i, j)-Δ^*-locally closed sets are proved in this chapter.

- In general every (i, j)-Δ^*-locally closed set in (X, τ_i, τ_j) is not τ_j-δ-closed.

- In any bitopological space (X, τ_i, τ_j) the following results are true.

  a) $A \in (i, j)-\Delta^*LC^*(X, \tau_i, \tau_j) \Rightarrow A \in (i, j)-\Delta^*LC(X, \tau_i, \tau_j)$

  b) $A \in (i, j)-\Delta^*LC^{**}(X, \tau_i, \tau_j) \Rightarrow A \in (i, j)-\Delta^*LC(X, \tau_i, \tau_j)$

  c) $A \in \tau_j-\Delta^*C(X, \tau_i, \tau_j) \Rightarrow A \in (i, j)-\Delta^*LC(X, \tau_i, \tau_j)$

  d) $A \in \tau_i-\Delta^*O(X, \tau_i, \tau_j) \Rightarrow A \in (i, j)-\Delta^*LC(X, \tau_i, \tau_j)$

- If (X, τ_i, τ_j) is a pairwise δ-door space then every subset of X is both (i, j)- Δ^*-locally closed and (j, i)- Δ^*-locally closed.

- **During the course of my work, it is essential to produce many counter examples to substantiate the results. Hence, for the topological spaces containing three elements and four elements, various open sets and closed sets are derived and the collections are tabulated in Appendix I and Appendix II. Also for the bitopological spaces of three elements, various closed sets are tabulated in Appendix III.**

- **These appendices are like dictionary of examples for those who work in a similar line.**