Chapter 3

Global Exponential Stability for Stochastic Cohen-Grossberg Neural Networks with Multiple Time-Varying Delays

3.1 Introduction

The past two decades have witnessed tremendous developments in the research field of neural networks, cellular neural networks, bidirectional associative neural networks and Cohen-Grossberg neural networks. Cohen - Grossberg models have been widely studied due to their extensive applications in classification of patterns, associative memories, image processing and optimization Cohen - Grossberg (1983, 1988). The time delays are commonly encountered in various engineering systems such as chemical processes, hydraulic and rolling mill systems, etc. These effects are unavoidably encountered in the implementation of neural networks, and may cause undesirable dynamic network behaviors such as oscillation and instability. Therefore, it is important to investigate the stability of delayed neural networks and that was explicitly introduced in Marcus (1989). A large number of the criteria on the stability of Cohen-Grossberg neural networks have been derived in the literature, Cao and Li (2005), Cohen Grossberg (1983), Huang et al. (2003), Rong (2005),Sun and Wan (2005), Wu et al. (2007 b) and Wang and Zou (2002).

The global asymptotic stability results for different classes of delayed neural networks were proposed in Cao and Li (2005), Rong (2005) and Wu et al. (2007 a , 2007 b). However, these results are only concerned with the asymptotic stability of such networks, without providing any conditions for exponential stability and any information about the decay rates of the delayed neural networks. Therefore, it is particularly important to consider the problems to determine the speed of neural computations using the exponential convergence rate. Considering this, many researchers have studied the exponential stability analysis problem for delayed neural networks and a great number of results on this topic have been reported in the literature Huang et al.(2003),Sun and Wan (2005) and Wang and Zou (2002) . Recently, LMI-based techniques have been successfully used to tackle various stability problems for neural networks with time delays Cao and Li (2005), Rong (2005) and Wu et al. (2007 a, 2007 b). In Chen and Rong (2003), the asymptotic stability of delayed Cohen-Grossberg neural networks was considered. However, the criteria are expressed in terms of algebraic inequalities and do not take into account the signs of entries of the connection matrices, hence it might ignore the difference between the neuronal excitatory and inhibitory effects.

It is worth noting that so far there are only a few papers that have taken stochastic phenomenon into account in neural networks, see Blythe et al.(2001), Liao and Mao (1996), Wan and Sun (2005) and Wang et al.(2007). Practically, such phenomenon always appears in the electrical circuit design of neural networks. The results in Liao and Mao (1996) and Mao (1997) suggested that a neural network could be stabilized or destabilized by some stochastic inputs. This implies that it is of practical significance to study the stability for delayed stochastic neural networks. However, the stability analysis of stochastic neural networks is more difficult than that of traditional neural networks. Wang et.al.(2006, 2007) studied the exponential stability of uncertain stochastic neural networks with discrete and distributed delays and robust stability for stochastic Hopfield neural networks with time delays. Huang and Cao (2007) studied the exponential stability of uncertain stochastic neural networks with multiple time-varying delays in terms of LMI. Recently, Wang et al.(2006 b) studied the stability analysis of stochastic Cohen-Grossberg neural networks with discrete and distributed delays.

Based on the above discussions, a class of stochastic Cohen-Grossberg neural networks with multiple time-varying delays is considered in this chapter. The main purpose of this chapter is to study the global exponential stability in the mean square sense for stochastic Cohen-Grossberg neural networks with multiple time-varying delays. By using the Lyapunov functional technique, global exponential stability conditions for the considered stochastic Cohen-Grossberg neural networks are given in terms of LMIs, which can be easily calculated by MATLAB LMI toolbox. A numerical example is also provided to demonstrate the effectiveness of the proposed stability results.

3.2 Global Stability Results

Throughout the chapter the notation $A > 0 (or A < 0)$ is used to denote that the matrix A is a symmetric and positive definite (or negative definite) matrix. The notation A^T and A^{-1} mean the transpose of A and the inverse of a square matrix. If A, B are symmetric matrices $A > B$ $(A \geq B)$ means that $A - B$ is positive definite (positive semi-definite).

Consider the following Cohen-Grossberg neural networks with multiple time-varying delays described by

$$x_i'(t) = -a_i(x_i(t))\Big[d_i(x_i(t)) - \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) - \sum_{j=1}^{n} b_{ij} f_j(x_j(t - \tau_{ij}(t))) + I_i\Big], \quad i = 1, 2, ..., n \tag{3.2.1}$$

where x_i is the state variable of the ith neuron and $a_i(\cdot)$ represents an amplification function and assumed to be positive, bounded and locally Lipschitz continuous, $d_i(\cdot)$ is the behaved function. The matrices a_{ij} and b_{ij} are the connection weight and delayed connection weight coefficients, respectively, f_j denotes the neuron activations, I_i is the constant input from outside the system and the delay $\tau_{ij}(t)$ is nonnegative, bounded, and differentiable with $0 \leq \tau_{ij}(t) \leq \tau$ and $\tau_{ij}'(t) \leq \eta_{ij} < 1$, $i, j = 1, 2, ..., n$.

Assumption$_1$ Each function $d_i : \mathbb{R} \to \mathbb{R}$ is locally Lipschitz and there exists γ_i such that $d_i'(x) \geq \gamma_i$ for all $x \in \mathbb{R}$ at which $d_i(\cdot)$ is differentiable. Let us denote $\Gamma = \text{diag}\{\gamma_1, \gamma_2, ..., \gamma_n\}$.

Assume $x^* = (x_1^*, x_2^*, ...x_n^*)^T$ is an equilibrium point of equation (3.2.1), one can derive from (3.2.1) that the transformation $y_i(t) = x_i(t) - x_i^*$ transforms system (3.2.1) into the following system:

$$y_i'(t) = -\alpha_i(y_i(t))\Big[\beta_i(y_i(t)) - \sum_{j=1}^{n} a_{ij} g_j(y_j(t)) - \sum_{j=1}^{n} b_{ij} g_j(y_j(t - \tau_{ij}(t)))\Big], \quad i = 1, 2, ..., n \tag{3.2.2}$$

where

$$\begin{aligned}
y &= [y_1, y_2, ..., y_n]^T \in \mathbb{R}^n, \\
\alpha(y(t)) &= \mathrm{diag}[\alpha_1(y_1(t)), \alpha_2(y_2(t)), ..., \alpha_n(y_n(t))] \in \mathbb{R}^{n\times n}, \\
\beta(y(t)) &= [\beta_1(y(t)), \beta_2(y_2(t)), ..., \beta_n(y_n(t))] \in \mathbb{R}^n, \\
g(y(t)) &= [g_1(y_1(t)), g_2(y_2(t)), ..., g_n(y_n(t))]^T \in \mathbb{R}^n, \\
\alpha_i(y_i(t)) &= a_i(y_i(t) + x_i^*), \\
\beta_i(y_i(t)) &= d_i(y_i(t) + x_i^*) - d_i(x_i^*), \\
g_j(y_j(t)) &= f_j(y_j(t) + x_j^*) - f_j(x_j^*).
\end{aligned}$$

In this chapter, Stochastic Cohen-Grossberg Neural Networks with multiple time-varying delays is considered.It is described by

$$\begin{aligned}
dy_i(t) &= -\alpha_i(y_i(t))\Big[\beta_i(y_i(t)) - \sum_{j=1}^{n} a_{ij} g_j(y_j(t)) - \sum_{j=1}^{n} b_{ij} g_j(y_j(t-\tau_{ij}(t)))\Big]dt \\
&\quad + \sum_{k=1}^{m} \sigma_{ik}(t, g_j(y_j(t)), g_j(y_j(t-\tau_{ij}(t))))dw_k(t) \qquad (3.2.3)
\end{aligned}$$

where $w_k(t) = (w_1(t), w_2(t), ..., w_m(t))^T$ is an m-dimensional Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, P)$ with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$. Let $\sigma(t, x, y) : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times m}$ is locally Lipschitz continuous and satisfies the linear growth condition as well.

The matrix form of equation (3.2.3) can be written as

$$\begin{aligned}
dy(t) &= -\alpha(y(t))[\beta(y(t)) - Ag(y(t)) - Bg(y(t-\tau(t)))]dt \qquad (3.2.4) \\
&\quad + \sigma(t, g(y(t)), g(y(t-\tau(t))))dw(t)
\end{aligned}$$

where $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$, $\tau(t) = \tau_{ij}(t)$ for $i, j = 1, 2, ..., n$.

Let $y(t;\xi)$ denote the state trajectory of the neural network (3.2.4) from the initial data $y(\theta) = \xi(\theta)$ on $-\tau \leq \theta \leq 0$ in $L^2_{\mathcal{F}_0}([-\tau, 0], \mathbb{R}^n)$. It can be easily seen that the system (3.2.4) admits a trivial solution $y(t; 0) \equiv 0$ corresponding to the initial data $\xi = 0$, see Burton (1985) and Hale (1977).

The main purpose of this chapter is to establish LMI-based stability criteria, which can be readily checked by using the MATLAB LMI toolbox, such that the global exponential stability is guaranteed for the stochastic Cohen-Grossberg neural network (3.2.4) with multiple time-varying delays.

3.3 Main Results

Let $C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ : \mathbb{R}_+)$ denote the family of all non-negative functions $V(y,t)$ on $\mathbb{R}^n \times \mathbb{R}_+$ which are continuously twice differentiable in y and once differentiable in t. For each $V \in C^{2,1}([-\tau,\infty] \times \mathbb{R}^n, \mathbb{R}_+)$, define an operator $\mathcal{L}V$ associated with stochastic delayed neural networks (3.2.4) from $\mathbb{R}^n \times \mathbb{R}_+$ to $\mathbb{R}$ by

$$\begin{aligned}\mathcal{L}V(y(t),t) &= V_t(y,t) + V_y(y,t)\Big[-\alpha(y(t))[\beta(y(t)) - Ag(y(t)) - Bg(y(t-\tau(t)))]\Big] \\ &\quad + \frac{1}{2} trace[\sigma^T V_{yy}(y,t)\sigma]\end{aligned}$$

where

$$V_t(y,t) = \frac{\partial V(y,t)}{\partial t}, V_y(y,t) = \Big(\frac{\partial V(y,t)}{\partial y_1}, \frac{\partial V(y,t)}{\partial y_2}, ..., \frac{\partial V(y,t)}{\partial y_n}\Big),$$

and

$$V_{yy}(y,t) = \Big(\frac{\partial^2 V(y,t)}{\partial y_i \partial y_j}\Big)_{n\times n}$$

where $i,j = 1,2,...,n$. Then by the generalized Ito's formula, one obtains

$$\mathbb{E}V(y(t),t) = V(y(0),0) + \mathbb{E}\int_0^t \mathcal{L}V(y(s),s)ds.$$

Definition 3.3.1. *For every $\phi \in C^b_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$, the trivial solution of system (3.2.4) is said to be globally exponentially stable in the mean square if there exist positive scalars $\alpha > 0$ and $\beta > 0$ such that*

$$\mathbb{E}|y(\phi,t)|^2 \leq \alpha e^{-\beta t}\mathbb{E}\|\phi\|^2.$$

The author introduce and prove the following theorem on global exponential stability of equation (3.2.4).

Theorem 3.3.1. *Assume $\gamma - c_2 > 0$ where γ is the rate of convergence and $c_2 = \frac{\lambda_{\max}(\Omega)}{\lambda_{\min}(P)}$. Assume that there exist matrices $P > 0, D_0 \geq 0$ and $D_i \geq 0$ $(i = 1,2,...,n)$ such that*

$$\begin{aligned}&trace[\sigma^T(t, g(y(t)), g(y(t-\tau_{ij}(t))))p_0 I \sigma(t, g(y(t)), g(y(t-\tau_{ij}(t))))] \\ &\leq (g(y(t)))^T D_0 (g(y(t))) + \sum_{i=1}^n \sum_{j=1}^n (g(y(t-\tau_{ij}(t))))^T D_i (g(y(t-\tau_{ij}(t)))),\end{aligned}$$

where $p_0 = \sum_{i=1}^{n} p_i$, $\bar{D}_i = diag(d_{i1}, d_{i2}, ..., d_{in})$ *and* $(g(y(t - \bar{\tau}_i(t)))) = ((g(y_1(t - \bar{\tau}_{i1}(t)))), (g(y_2(t - \bar{\tau}_{i2}(t)))), ..., (g(y_n(t - \bar{\tau}_{in}(t))))$. *In addition, system (3.2.4) is globally exponentially stable in the mean square, if there exist positive diagonal matrices* $Q_i = diag(q_{i1}, q_{i2}, ..., q_{in})$*, such that the following inequality holds:*

$$\Omega = 2\gamma P + PA + A^T P - 2P\Gamma + D_0 + e^{\gamma\tau} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{1-\eta_{ij}} D_i + \sum_{i=1}^{n} \left(e^{\gamma\tau} Q_i + PW_i Q_i^{-1} W_i^T P \right) < 0, \tag{3.3.1}$$

where $\Gamma = diag(\gamma_1, \gamma_2, ..., \gamma_n)$, $A = (a_{ij})_{n\times n}$, *and* W_i *is an* $n\times n$ *square matrix, whose* i*th row is composed of* $(b_{i1}, b_{i2}, ..., b_{in})$ *and other rows are all zeros,* $i, j = 1, 2, ..., n$.

Proof. We use the following Lyapunov functional to derive the stability result

$$\begin{aligned} V(y,t) \;=\; & 2e^{\gamma t} \sum_{i=1}^{n} p_i \int_{x_i^*}^{y_i(t)} \frac{g_i(s)}{\alpha_i(s)} ds + \sum_{i=1}^{n} \sum_{j=1}^{n} \int_{t-\tau_{ij}(t)}^{t} e^{\gamma(s+\tau)} (g_j(y_j(s)))^T q_{ij} (g_j(y_j(s))) ds \\ & + \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{1-\eta_{ij}} \int_{t-\tau_{ij}(t)}^{t} e^{\gamma(s+\tau)} (g_j(y_j(s)))^T d_{ij} (g_j(y_j(s))) ds. \end{aligned}$$

By Ito's formula, $\mathcal{L}V$ can be calculated along the trajectories of the system (3.2.4), then we have

$$\begin{aligned} \mathcal{L}V(y(t),t) \;\le\; & 2\gamma e^{\gamma t} g^T(y(t)) P y(t) + 2e^{\gamma t} \sum_{i=1}^{n} p_i (g_i(y_i(t))) [-\beta_i(y_i(t)) + \sum_{j=1}^{n} a_{ij} g_j(y_j(t)) \\ & + \sum_{j=1}^{n} b_{ij} g_j(y_j(t - \tau_{ij}(t)))] + \sum_{i=1}^{n} \sum_{j=1}^{n} q_{ij} \Big(e^{\gamma(t+\tau)} (g_j(y_j(t)))^2 \\ & - e^{\gamma t} (g_j(y_j(t - \tau_{ij}(t)))^2 \Big) + \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{1-\eta_{ij}} e^{\gamma(t+\tau)} (g_j(y_j(t))) d_{ij} (g_j(y_j(t))) \\ & - \sum_{i=1}^{n} \sum_{j=1}^{n} e^{\gamma t} (g_j(y_j(t - \tau_{ij}(t)))) d_{ij} (g_j(y_j(t - \tau_{ij}(t)))) \\ & + trace[\sigma^T(t, g(y(t)), g(y(t - \tau_{ij}(t)))) p_0 I \sigma(t, g(y(t)), g(y(t - \tau_{ij}(t))))] \end{aligned}$$

$$
\begin{aligned}
\leq\ & e^{\gamma t}\Big[2(f(x(t))-f(x^*))^T\gamma P(x(t)-x^*)-2(f(x(t))-f(x^*))^T P\mathit{\Gamma}(x(t)-x^*)\\
&+2(f(x(t))-f(x^*))^T PA(f(x(t))-f(x^*))\\
&+2\sum_{i=1}^{n}p_i(f(x(t))-f(x^*))[b_{i1},b_{i2},...,b_{in}]\\
&\times(f(x(t-\bar{\tau}_i(t)))-f(x^*))+\sum_{i=1}^{n}\Big[e^{\gamma\tau}(f(x(t))-f(x^*))^T Q_i(f(x(t))-f(x^*))\\
&-(f(x(t-\bar{\tau}_i(t)))-f(x^*))^T Q_i(f(x(t-\bar{\tau}_i(t)))-f(x^*))\Big]\\
&+\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\eta_{ij}}e^{\gamma\tau}(f(x(t-f(x^*)))^T D_i(f(x(t)-f(x^*)))\\
&-\sum_{i=1}^{n}(f(x(t-\bar{\tau}_i(t)))-f(x^*))^T D_i(f(x(t-\bar{\tau}_i(t)))-f(x^*))\\
&+(f(x(t))-f(x^*))^T D_0(f(x(t))-f(x^*))\\
&+\sum_{i=1}^{n}(f(x(t-\bar{\tau}_i(t)))-f(x^*))^T D_i(f(x(t-\bar{\tau}_i(t)))-f(x^*))\Big] \qquad (3.3.2)
\end{aligned}
$$

where $f(x(t-\bar{\tau}_i(t))=(f(x_1(t-\tau_{i1}t)),f(x_2(t-\tau_{i2}(t))),...,f(x_n(t-\tau_{in}(t))))^T$, $D_i=diag(d_{i1},d_{i2},...,d_{in})$ and $Q_i=diag(q_{i1},q_{i2},...,q_{in})$, $i=1,2,...,n$.

It can be easily shown that the following conditions holds:

$$
\begin{aligned}
2\sum_{i=1}^{n}p_i(f(x_i(t)) \quad - \quad & f(x_i^*))[b_{i1},b_{i2},...,b_{in}](f(x(t-\bar{\tau}_i(t)))-f(x^*))\\
&=\sum_{i=1}^{n}2(f(x(t))-f(x^*))^T PW_i(f(x(t-\bar{\tau}_it))-f(x^*))\\
&\leq(f(x(t))-f(x^*))^T\Big(\sum_{i=1}^{n}PW_iQ_i^{-1}W_i^TP\Big)(f(x(t))-f(x^*))\\
&+\sum_{i=1}^{n}(f(x(t-\bar{\tau}_i(t)))-f(x^*))^T Q_i(f(x(t-\bar{\tau}_i(t)))-f(x^*)) \qquad (3.3.3)
\end{aligned}
$$

Using (3.3.3) in (3.3.2), we have

$$\begin{aligned}
\mathcal{L}V(y(t),t) \;\leq\;& e^{\gamma t}\Big[2(f(x(t))-f(x^*))^T\gamma P(x(t)-x^*)-2(f(x(t))-f(x^*))^T P\Gamma(x(t)-x^*) \\
&+2(f(x(t))-f(x^*))^T PA(f(x(t))-f(x^*)) \\
&+(f(x(t))-f(x^*))^T\Big[\sum_{i=1}^{n} PW_iQ_i^{-1}W_i^TP\Big](f(x(t))-f(x^*)) \\
&+\sum_{i=1}^{n}(f(x(t-\bar{\tau}_i(t)))-f(x^*))^TQ_i(f(x(t-\bar{\tau}_i(t)))-f(x^*)) \\
&+\sum_{i=1}^{n}\Big[e^{\gamma\tau}(f(x(t))-f(x^*))^TQ_i(f(x(t))-f(x^*)) \\
&-(f(x(t-\bar{\tau}_i(t)))-f(x^*))^TQ_i(f(x(t-\bar{\tau}_i(t)))-f(x^*))\Big] \\
&+(f(x(t))-f(x^*))^TD_0(f(x(t))-f(x^*)) \\
&+\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\eta_{ij}}e^{\gamma\tau}(f(x(t)-f(x^*)))^TD_i(f(x(t)-f(x^*)))\Big]
\end{aligned}$$

$$\begin{aligned}
\leq\;& e^{\gamma t}\Big[2(f(x(t))-f(x^*))^T\gamma P(x(t)-x^*)-2(f(x(t))-f(x^*))^T P\Gamma(x(t)-x^*) \\
&+2(f(x(t))-f(x^*))^T PA(f(x(t))-f(x^*)) \\
&+(f(x(t))-f(x^*))^T\Big[\sum_{i=1}^{n} PW_iQ_i^{-1}W_i^TP+e^{\gamma\tau}Q_i\Big](f(x(t))-f(x^*)) \\
&+2(f(x(t))-f(x^*))^T\gamma P(f(x(t))-f(x^*))-2(f(x(t))-f(x^*))^T\gamma P(f(x(t))-f(x^*)) \\
&+2(f(x(t))-f(x^*))^T P\Gamma(f(x(t))-f(x^*))-2(f(x(t))-f(x^*))^T P\Gamma(f(x(t))-f(x^*)) \\
&+(f(x(t))-f(x^*))^TD_0(f(x(t))-f(x^*)) \\
&+\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\eta_{ij}}e^{\gamma\tau}(f(x(t)-f(x^*)))^TD_i(f(x(t)-f(x^*)))\Big]. \qquad (3.3.4)
\end{aligned}$$

According to Liao et al.(2005), Lemma 2, the following inequality holds:

$$\begin{aligned}
-2(f(x(t))-f(x^*))^TPC(x(t)-x^*)+2(f(x(t))-f(x^*))^TPC(f(x(t))-f(x^*)) &\leq 0, \\
-2(f(x(t))-f(x^*))^T\gamma P(x(t)-x^*)+2(f(x(t))-f(x^*))^T\gamma P(f(x(t))-f(x^*)) &\leq 0.
\end{aligned} \qquad (3.3.5)$$

The equation (3.3.4) using (3.3.5), can be rewritten as

$$
\begin{aligned}
\mathcal{L}V(y(t),t) \leq\ & (f(x(t)) - f(x^*))^T e^{\gamma t}\Big[2\gamma P - 2P\mathit{\Gamma} + PA + A^T P \\
& + \sum_{i=1}^{n}(PW_iQ_i^{-1}W_i^TP + e^{\gamma\tau}Q_i) \\
& + D_0 + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\eta_{ij}}e^{\gamma\tau}D_i\Big](f(x(t)) - f(x^*)).
\end{aligned}
$$

By applying the generalized Ito's formula, it yields

$$
\begin{aligned}
\mathbb{E}V(y(t),t) &\leq \mathbb{E}V(y(0),0) + \lambda_{\max}(\mathit{\Omega})\mathbb{E}\int_0^t e^{\gamma s}\|g(y(s))\|^2 ds \\
&\leq \mathbb{E}V(y(0),0) + \lambda_{\max}(\mathit{\Omega})\mathbb{E}\int_0^t e^{\gamma s}\|y(s)\|^2 ds.
\end{aligned}
$$

On the other hand, from the definition of $V(y(t),t)$,

$$
\begin{aligned}
\mathbb{E}V(y(0),0) &\leq \lambda_{\max}(P)\mathbb{E}\|\phi\|^2 + \lambda_{\max}(Q_i)\tau e^{\gamma\tau}\mathbb{E}\|\phi\|^2 + \lambda_{\max}(\bar{D}_i)\tau e^{\gamma\tau}\mathbb{E}\|\phi\|^2 \\
&= \psi_1\mathbb{E}\|\phi\|^2, \\
\mathbb{E}V(y(t),t) &\geq e^{\gamma t}\lambda_{\min}(P)\mathbb{E}\|y(t)\|^2.
\end{aligned}
$$

Therefore, we have

$$
e^{\gamma t}\mathbb{E}\|y(t)\|^2 \leq \frac{\psi_1}{\lambda_{\min}(P)}\mathbb{E}\|\phi\|^2 + \frac{\lambda_{\max}(\mathit{\Omega})}{\lambda_{\min}(P)}\mathbb{E}\int_0^t e^{\gamma s}\|y(s)\|^2 ds.
$$

By the Gronwall integral inequality, we have

$$
\mathbb{E}\|y(t)\|^2 \leq c_1 e^{-(\gamma - c_2)t},
$$

where $c_1 = \frac{\psi_1}{\lambda_{\min}(P)}\mathbb{E}\|\phi\|^2$ and $c_2 = \frac{\lambda_{\max}(\mathit{\Omega})}{\lambda_{\min}(P)}$.

Therefore, if (3.2.3) holds, then $\mathcal{L}V(y(t),t) < 0$ for any $(f(x(t)) - f(x^*)) \neq 0$. $\mathcal{L}V(y(t),t) = 0$ if and only if $(f(x(t)) - f(x^*)) = 0$. This completes the proof of the Theorem. □

Remark 3.3.1. *If the time delay $\tau_{ij}(t) = \tau_j(t)$ in (1), $i, j = 1, 2, ..., n$, we will have the following result.*

Corollary 3.3.1. *The equilibrium point of (3.2.4) with $\tau_{ij} = \tau_j$ is globally exponentially stable if there exist positive diagonal matrix $Q = diag(q_1, q_2, ..., q_n)$ such that the following inequality holds:*

$$2\gamma P - 2P\Gamma + PA + A^T P + PBQ^{-1}B^T P + e^{\gamma\tau}Q + D_0 + \sum_{i=1}^{n} \frac{1}{1-\eta_i} e^{\gamma\tau} D_i < 0 \quad (3.3.6)$$

where $B = (b_{ij})_{n\times n}$ and others are the same as those defined in Theorem 3.3.1.

Proof. We use the following Lyapunov functional to derive the stability result,

$$\begin{aligned} V(y,t) &= \sum_{i=1}^{n} 2p_i \int_{x_i^*}^{y_i(t)} \frac{g_i(s)}{\alpha_i(s)} ds + \int_{t-\tau_j(t)}^{t} g_j(y_j(s)) q_j g_j(y_j(s)) ds \\ &\quad + \sum_{j=1}^{n} \frac{1}{1-\eta_j} \int_{t-\tau_j(t)}^{t} g_j(y_j(s)) d_j g_j(y_j(s)) ds \end{aligned}$$

where $q_j > 0$ and $d_j > 0$ $i = 1, 2, ..., n$. In a similar manner to the proof of Theorem 3.3.1, condition (3.3.6) is easy to be derived. The details are omitted. □

Remark 3.3.2. *Theorem* 3.3.1 *and Corollary* 3.3.1 *can be expressed in the form of linear matrix inequality as*

$$\begin{bmatrix} \Psi & PW_1 & \ldots & PW_n \\ W_1^T P & -Q_1 & \ldots & 0 \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ W_n^T P & 0 & \ldots & -Q_n \end{bmatrix} < 0$$

and

$$\begin{bmatrix} 2\gamma P - 2P\Gamma + PA + A^T P + e^{\gamma\tau} Q + D_0 + e^{\gamma\tau}\sum_{i=1}^{n} D_i & PB \\ B^T P & -Q \end{bmatrix} < 0$$

where $\Psi = 2\gamma P - 2P\Gamma + PA + A^T P + e^{\gamma\tau}\sum_{i=1}^{n} Q_i + D_0 + e^{\gamma\tau}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\eta_{ij}} D_i$. *Therefore, the conditions in Theorem* 3.3.1 *and Corollary 3.3.1 are easy to verify.*

Remark 3.3.3. *Theorem* 3.3.1 *gives a novel condition to check the global exponential stability in the mean square of system* (3.2.4)*. Recently, the stability problem was discussed in Blythe (2001) and Liao and Mao (1996) for stochastic Hopfield neural networks with constant delay. In addition, compared with Blythe (2001) and Liao and Mao (1996) and Wan and Sun (2005), multiple time-varying delays is taken into account in this chapter. This chapter differs from Huang and Cao (2007) in which the sign of entries in connection weight matrix and the nonlinear diffusion term involved with the general activation function. Therefore, the result proposed here, is less conservative and less restrictive than some early results.*

Remark 3.3.4. *In this chapter, a new class of stochastic Cohen-Grossberg neural networks with multiple time-varying delays has been studied. The advantage of this approach is that sufficient conditions have been derived to obtain less conservative result. In Wang et al.(2006 b) the authors studied the stability analysis for a class of stochastic Cohen-Grossberg neural networks with discrete and distributed delays. It should be noted that the stochastic neural networks studied in this chapter has the time-varying delays as in Huang (2007). Therefore, our results and those established in Huang (2007) and Nang et al.(2006 b) are complementary to each other.*

Remark 3.3.5. *In Cao and Li (2005), Rong (2005), Wu et al.(2007 b) and Wu et al.(2007 a), the Cohen-Grossberg neural networks with time delays were investigated, and several LMI based conditions were proposed to guarantee the stability of equilibrium point of Cohen-Grossberg neural networks. However, the stochastic term was not taken into account in the models. Therefore, the developed results in this* *chapter are more general than those reported in Cao and Li (2005), Rong (2005) and Wu et al.(2007).*

Remark 3.3.6. *The results in Chen and Rong (2003), Huang et al.(2003), Sun and Wan(2005) and Wang and Zou (2002) did not consider the sign of entries in connection weight matrix, therefore, the neuron's excitatory and inhibitory effect on neural networks was not considered. In contrast, the delayed connection weight matrix could be decomposed into sum of the singular matrices in which the sign entries in connection weight matrix, and the neuron's excitatory and inhibitory effect on neural networks is considered.*

3.4 Numerical Example

In this section, one numerical example is provided to demonstrate the effectiveness and applicability of the proposed method.

Example 3.4.1

Consider the two-neuron stochastic neural network with delays (3.2.1), where

$$\Gamma = \begin{bmatrix} 7 & 0 \\ 0 & 6.5 \end{bmatrix} \quad A = \begin{bmatrix} 0.5330 & 1.0328 \\ 1.0328 & 0.3621 \end{bmatrix} \quad B = \begin{bmatrix} -0.0184 & -0.1375 \\ -0.6138 & -0.0802 \end{bmatrix}$$

It is assumed that $\tau = 0.5$, $\tau_{11}(t) = \tau_{12}(t) = 0.5 + 0.5sint$, $\tau_{21}(t) = \tau_{22}(t) = 0.5 - 0.5cost$ and

$$\sigma(t, y(t), g(y(t)), g(y(t-\tau(t)))) = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}, \text{ where}$$

$\sigma_{11} = \sigma_{22} = 0.5tanhy_1(t) + 0.5tanhy_1(t - \tau_{11}(t)) + 0.5tanhy_1(t - \tau_{12}(t))$

$\sigma_{12} = \sigma_{21} = 0.4tanhy_2(t) + 0.4tanhy_2(t - \tau_{21}(t)) + 0.4tanhy_2(t - \tau_{22}(t))$.

Now the matrix B can be expressed in the following two matrices

$$W_1 = \begin{bmatrix} -0.0184 & -0.1375 \\ 0 & 0 \end{bmatrix} \quad \text{and } W_2 = \begin{bmatrix} 0 & 0 \\ -0.6138 & -0.0802 \end{bmatrix}.$$

By setting $P = I$, $D_0 = D_1 = D_2 = diag(1.0, 0.64)$ and taking $\gamma = 0.324$, and by using the MATLAB LMI toolbox, one can obtain the following feasible solution for the LMIs in Theorem 3.3.1:

$$Q_1 = \begin{bmatrix} 0.0802 & 0 \\ 0 & 0.2755 \end{bmatrix} \quad Q_2 = \begin{bmatrix} 0.3591 & 0 \\ 0 & 0.1996 \end{bmatrix}.$$

Therefore, the concerned stochastic Cohen-Grossberg neural networks with multiple time-varying delays is globally exponentially stable in the mean square sense. Since the time delays are variant in t, the stability criteria in Wang et al.(2006, 2007) cannot be applicable to the example.

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