

# **Introduction**

## INTRODUCTION

*"If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is".*

**- John Von Neumann**

Soft set theory is one of the recent topics gaining significance in finding rational and logical solutions to various real life problems which involve uncertainty, impreciseness and vagueness.

Uncertainty is present in almost every sphere of our daily life. Traditional mathematical tools are not sufficient to handle all the practical problems in fields such as Medical Science, Social Science, Engineering, Economics etc., which involve various types of uncertainty. Zadeh (1965) was the first to come up with his remarkable theory of fuzzy sets for dealing these types of uncertainties where conventional tools fail. His theory brought a grand paradigmatic change in mathematics.

Later, there are theories namely the Intuitionistic Fuzzy Sets, Vague Sets, Rough Sets, Interval Mathematics etc., for handling uncertainty. Molodtsov (1999) initiated the theory of soft sets as a new mathematical tool for dealing uncertainty.

"Given an initial universe set  $U$  and a set of parameters  $E$ , a soft set over  $U$  is a pair  $(F, A)$  where  $A \subseteq E$  and  $F$  is a mapping given by  $F : A \rightarrow P(U)$ , the power set of  $U$ ".

The topological structures of set theories dealing with uncertainties were first studied by Chang (1968). Chang introduced the notion of fuzzy topology and also studied some of its basic properties.

Shabir and Naz (2011) introduced the notion of soft topological spaces (Definition 2.1) which are defined over an initial universe with a fixed set of

parameters. They studied some basic concepts of soft topological spaces and also some related concepts such as soft interior, soft closure, soft subspace and soft separation axioms.

Aygunoglu – Aygun (2011) introduced soft product topology and defined the concept of compactness in soft spaces named as soft compactness.

Peyghan et al. (2012) introduced the concept of soft connectedness and studied some properties related to these spaces.

Juthika (2013) introduced semiopen and semiclosed soft sets in soft topological spaces and generalized the notions of interior and closure to soft topological spaces. Also various forms of soft functions like semi continuous, irresolute, semiopen and semiclosed soft functions are introduced and characterized including those of soft semicompactness and soft semiconnectedness.

The main aim of this thesis is to study soft sets and soft topological spaces.

The plan of study is as follows :

1. Soft sets and soft topological spaces
2. Soft mappings on soft topological spaces.
3. Soft Hausdorff spaces
4. Soft compact spaces
5. Soft connected spaces
6. Generalizations of soft open and closed sets, soft functions, soft compactness and soft connectedness.

The first chapter deals with preliminary definitions and notations regarding soft sets.

The second chapter is devoted to the study of soft topological spaces. In this chapter, the concepts of soft open, soft closed, soft closure and soft interior of a soft set, soft subspace, soft neighborhood and soft continuity are studied with interesting properties. The important results are given in theorems 2.22 and 2.26.

Third chapter deals with soft mappings on soft topological spaces. In this chapter, soft continuous mapping, soft open mapping, soft closed mapping, soft homeomorphism, soft  $pu$ -continuous mapping are introduced with examples. An interesting characterization regarding soft continuous mapping is given in theorem 3.2. The notion of soft  $pu$ -continuity of functions is induced by two mappings  $u : U \rightarrow V$  and  $p : A \rightarrow B$  on soft topological spaces  $(U, \tau, A)$  and  $(V, \tau^*, B)$  (Definition 3.15). Interesting characterizations proved regarding soft  $pu$ -continuous functions are given in theorems 3.16, 3.17 and 3.18.

Chapter IV deals with soft Hausdorff spaces. In this chapter the definition of soft Hausdorff space is given with an example. Also the definition of diagonal soft set is given and the concept of soft Hausdorffness is characterized with this diagonal soft set (Theorem 4.4). Following are the important results proved in this chapter :

1. If  $(X, \tau)$  is soft Hausdorff space,  $(Y, \tau^*)$  is a soft topological space and  $(\varphi, \psi) : (X, \tau) \rightarrow (Y, \tau^*)$  is injective, surjective and soft open, then  $(Y, \tau^*)$  is soft Hausdorff space (Theorem 4.5)
2. The property of being a soft Hausdorff space is hereditary (Theorem 4.6).
3.  $X$  and  $Y$  are soft Hausdorff spaces iff  $X \times Y$  is a soft Hausdorff space (Theorem 4.9).

Soft compact spaces are studied in chapter V. The concept of soft compactness is introduced and some interesting results are proved (Theorems 5.4, 5.5 and 5.6).

Chapter VI deals with soft connected spaces. In this chapter the concept of soft connectedness is introduced with interesting properties. Also some important results which involve the concept of soft pu-continuous functions are obtained. Some of them are

1. A soft topological space  $(X, \tau, E)$  is soft connected iff the only soft sets in  $SS(X)_E$  that are both open and soft closed over  $X$  are  $\Phi_E$  and  $\tilde{X}$  (Theorem 6.3).
2. Let  $f_{pu}$  be a soft pu-continuous function carrying the soft connected space  $(U, \tau, A)$  onto the soft space  $(V, \tau', B)$ . Then  $(V, \tau', B)$  is soft connected (Theorem 6.7).
3. The union of a collection of soft connected subspaces of  $(X, \tau, E)$  that have non-null intersection is soft connected (Theorem 6.13).

Chapter VII is devoted to the study of generalization of soft open and closed sets, soft functions, soft compactness and soft connectedness. In this chapter semiopen and semiclosed soft sets in soft topological spaces are introduced and then these are used to generalize the notions of interior and closure. Further, the properties of semiopen soft sets, semiclosed soft sets, semi interior and semi closure of soft sets in soft topological spaces are studied. The definitions of soft semicontinuous, soft irresolute, soft semiopen function, soft semiclosed function are introduced (Definition 7.8). Interesting characterizations regarding soft semicontinuous functions are given in Theorems 7.10 and 7.11. The concepts of soft semi compactness and soft semi connectedness are introduced and characterized.