Micro g^*-Closed Sets in Micro Topological Spaces

By

Sandhiya S

(20PMA013)

Supervisor

Dr. N. Balamani

Thesis Submitted to

Avinashilingam Institute for Home Science and Higher Education for Women

Coimbatore-641 043

In Partial Fulfilment of the Requirement for the Degree of

Master of Science in Mathematics

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N. Balamani
17.05.2022
Signature of the Head of the Department

N. Balamani
17.05.2022
Signature of the Supervisor

DECLARATION

I declare that the dissertation entitled **"Micro g^*-Closed Sets in Micro Topological Spaces"** submitted by me for the degree of **Master of Science** is a record of research work carried out by me during the period from December 2021 to May 2022 under the guidance of **Dr. (Tmt) N. Balamani,** Assistant Professor (SS) and Head (i/c), Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore and has not formed the basis for the award of any Degree, Diploma, Associates, Fellowship or other titles in the University or any other University or Institute of Higher Learning.

S. Sandhya
Signature of the Candidate

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ACKNOWLEDGEMENT

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CONTENTS

INTRODUCTION

INTRODUCTION

Topology is a branch of mathematics which is good at extracting qualitative features from complicated geometric structures. General topology known as point set topology is the branch of topology dealing with the set theoretic definitions and constructions used in topology. The concept of Nano topology was introduced by Carmel Richard [2013] which is defined in terms of the lower and upper approximations and the boundary region of a subset of a universe. The notion of approximations and boundary region of a set was originally proposed by Pawlak [2004] in order to introduce the concept of rough set theory.

Sakkraiveeranan Chandrasekar [2019] introduced the concept of Micro topology which is a simple extension of Nano topology. The notion of Micro open sets are the powerful tool for defining Micro topological spaces. The weaker form of Micro open sets namely, Micro semi open sets and Micro pre open sets were introduced by him. Chandrasekar and Swathi [2018] introduced Micro α-open sets which are weaker than Micro open sets in Micro topological spaces. They have also introduced the concept of Micro α-continuous map in Micro topological spaces.

The concept of Micro continuous, Micro semi-continuous and Micro pre-continuous maps were defined by Sakkraiveeranan Chandrasekar [2019]. Ibrahim [2020 b] introduced Micro generalized closed sets in Micro topological spaces. Using this concept many researchers have introduced and studied various types of Micro generalized closed sets. He introduced Micro $T_{1/2}$- space in Micro topological spaces.

Bhavani [2021] introduced strong forms of generalized closed sets called Micro $\ddot{g}$ - closed sets in Micro topological spaces. The concept of Micro α-generalized closed set was introduced by Anandhi and Balamani [2022 a] and its properties were analyzed by them. A generalization to the Micro continuous maps called Micro g-continuous was initiated by Taha et.al [2021]. They have also introduced Micro g- irresolute maps.

The present study focuses on the following concepts:

(i) Micro g^*-closed sets in Micro topological spaces
(ii) Micro g^*-continuous maps and Micro g^*-irresolute maps in Micro topological spaces

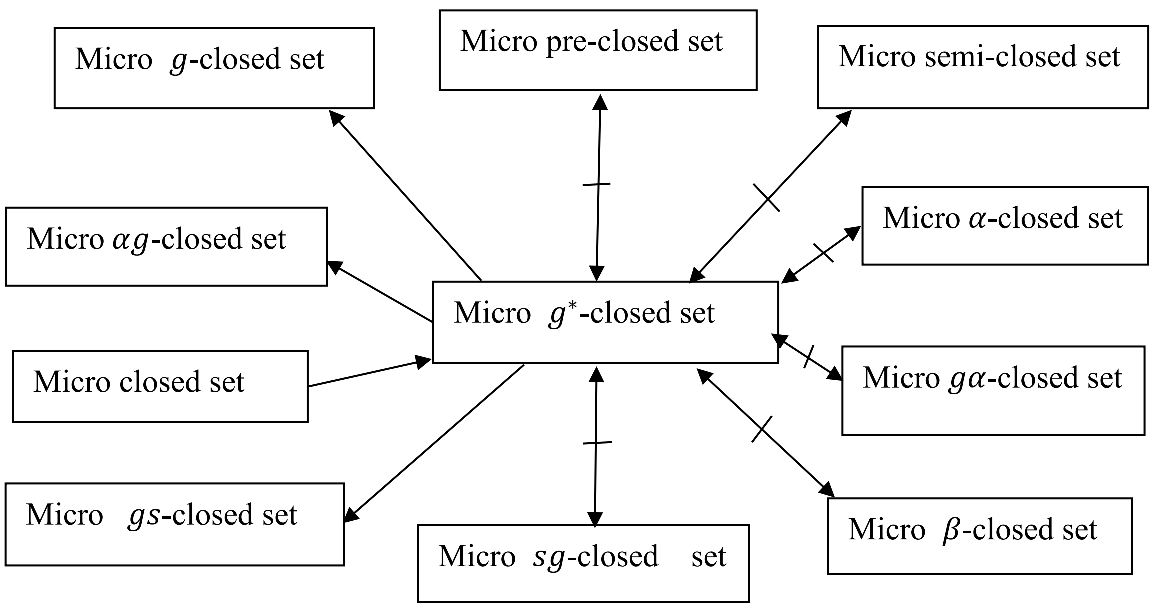
Chapter 1 deals with preliminary definitions in Micro topological spaces that are needed for the present study.

In **Chapter 2**, a new class of Micro generalized closed sets called Micro g^*-closed sets in Micro topological spaces is introduced. A comparison of Micro g^*-closed sets with other existing Micro closed sets, properties of Micro g^*-closed sets and characterization of Micro g^*-closed sets are studied. A similar study is also carried out for Micro g^*-open sets. Micro g^*-closure operator and Micro g^*-interior operator are defined and their properties are analyzed. As an application of Micro g^*-closed sets four new spaces namely, Micro T_b- space, Micro $T^*_{1/2}$-space, Micro ${}_{\alpha}T_c$- space and Micro ${}^*T_{1/2}$- space are introduced and their properties and interrelations are studied.

Important definitions and results:

A subset A of a Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be a **Micro g^*-closed set** if $Mic\text{-}cl(A) \subseteq L$ whenever $A \subseteq L$ and L is Micro g-open in U.

The following diagrams exhibit the relations between Micro g^*-closed sets with other existing Micro closed sets.



where A ⟶ B represents A implies B an A ⟷ B represents A and B are independent.

A Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be a

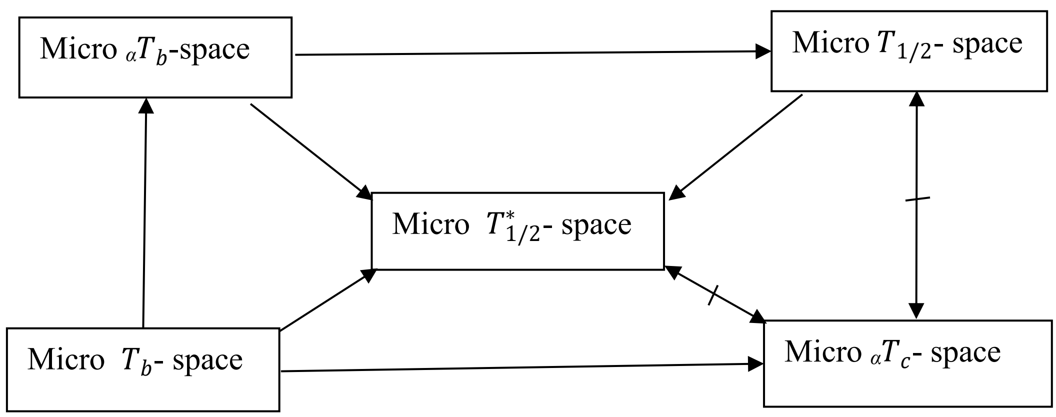
(i) Micro T_b- space (briefly Mic- T_b- space) if every Micro gs-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro closed in $(U, \tau_R(X), \mu_R(X)$.

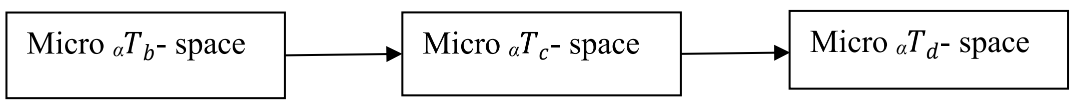
(ii) Micro $T^*_{1/2}$- space (briefly Mic- $T^*_{1/2}$- space) if every Micro g^*-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro closed in $(U, \tau_R(X), \mu_R(X))$.

(iii) Micro ${}_\alpha T_c$- space (briefly Mic- ${}_\alpha T_c$- space) if every Micro αg-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$.

(iv) Micro ${}^* T_{1/2}$- space (briefly Mic-${}^*T_{1/2}$-space) if every Micro g-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$.

The following diagram shows the dependency and independency relations of newly defined Micro spaces with already existing Micro spaces.





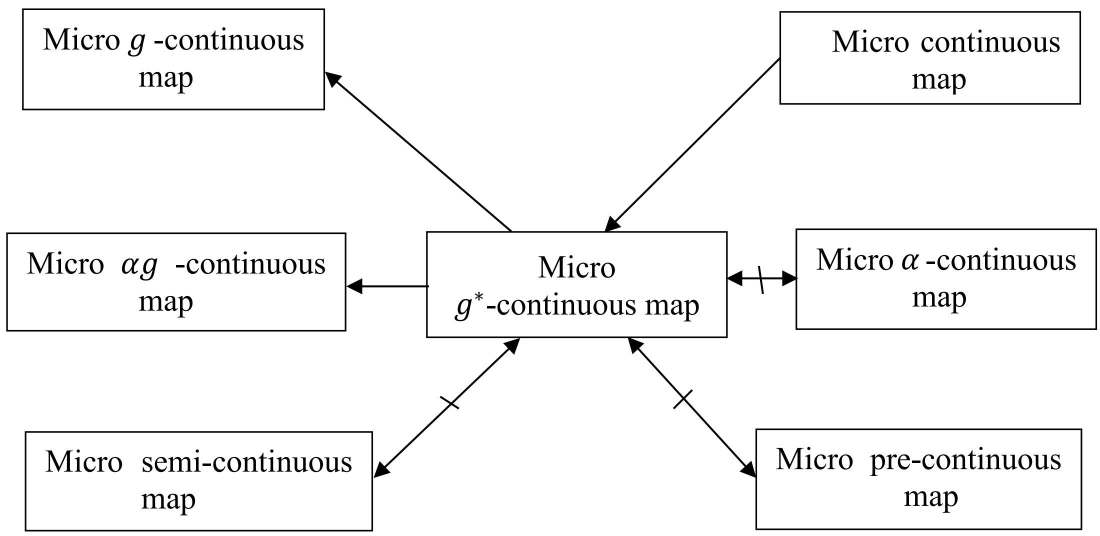
In **Chapter 3**, using Micro g^*-closed sets the concept of Micro g^*-continuity and Micro g^*-irresolute maps are defined and their properties and characterizations are discussed. The impact of newly defined Micro spaces with respect to Micro continuous mappings and composition of mappings are investigated.

Important definitions and results:

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ is called a **Micro g^*-continuous map** if $f^{-1}(P)$ is Micro g^*-closed in U for every Micro closed set P in V.

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ is called a **Micro g^*-irresolute map** if $f^{-1}(P)$ is Micro g^*-closed in U for every Micro g^*-closed set P in V.

The following diagram shows the interrelations between Micro g^*-continuous maps with already existing Micro continuous maps.



REVIEW OF LITERATURE

REVIEW OF LITERATURE

Topology is one of the widely studied areas of mathematics emerged through the works of the great mathematician Henri Poincare in the 19^{th} century. Topological structures on the collection of data are suitable mathematical models for mathematizing not only quantitative data but also qualitative data. Topological spaces show up naturally in almost every branch of Mathematics and that has made it as one of the great unifying ideas of Mathematics.

New form of topology called Nano topology was introduced by Carmel Richard [2013]. Nano means something very small. It comes from the Greek word 'nanos' which means "dwarf". The elements of the Nano Topology are called Nano-open sets. The notion of Nano Closure and Nano Interior are also introduced by him and also proved that the Nano closure operator is the Kuratowski closure operator.

Micro topology is the extension of Nano topology. Micro topology was introduced by Sakkraiveeranan Chandrasekar [2019]. Pawlak [2004] in the paper "Some Issues on Rough Sets" introduced the notion of lower approximation, upper approximation and boundary region of a subset of the universe. An equivalence relation known as indiscernibility relation is the mathematical basis for the theory. A rough set can be described by a pair of definable sets called lower and upper approximations. The lower approximations is the greatest definable set contained in the given set of objects, while the upper approximation is the smallest definable set containing the given set.

The concept of Nano topology was introduced by Carmel Richard [2013] in his research work entitled "Studies on Nano Topological Spaces". He established certain weak forms of Nano open sets namely, Nano α-open sets, Nano semi open sets and Nano pre-open sets. He also introduced strong form of Nano open sets called Nano θ-open sets. As a generalization, Nano generalized closed sets, Nano semi generalized closed sets and Nano θ-generalized closed sets were introduced by him. A brief study of Nano regular open sets was also made by him.

Sakkraiveeranan Chandrasekar [2019] introduced Micro topology and also he examined the concepts of Micro pre-open sets and Micro semi-open sets and derived some of their properties, in the paper "On Micro Topological Spaces". He also initiated the concept of Micro continuous, Micro pre-continuous and Micro semi-continuous maps in Micro topological spaces.

Chandrasekar and Swathi [2018] initiated the concept of Micro α-Open Sets in Micro topological spaces. They also studied about Micro α-continuity in Micro topological spaces. "On Micro α-Open Sets and Micro α-Continuous Functions in Micro Topological Spaces" was introduced by Reem O. Rasheed and Taha H. Jasim [2020] and investigated their properties.

Subramanian Jeyashri and Selvaraj Ganesan [2020] initiated Micro Generalized Locally Closed Sets in Micro topological spaces. They introduced mg-closure and mg-interior operators. Later, they have studied MLC-continuous, MGLC-continuous maps and MGLC-irresolute maps.

Micro β-Open Sets in Micro topology was introduced by Ibrahim [2020 a]. Different forms of β-open sets have also been studied by him.

In the article "On Micro $T_{1/2}$-Space" Ibrahim [2020 b] introduced Micro generalized closed sets and obtained some of its basic properties also he introduced a space called Micro $T_{1/2}$-space in which every Micro g-closed set is closed. The notions of Micro difference sets and Micro Kernel of sets are investigated and some of their properties are analysed. Later Taha H. Jasim et al. [2021] presented an article "On Micro generalized closed sets and Micro generalized continuity" and investigated the important properties of Micro generalized closed sets. They have also defined and analyzed Micro generalized continuous maps and Micro generalized irresolute maps.

In the article "On Strong Forms of Generalized Closed Sets in Micro Topological Spaces" Bhavani [2021] introduced various types of generalizations of Micro closed sets in Micro topological spaces namely Micro semi-generalized closed sets and Micro generalized semi-closed sets in Micro topological spaces.

Jamil Mahmoud Jamil [2021] introduced the concept of "Micro sω-open sets and sωρ-open sets in Micro Topological Spaces". Furthermore, he presented new functions by utilizing these sets namely Micro sω-continuous and Micro sωρ-continuous functions and some of the relations among these functions are analyzed.

θ-Continuity in Micro Topological Spaces was introduced by Rana H. Jassim et.al [2021]. They have studied the concepts of θ-Micro open sets and θ-Micro continuous maps.

Saja S. Mohsen [2021] introduced Micro-generalized Pre-minimal closed sets in Micro topological spaces. Some fundamental properties of these sets are also researched by him.

In the article " mω -Closed Sets in Micro Topological Spaces" by Selvaraj Ganesan [2021] were introduced mω-closed sets, Tmω-spaces and gTmω-spaces. Further he also introduced a new class of continuous map called mω-continuous and analyzed some of its properties.

Anandhi and Balamani [2022 a] initiated and studied the properties of Micro α-generalized closed sets in Micro topological spaces. They have derived the interrelations and properties of Micro αg - closed sets in the paper "Micro α-Generalized Closed Sets in Micro Topological Spaces". Further, Anandhi and Balamani [2022 b] studied separation axioms on Micro αg-closed sets. Anandhi and Balamani [2022 c] derived Micro αg-continuous maps and its properties in Micro topological spaces.

As an extension of Micro topology, N-Micro topology was introduced by Ekram Abdul-Kadir Saleh and Taha Hameed Jasim [2020]. They have studied N-Micro open set, Micro i-open set, N-Micro α-open set and N-Micro semi open set. They have also defined and studied N-Micro continuous functions.

Micro Ideal generalized closed sets in Micro Ideal topological spaces was initiated by Selvaraj Ganesan [2020]. He introduced a new type of generalized closed and open sets called $m\mathfrak{I}_g$ -closed set and $m\mathfrak{I}_g$-open set in Micro Ideal topological spaces and some of their properties are investigated.

CHAPTER 1

CHAPTER 1

PRELIMINARIES

Definition 1.1 [Pawlak, 2004]

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be in indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = U_{x \in U}\{R(x): R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by $x \in U$.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.That is $U_R(X) = \cup_{x \in U}\{R(x): R(x) \cap X \neq \phi\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.That is, $B_R(X) = U_R(X) - L_R(X)$

Definition 1.2 [Carmel Richard, 2013]

Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$
2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$
3. The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$

That is, $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets and the complement of a Nano open set is called a Nano closed set.

Definition 1.3 [Sakkraiveeranan Chandrasekar, 2019]

Let $(U, \tau_R(X))$ be a Nano topological space. Then $\mu_R(X) = \{N \cup (N' \cap \mu): N, N' \in \tau_R(X)\ and\ \mu \notin \tau_R(X)\}$ and $\mu_R(X)$ satisfies the following axioms:

1. U and $\phi \in \mu_R(X)$
2. The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$
3. The intersection of the elements of any finite sub-collection of $\mu_R(X)$ is in $\mu_R(X)$

Then $\mu_R(X)$ is called the Micro topology on U with respect to X. The triplet $(U, \tau_R(X), \mu_R(X))$ is called Micro topological space and the elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 1.4 [Sakkraiveeranan Chandrasekar, 2019]

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. The intersection of all Micro closed sets containing A is called Micro closure of A and is denoted by Mic- $cl(A)$. The union of all Micro open sets contained in A is called Micro interior of A and is denoted by Mic-int (A).

Definition 1.5 [Sakkraiveeranan Chandrasekar, 2019]

A subset A of a Micro topological space $(U, \tau_R(X), \mu_R(X))$ is called Micro pre-open (briefly Mic-pre-open) if $A \subseteq$ Mic-int (Mic-$cl(A)$) and Micro pre-closed (briefly Mic-pre-closed) if Mic-cl(Mic-$int(A)$) $\subseteq A$.

Definition 1.6 [Sakkraiveeranan Chandrasekar, 2019]

A subset A of a Micro topological space $(U, \tau_R(X), \mu_R(X))$ is called Micro semi-open (briefly Mic-semi-open) if $A \subseteq$ Mic- cl (Mic- $int(A)$) and Micro semi-closed (briefly Mic-semi-closed) if Mic- int(Mic-$cl(A)$) $\subseteq A$.

Definition 1.7 [Chandrasekar and Swathi, 2018]

A subset A of a Micro topological space $(U, \tau_R(X), \mu_R(X))$ is called Micro α-open (briefly Mic- α-open) if $A \subseteq$ Mic-int (Mic- cl (Mic-$int(A)$)) and Micro α-closed (briefly Mic- α-closed) if Mic-cl(Mic- int(Mic-$cl(A)$)) $\subseteq A$.

Definition 1.8 [Ibrahim, 2020 a]

A subset A of a Micro topological space $(U, \tau_R(X), \mu_R(X))$ is called Micro β-open (briefly Mic- β-open) if $A \subseteq$ Mic-cl(Mic-int (Mic-$cl(A)$)) and Micro β-closed (briefly Mic- β-closed) if Mic-int(Mic-cl(Mic-$int(A)$)) $\subseteq A$

.

Definition 1.9 [Ibrahim, 2020 b]

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro generalized closed (briefly Mic-g-closed) if $Mic\text{-}cl(A) \subseteq L$ whenever $A \subseteq L$ and L is Micro open in U.

Definition 1.10 [Ibrahim, 2020 b]

A Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be Micro $T_{1/2}$- space (briefly Mic- $T_{1/2}$- space) if every Micro g-closed subset of $(U, \tau_R(X), \mu_R(X))$ is closed in $(U, \tau_R(X), \mu_R(X))$.

Definition 1.11 [Bhavani, 2021]

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro semi-generalized closed (briefly Mic-sg-closed) if $Mic\text{-}scl(A) \subseteq L$ whenever $A \subseteq L$ and L is Micro semi-open in U.

Definition 1.12 [Bhavani, 2021]

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro generalized semi-closed (briefly Mic- gs -closed) if $Mic\text{ - }scl(A) \subseteq L$ whenever $A \subseteq L$ and L is Micro open in U.

Definition 1.13 [Bhavani, 2021]

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro $\ddot{g}$-closed (briefly Mic-$\ddot{g}$-closed) if $Mic\text{-}cl(A) \subseteq L$ whenever $A \subseteq L$ and L is Micro sg-open in U.

Definition 1.14 [Anandhi and Balamani, 2022 a]

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro generalized α-closed (briefly Mic- $g\alpha$ -closed) if Mic- $\alpha cl(A) \subseteq L$ whenever $A \subseteq L$ and L is Micro α-open in U.

Definition 1.15 [Anandhi and Balamani, 2022 a]

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro α-generalized closed (briefly Mic- αg -closed) if Mic- $\alpha cl(A) \subseteq L$ whenever $A \subseteq L$ and L is Micro open in U.

Definition 1.16 [Anandhi and Balamani, 2022 b]

A Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be Micro ${}_{\alpha}T_b$- space (briefly Mic-${}_{\alpha}T_b$-space) if every Micro αg-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro closed in $(U, \tau_R(X), \mu_R(X))$.

Definition 1.17 [Anandhi and Balamani, 2022 b]

A Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be Micro ${}_\alpha T_d$- space (briefly Mic-${}_\alpha T_d$-space) if every Micro αg-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro g-closed in $(U, \tau_R(X), \mu_R(X))$.

Definition 1.18 [Sakkraiveeranan Chandrasekar, 2019]

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ is called a Micro continuous map if $f^{-1}(P)$ is Micro open in U for every Micro open set P in V.

Definition 1.19 [Sakkraiveeranan Chandrasekar, 2019]

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ is called a Micro pre-continuous map if $f^{-1}(P)$ is Micro pre-closed in U for every Micro pre-closed set P in V.

Definition 1.20 [Sakkraiveeranan Chandrasekar, 2019]

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ is called a Micro semi-continuous map if $f^{-1}(P)$ is Micro semi-closed in U for every Micro closed set P in V.

Definition 1.21 [Ibrahim, 2020 a]

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ is called a Micro β-continuous map if $f^{-1}(P)$ is Micro β-closed in U for every Micro closed set P in V.

Definition 1.22 [Taha et. al., 2021]

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ is called a Micro g-continuous map if $f^{-1}(P)$ is Micro g-closed in U for every Micro closed set P in V.

Definition 1.23 [Reem O. Rasheed and Taha H. Jasim, 2021]

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ is called a Micro α-continuous map if $f^{-1}(P)$ is Micro α-closed in U for every Micro closed set P in V.

Definition 1.24 [Anandhi and Balamani, 2022 c]

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ is called a Micro αg-continuous map if $f^{-1}(P)$ is Micro αg-closed in U for every Micro closed set P in V.

Definition 1.25 [Taha et. al., 2021]

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ is called a Micro g-irresolute map if $f^{-1}(P)$ is Micro g-closed in U for every Micro g-closed set P in V.

Result 1.26

(i) Every Micro closed set is Micro α-closed. **[Chandrasekar and Swathi, 2018]**

(ii) A is a Micro closed set if and only if $A = Mic$ - $cl(A)$. **[Sakkraiveeranan Chandrasekar, 2019]**

(iii) Every Micro closed (open) set is Micro g-closed (g-open). **[Ibrahim, 2020 b]**

CHAPTER 2

CHAPTER 2

Micro g^*-Closed Sets in Micro Topological Spaces

2.1 Introduction

The Micro topology was introduced by Sakkraiveeranan Chandrasekar [2019], which is an extension of Nano topology. He introduced Micro pre-open sets and Micro semi-open sets in Micro topological spaces. Chandrasekar and Swathi [2018] introduced Micro α -open sets in Micro topological spaces. Ibrahim [2020 b] introduced Micro generalized closed sets in Micro topological spaces. Anandhi and Balamani [2022 a] initiated Micro αg-closed sets in Micro topological spaces.

In this chapter a new class of Micro generalized closed sets called Micro g^*-closed sets which contains the class of Micro closed sets and contained in the class of Micro g-closed sets is introduced in Micro topological spaces. Properties and characterizations are discussed and a comparative study between Micro g^*-closed sets and already existing various Micro closed sets is carried out. Also Micro g^*-open sets and its properties are studied. Moreover Micro closure and Micro interior operators are introduced and their properties are examined. Further four new Micro spaces namely, Micro T_b- space, Micro $T^*_{1/2}$- space, Micro ${}_\alpha T_c$- space and Micro ${}^*T_{1/2}$- space are introduced and their interrelations are derived.

2.2 Micro g^*-closed sets

In this section a new class of Micro generalized closed sets called Micro g^*-closed sets in Micro topological spaces is introduced and interrelation between this set and already existing various Micro closed sets are obtained.

Definition 2.2.1

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro g^*-closed if $Mic\text{-}cl(A) \subseteq L$ whenever $A \subseteq L$ and L is Micro g-open in U.

Example 2.2.2

Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b, c\}\}$. Let $X = \{a, b\} \subseteq U$. Then, Nano topology $\tau_R(X) = \{\phi, \{a\}, U\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then, Micro topology $\mu_R(X) = \{U, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then $\phi, \{c\}, \{a, c\}, \{b, c\}, U$ are Micro g^*-closed sets.

Proposition 2.2.3

Every Micro closed set is Micro g^*-closed but not conversely.

Proof: Let A be a Micro closed set and $A \subseteq L$, where L is Micro g-open in U. By Remark 2.11 (i), $Mic\text{-}cl(A) = A$. Since $A \subseteq L \Rightarrow Mic\text{-}cl(A) \subseteq L$ where L is Micro g-open. Hence A is Micro g^*-closed.

Example 2.2.4

Let $U = \{1,2,3,4\}$, $U/R = \{\{1\},\{3\},\{2,4\}\}$. Let $X = \{2,4\} \subseteq U$. Then, Nano topology $\tau_R(X) = \{U, \phi, \{2,4\}\}$. Let $\mu = \{1\} \notin \tau_R(X)$. Then, Micro topology $\mu_R(X) = \{\phi, \{1\}, \{2,4\}, \{1,2,4\}, U\}$. Micro closed sets are $\phi, \{3\}, \{1,3\}, \{2,3,4\}, U$. Micro g^*-closed sets are $\phi, \{3\}, \{1,3\}, \{2,3,\}, \{3,4\}, \{1,2,3\}, \{1,3,4\}, \{2,3,4\}, U$. Here the subset $\{2,3\}$ is Micro g^*-closed, but not Micro closed.

Proposition 2.2.5

Every Micro g^*-closed set is Micro g-closed but not conversely.

Proof: Let A be a Micro g^*-closed set and L be any Micro open set containing A in U. Since every Micro open set is Micro g-open and A is Micro g^*-closed, $Mic\text{-}cl(A) \subseteq L$. Hence A is Micro g-closed.

Example 2.2.6

Let $U = \{a,b,c\}$, $U/R = \{\{c\},\{a,b\}\}$. Let $X = \{a,b\} \subseteq U$. Then, Nano topology $\tau_R(X) = \{U, \phi, \{a,b\}\}$. Let $\mu = \{c\} \notin \tau_R(X)$. Then, Micro topology $\mu_R(X) = \{\phi, \{c\}, \{a,b\}, U\}$. Micro g^*-closed sets are $\phi, \{c\}, \{a,b\}, U$. Micro g-closed sets are $\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, U$. Here the subset $\{a\}$ is Micro g-closed but not Micro g^*-closed.

Proposition 2.2.7

Every Micro g^*-closed set is Micro αg-closed but not conversely.

Proof: Let A be a Micro g^*-closed set and L be any Micro open set containing A in U. Since every Micro open set is Micro g-open and A is Micro g^*-closed, Mic-$cl(A) \subseteq L$. For every subset A of U, Mic-$\alpha cl(A) \subseteq$ Mic- $cl(A)$ and so Mic-$\alpha cl(A) \subseteq L$. Hence A is Micro αg-closed.

Example 2.2.8

Let $U = \{a,b,c,d\}$, $U/R = \{\{c\},\{d\},\{a,b\}\}$. Let $X = \{a,b\} \subseteq U$. Then, Nano topology $\tau_R(X) = \{U, \phi, \{a,b\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, Micro topology $\mu_R(X) = \{\phi, \{a\}, \{a,b\}, U\}$. Micro g^*-closed sets are $\phi, \{c,d\}, \{a,c,d\}, \{b,c,d\}, U$. Micro αg -closed sets are

$\phi, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, U$. Here the subset $\{a, d\}$ is Micro αg-closed but not Micro g^*-closed.

Proposition 2.2.9

Every Micro g^*-closed set is Micro gs-closed but not conversely.

Proof: Let A be a Micro g^*-closed set and L be any Micro open set containing A in U. Since every Micro open set is Micro g-open and A is Micro g^*-closed, Mic-$cl(A) \subseteq L$. For every subset A of U, Mic-$scl(A) \subseteq$ Mic-$cl(A)$. Thus Mic-$scl(A) \subseteq L$. Hence A is Micro gs-closed.

Example 2.2.10

Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b, c\}\}$. Let $X = \{a, b\} \subseteq U$. Then, Nano topology $\tau_R(X) = \{U, \phi, \{a\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then, Micro topology $\mu_R(X) = \{\phi, \{a\}, \{b\}, \{a, b\}, U\}$. Micro g^*-closed sets are $\phi, \{c\}, \{a, c\}, \{b, c\}, U$. Micro gs-closed sets are $\phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, U$. Here the subset $\{a\}$ is Micro gs-closed but not Micro g^*-closed.

Remark 2.2.11

The following example shows that Micro g^*-closed set is independent from Micro pre-closed set.

Example 2.2.12

Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{c, d\} \subseteq U$. Then, Nano topology $\tau_R(X) = \{U, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, Micro topology $\mu_R(X) = \{\phi, \{a\}, \{c, d\}, \{a, c, d\}, U\}$. Micro pre-closed sets are $\phi, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, U$. Micro g^*-closed sets are $\phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, U$. Here the subset $\{c\}$ is Micro pre-closed but not Micro g^*-closed and the subset $\{a, b, d\}$ is Micro g^*-closed but not Micro pre-closed.

Remark 2.2.13

The following example shows that Micro g^*-closed set is independent from Micro semi-closed set.

Example 2.2.14

Let $U = \{a, b, c, d\}$, $U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{a, b, c\} \subseteq U$. Then, Nano topology $\tau_R(X) = \{U, \phi, \{a, b, c\}\}$. Let $\mu = \{a, b\} \notin \tau_R(X)$. Then, Micro topology $\mu_R(X) = \{\phi, \{a, b\}, \{a, b, c\}, U\}$. Micro semi-closed sets are $\phi, \{c,\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, U$. Micro g^*-closed sets are $\phi, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}, U$. Here the subset $\{c\}$ is Micro semi-closed but not Micro g^*-closed and the subset $\{a, d\}$ is Micro g^*-closed but not Micro semi-closed.

Remark 2.2.15

The following example shows that Micro g^*-closed set is independent from Micro α-closed set.

Example 2.2.16

Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{\text{c}\}, \{\text{b, d}\}\}$. Let $X = \{b, d\} \subseteq U$. Then, Nano topology $\tau_R(X) = \{U, \phi, \{b, d\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then, Micro topology $\mu_R(X) = \{\phi, \{b\}, \{b, d\}, U\}$. Micro α -closed sets are $\phi, \{a\}, \{c\}, \{d\}, \{a, c\}, \{c, d\}, \{a, c, d\}, U$. Micro g^* -closed sets are $\phi, \{a, c\}, \{a, b, c\}, \{a, c, d\}, U$. Here the subset $\{a\}$ is Micro α-closed but not Micro g^*-closed and the subset $\{a, b, c\}$ is Micro g^*-closed but not Micro α-closed.

Remark 2.2.17

The following example shows that Micro g^*-closed set is independent from Micro β-closed set.

Example 2.2.18

Let $U = \{a, b, c, d\}$, $U/R = \{\{c\}, \{\text{d}\}, \{\text{a, b}\}\}$. Let $X = \{c\} \subseteq U$. Then, Nano topology $\tau_R(X) = \{U, \phi, \{c\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then, Micro topology $\mu_R(X) = \{\phi, \{b\}, \{c\}, \{b, c\}, U\}$. Micro β -closed sets are $\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, U$. Micro g^*-closed sets are $\phi, \{a, d\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}, U$. Here the subset $\{a, b\}$ is Micro β-closed but not Micro g^*-closed and the subset $\{b, c, d\}$ is Micro g^*-closed but not Micro β-closed.

Remark 2.2.19

The following example shows that Micro g^*-closed set is independent from Micro $g\alpha$-closed set.

Example 2.2.20

Let $U = \{a, b, c\}$, $U/R = \{\{c\}, \{\text{a, b}\}\}$. Let $X = \{a, b\} \subseteq U$. Then, Nano topology $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, Micro topology $\mu_R(X) = \{\phi, \{a\}, \{a, b\}, U\}$. Micro $g\alpha$ -closed sets are $\phi, \{b\}, \{c\}, \{b, c\}, U$. Micro g^*-closed sets are $\phi, \{c\}, \{\text{a, c}\}, \{b, c\}, U$. Here the subset $\{b\}$ is Micro $g\alpha$ -closed but not Micro g^*-closed and the subset $\{a,c\}$ is Micro g^*-closed but not Micro $g\alpha$-closed.

Remark 2.2.21

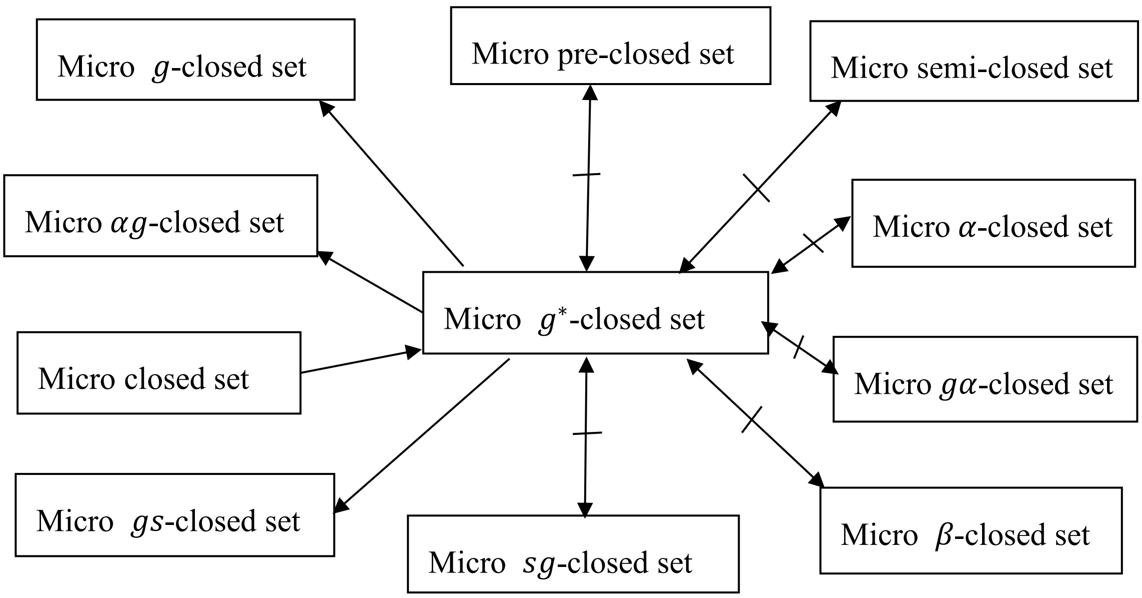
The following example shows that Micro g^*-closed set is independent from Micro sg-closed set.

Example 2.2.22

Let $U = \{a,b,c,d\}$, $U/R = \{\{a\},\{b\},\{c,d\}\}$. Let $X = \{c,d\} \subseteq U$. Then, Nano topology $\tau_R(X) = \{U,\phi,\{c,d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, Micro topology $\mu_R(X) = \{\phi,\{a\},\{c,d\},\{a,c,d\},U\}$. Micro g^*-closed sets are $\phi,\{b\},\{a,b\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{b,c,d\},U$. Micro sg-closed sets are $\phi,\{b\},\{a,b\},\{b,c\},\{a,b,c\},\{a,c,d\},\{a,b,d\},\{b,c,d\},U$. Here the subset $\{b,d\}$ is Micro g^*-closed but not Micro sg-closed and the subset $\{a,c,d\}$ is Micro sg-closed but not Micro g^*-closed.

Remark 2.2.23

The following diagram shows the dependency and independency relations of Micro g^*-closed sets with already existing Micro closed sets in Micro topological spaces.



2.3 Properties of Micro g^*-closed sets

In this section some of the fundamental properties of Micro g^*-closed sets are analyzed.

Theorem 2.3.1

If A and B be are Micro g^*-closed subsets of $(U, \tau_R(X), \mu_R(X))$ then $A \cup B$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$.

Proof: Let A and B be any two Micro g^*-closed sets in $(U, \tau_R(X), \mu_R(X))$ and L be any Micro g open set containing $A \cup B$. Then $A \subseteq L$ and $B \subseteq L$. Since A and B are Micro g^*-closed sets, $Mic\text{-}cl(A) \subseteq L$ and $Mic\text{-}cl(B) \subseteq L$. Always $Mic\text{-}cl(A \cup B) \subseteq Mic\text{-}cl(A) \cup Mic\text{-}cl(B) \subseteq L$. Hence $A \cup B$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$.

Remark 2.3.2

The intersection of any two Micro g^*-closed sets in $(U, \tau_R(X), \mu_R(X))$ need not be a Micro g^*-closed set in $(U, \tau_R(X), \mu_R(X))$ as seen from the following example.

Example 2.3.3

Let $U = \{a, b, c, d\}$, $U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{c\} \subseteq U$. Then, Nano topology $\tau_R(X) = \{U, \phi, \{c\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then, Micro topology $\mu_R(X) = \{\phi, \{b\}, \{c\}, \{b, c\}, U\}$. Micro g^*-closed sets are $\phi, \{a, d\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}, U$. Here the subsets $\{a, b, d\}$ and $\{b, c, d\}$ are Micro g^*-closed sets but their intersection $\{a, b, d\} \cap \{b, c, d\} = \{b, d\}$ is not Micro g^*-closed.

Theorem 2.3.4

Let A and B be subsets of $(U, \tau_R(X), \mu_R(X))$ such that $A \subseteq B \subseteq Mic\text{-}cl(A)$. If A is a Micro g^*-closed set in $(U, \tau_R(X), \mu_R(X))$, then B is also a Micro g^*-closed set in $(U, \tau_R(X), \mu_R(X))$.

Proof: Let A and B be subsets of $(U, \tau_R(X), \mu_R(X))$ such that $A \subseteq B \subseteq Mic\text{-}cl(A)$. Suppose that A is a Micro g^*-closed set. Let L be a Micro g-open set of $(U, \tau_R(X), \mu_R(X))$ such that $B \subseteq L$. Then $A \subseteq L$ and since A is Micro g^*-closed, $Mic\text{-}cl(A) \subseteq L$. Also since $B \subseteq Mic\text{-}cl(A)$, $Mic\text{-}cl(B) \subseteq Mic\text{-}cl(Mic\text{-}cl(A)) = Mic\text{-}cl(A) \subseteq L$. Hence $Mic\text{-}cl(B) \subseteq L$. Therefore B is a Micro g^*-closed set in $(U, \tau_R(X), \mu_R(X))$.

Theorem 2.3.5

If A is both Micro g-open and Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$, then A is Micro closed in $(U, \tau_R(X), \mu_R(X))$.

Proof: Let A be Micro g-open and Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$. Then by the definition of Micro g^*-closed set, $Mic\text{-}cl(A) \subseteq A$. Always $A \subseteq Mic\text{-}cl(A)$. Therefore $Mic\text{-}cl(A) = A$. Hence A is Micro closed.

Theorem 2.3.6

Let A be a Micro g^*-closed set of $(U, \tau_R(X), \mu_R(X))$ if and only if $Mic\text{-}cl(A) - A$ does not contain any non empty Micro g-closed set.

Proof: (Necessity) Let F be a Micro g-closed set of $(U, \tau_R(X), \mu_R(X))$ such that $F \subseteq Mic\text{-}cl(A) - A$. Then $A \subseteq U - F$. Since A is Micro g^*-closed and $U - F$ is Micro g-open, $Mic\text{-}cl(A) \subseteq U - F$. This implies $F \subseteq U - Mic\text{-}cl(A)$. So $F \subseteq (U - Mic\text{-}cl(A)) \cap (Mic\text{-}cl(A) - A) \subseteq (U - Mic\text{-}cl(A)) \cap Mic\text{-}cl(A) = \phi$. Therefore, $F = \phi$.

Sufficiency: Suppose that $Mic\text{-}cl(A) - A$ contains no non empty Micro g-closed set. Let $A \subseteq L$ and L be Micro g-open. If Mic- $cl(A)$ is not a subset of L then $Mic\text{-}cl(A) \cap L^c$ is a non empty Micro g-closed subset of $Mic\text{-}cl(A) - A$ which is a contradiction. Therefore, $Mic\text{-}cl(A) \subseteq L$ and hence A is Micro g^*-closed.

Theorem 2.3.7

Let A be a Micro g^*-closed set in $(U, \tau_R(X), \mu_R(X))$. Then $Mic\text{-}cl(A) - A$ contains no non empty Micro closed set.

Proof: Suppose that A is a Micro g^*-closed set in $(U, \tau_R(X), \mu_R(X))$. Let F be a Micro closed set contained in $Mic\text{-}cl(A) - A$ i.e., $F \subseteq Mic\text{-}cl(A) - A$. Now F^c is Micro open and hence Micro g-open in U such that $A \subseteq F^c$. Since A is Micro g^*-closed, $Mic\text{-}cl(A) \subseteq F^c$. Thus $F \subseteq [Mic\text{-}cl(A)]^c$. Also $F \subseteq Mic\text{-}cl(A) - A \Rightarrow F \subseteq Mic\text{-}cl(A)$. Therefore $F \subseteq [Mic\text{-}cl(A)]^c \cap [Mic\text{-}cl(A)] = \phi$. Hence $F = \phi$.

Theorem 2.3.8

If $Mic\text{-}cl(\{x\}) \cap A \neq \phi$ holds for every $x \in Mic\text{-}cl(A)$, then $Mic\text{-}cl(A) - A$ does not contain a non empty Micro closed set.

Proof: Suppose there exists a non empty Micro closed set F such that $F \subseteq Mic\text{-}cl(A) - A$. Let $x \in F$. Then $x \in Mic\text{-}cl(A)$. It follows that $F \cap A = Mic\text{-}cl(A) - A \cap A \supseteq Mic - cl(\{x\}) \cap A \neq \phi$. Hence $F \cap A \neq \phi$, which is a contradiction. Thus $F = \phi$.

Theorem 2.3.9

Let A be a Micro g^*-closed set in $(U, \tau_R(X), \mu_R(X))$. Then A is a Micro closed set if and only if $Mic\text{-}cl(A) - A$ is Micro g-closed.

Proof: (Necessity) Suppose that A is a Micro g^*-closed set. Let A be a Micro closed subset of $(U, \tau_R(X), \mu_R(X)$). Then $Mic\text{-}cl(A) = A$. Therefore $Mic\text{-}cl(A) - A = \phi$ is Micro g-closed.

Sufficiency: Let $Mic\text{-}cl(A) - A$ be a Micro g-closed set. Since A is Micro g^*-closed, by Theorem 2.3.7 $Mic\text{-}cl(A) - A$ contains no non empty Micro closed set which implies $Mic\text{-}cl(A) - A = \phi$. That is $Mic\text{-}cl(A) = A$. Hence A is Micro closed.

Theorem 2.3.10

Let $(U, \tau_R(X), \mu_R))$ be a Micro topological space. Then for each $\in U$, either $\{x\}$ is Micro g-open or $U - \{x\}$ is Micro g^*-closed.

Proof: Let $x \in U$ and suppose that $\{x\}$ is not Micro g-closed in $(U, \tau_R(X), \mu_R))$. Then $U - \{x\}$ is not Micro g-open in U. Hence U is the only Micro g-open set containing $U - \{x\}$. That is $U - \{x\} \subseteq U$. Therefore $Mic\text{-}cl\ (U - \{x\}) \subseteq U$ which implies that $U - \{x\}$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$.

2.4 Micro g^*-closure operator

In this section the concept of Micro g^*-closure operator is introduced and its basic properties are studied.

Definition 2.4.1

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space and $A \subseteq U$. Then the Micro g^*-closure of a set A is denoted by $Mic\text{-}g^*\text{-}cl(A)$ and is defined as $Mic\text{-}g^*\text{-}cl(A) = \cap$ {B: B is a Micro-g^*-closed set in U and $A \subseteq B\}$.

Remark 2.4.2

For a subset A of a Micro topological space $(U, \tau_R(X), \mu_R(X))$, $A \subseteq Mic\text{-}g^*\text{-}cl(A) \subseteq Mic\text{-}cl(A)$.

Proof: Follows Proposition 2.2.3 and Definition 2.4.1

Remark 2.4.3

Both inclusion relations in Remark 2.4.2 may be proper as seen from the following example.

Example 2.4.4

Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{c, d\} \subseteq U$. Then, Nano topology $\tau_R(X) = \{U, \phi, \{c, d\}\}$. Let $= \{a\} \notin \tau_R(X)$. Then, Micro topology $\mu_R(X) = \{\phi, \{a\}, \{c, d\}, \{a, , c, d\}, U\}$. Let $A = \{d\}$. Then $Mic\text{-}g^*\text{-}cl(A) = \{a, d\}$ and $Mic\text{-}cl(A) = \{b, c, d\}$ and so $A \subset Mic\text{-}g^*\text{-}cl(A) \subset Mic\text{-}cl(A)$.

Proposition 2.4.5

Let A and B be any two subsets of $(U, \tau_R(X), \mu_R(X))$. Then the following statements are true

(a) $Mic\text{-}g^*\text{-}cl(\phi) = \phi$ and $Mic\text{-}g^*\text{-}cl(U) = U$

(b) $A \subseteq Mic\text{-}g^*\text{-}cl(A)$

(c) If B is any Micro g^*-closed set containing A, then $Mic\text{-}g^*\text{-}cl(A) \subseteq B$

(d) If $A \subseteq B$, then $Mic\text{-}g^*\text{-}cl(A) \subseteq Mic\text{-}g^*\text{-}cl(B)$

(e) $Mic\text{-}g^*\text{-}cl(A)\, U\; Mic\text{-}g^*\text{-}cl(B) \subseteq Mic\text{-}g^*\text{-}cl\,(A\, U\, B)$

(f) $Mic\text{-}g^*\text{-}cl(A \cap B) \subseteq Mic\text{-}g^*\text{-}cl(A) \cap Mic\text{-}g^*\text{-}cl(B)$

(g) $Mic\text{-}g^*\text{-}cl(Mic\text{-}g^*\text{-}cl(A)) = Mic\text{-}g^*\text{-}cl(A)$

Proof: (a) Follows from the Definition 2.4.1

(b) By the definition of $Mic\text{-}g^*\text{-}cl(A)$, it is obvious that $A \subseteq Mic\text{-}g^*\text{-}cl(A)$.

(c) Let B be any Micro g^*-closed set containing A. Since $Mic\text{-}g^*\text{-}cl(A)$ is the intersection of all Micro g^*-closed sets containing A, $Mic\text{-}g^*\text{-}cl(A)$ is contained in every Micro g^*-closed set containing A. Hence $Mic\text{-}g^*\text{-}cl(A) \subseteq B$.

(d) Follows from the Definition 2.4.1

(e) Since $A \subseteq A\, U\, B$ and $B \subseteq A\, U\, B$ by (d) $Mic\text{-}g^*\text{-}cl(A) \subseteq Mic\text{-}g^*\text{-}cl(A\, U\, B)$ and $Mic\text{-}g^*\text{-}cl(B) \subseteq Mic\text{-}g^*\text{-}cl(A\, U\, B)$. Hence $Mic\text{-}g^*\text{-}cl(A)\, U\, Mic\text{-}g^*\text{-}cl(B) \subseteq Mic\text{-}g^*\text{-}cl(A\, U\, B)$.

(f) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$ by (d) $Mic\text{-}g^*\text{-}cl(A \cap B) \subseteq Mic\text{-}g^*\text{-}cl(\mathrm{A})$ and $Mic\text{-}g^*\text{-}cl(A \cap B) \subseteq Mic\text{-}g^*\text{-}cl(B)$. Hence $Mic\text{-}g^*\text{-}cl(A \cap B) \subseteq Mic\text{-}g^*\text{-}cl(A) \cap Mic\text{-}g^*\text{-}cl(B)$.

(g) Follows from the definition 2.4.1.

Remark 2.4.6

Since $Mic\text{-}g^{*}\text{-}cl(A) \cup Mic\text{-}g^{*}\text{-}cl(B) \neq Mic\text{-}g^{*}\text{-}cl(A \cup B)$, Micro g^{*}closure is not a Kuratowski closure operator on $(U, \tau_R(X), \mu_R(X))$.

Remark 2.4.7

In general, the reverse inclusion of (f) in Proposition 2.4.5 is not true.

Example 2.4.8

Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{c, d\} \subseteq U$. Then, Nano topology $\tau_R(X) = \{U, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, Micro topology $\mu_R(X) = \{\phi, \{c\}, \{c, d\}, \{a,, c, d\}, U\}$. If $A = \{a\}$ and $B = \{b\}$ then $Mic\text{-}g^{*}\text{-}cl(A) = \{a, b\}$ and $Mic\text{-}g^{*}\text{-}cl(B) = \{b\}$, $A \cap B = \phi$, $Mic\text{-}g^{*}\text{-}cl(A \cap B) = \phi$. But $Mic\text{-}g^{*}\text{-}cl(A) \cap Mic\text{-}g^{*}\text{-}cl(B) = \{b\}$.

Proposition 2.4.9

Let A be any subset of $(U, \tau_R(X), \mu_R(X))$. If A is Mic-g^{*}-closed in $(U, \tau_R(X), \mu_R(X))$ then $Mic\text{-}g^{*}\text{-}cl(A) = A$.

Proof: Assume that a set A is Mic-g^{*}-closed in $(U, \tau_R(X), \mu_R(X))$. By Definition 2.4.1, $A \subseteq Mic\text{-}g^{*}\text{-}cl(A)$. Also A is a Micro g^{*}-closed set containing A, From Proposition 2.4.5 (c), $Mic\text{-}g^{*}\text{-}cl(A) \subseteq A$. Hence $Mic\text{-}g^{*}\text{-}cl(A) = A$.

Remark 2.4.10

Converse part of Proposition 2.4.9 need not be true as observed from the following example.

Example 2.4.11

Let $U = \{a, b, c, d\}$, $U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{c\} \subseteq U$. Then, Nano topology $\tau_R(X) = \{U, \phi, \{c\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then, Micro topology $\mu_R(X) = \{\phi, \{b\}, \{c\}, \{b, c\}, U\}$. If $A = \{c, d\}$ then $Mic\text{-}g^{*}\text{-}cl(\{c, d\}) = \cap$ {all Micro g^{*}-closed sets containing $\{c, d\}$} $= \{a, c, d\} \cap \{b, c, d\} \cap U = \{c, d\}$ but A is not Micro g^{*}-closed.

Remark 2.4.12

Let $A \subseteq U$. Then $Mic\text{-}g^{*}\text{-}cl(A)$ need not be Micro g^{*}-closed.

Example 2.4.13

Let $U = \{a, b, c, d\}$, $U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{c\} \subseteq U$. Then, Nano topology $\tau_R(X) = \{U, \phi, \{c\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then, Micro topology $\mu_R(X) = \{\phi, \{b\}, \{c\}, \{b, c\}, U\}$. Let $A = \{b\}$. $Mic\text{-}g^*\text{-}cl(\{b\}) = \cap$ {all Micro g^*-closed sets containing $\{b\}\} = \{a, b, d\} \cap \{b, c, d\} \cap U = \{b\} \neq$ Micro g^*-closed.

2.5 Micro g^*-open sets

In this section the concept of Micro g^*-open sets is introduced and its properties are analyzed.

Definition 2.5.1

A subset A of a Micro topological space $(U, \tau_R(X), \mu_R(X))$ is called Micro g^*-open (briefly Mic-g^*-open) if its complement A^c is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$.

Example 2.5.2

Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$. Let $X = \{a, b\} \subseteq U$. Then, Nano topology $\tau_R(X) = \{U, \phi, \{a\}\{b, d\}, \{a, b, d\}\}$. Let $\mu = \{c\} \notin \tau_R(X)$. Then, Micro topology $\mu_R(X) = \{\phi, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{b, c, d\}, \{a, b, d\}, U\}$. Then $\phi, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, b, d\}, \{b, c, d\}, U$ are Micro g^*-open sets.

Proposition 2.5.3

Every Micro open set in $(U, \tau_R(X), \mu_R(X))$ is Micro g^*-open but not conversely.

Proof: Let A be a Micro open set in U. Then A^c is Micro closed in U. By Proposition 2.2.3, A^c is Micro g^*-closed in U. Hence A is Micro g^*-open in U.

Example 2.5.4

Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b, c\}\}$. Let $X = \{a, b\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi\}$. Let $\mu = \{a, b\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a, b\}, U\}$. Micro open sets are $\phi, \{a, b\}, U$. Micro g^*-open sets are $\phi, \{a\}, \{b\}, \{a, b\}, U$. Here the subset $\{a\}$ is Micro g^*-open but not Micro open.

Proposition 2.5.5

Every Micro g^*-open set in $(U, \tau_R(X), \mu_R(X))$ is Micro g-open but not conversely.

Proof: Let A be a Micro g^*-open set in U. Then A^c is Micro g^*-closed in U. By Proposition 2.2.5, A^c is Micro g-closed in U. Hence A is Micro g-open in U.

Example 2.5.6

Let $U = \{a, b, c, d\}$, $U/R = \{\{c\}, \{d\}, \{a, b\}, \}$. Let $X = \{a, b\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a\}, \{a, b\}, U\}$. Micro g^* -open sets are $\phi, \{a\}, \{b\}, \{a, b\}, U$. Micro g -open sets are $\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, U$. Here the subset $\{c\}$ is Micro g-open but not Micro g^*-open.

Proposition 2.5.7

Every Micro g^*-open set is Micro αg-open.

Proposition 2.5.8

Every Micro g^*-open set is Micro gs-open.

Lemma 2.5.9 For a subset A of $(U, \tau_R(X), \mu_R(X))$, $Mic\text{-}cl(U - A) = U - Mic\text{-}int(A)$.

Theorem 2.5.10

A subset A of a Micro topological space $(U, \tau_R(X), \mu_R(X))$ is Micro g^*-open if and only if $L \subseteq Mic\text{-}int(A)$ whenever $L \subseteq A$ and L is Micro g-closed.

Proof : Assume that A is Micro g^*-open. Then A^c is Micro g^* -closed. Let L be a Micro g-closed set in U contained in A. Then L^c is a Micro g-open set in U containing A^c. Since A^c is Micro g^* -closed, $Mic\text{-}cl(A^c) \subseteq L^c$ equivalently $L \subseteq Mic\text{-}int(A)$.

Conversely assume that L is contained in $Mic\text{-}int(A)$ whenever L is contained in A and L is Micro g-closed in U. Let A^c be contained in L, where L is Micro g-open. Then L^c is contained in A. By hypothesis $L^c \subseteq Mic\text{-}int(A)$. This implies $[Mic\text{-}int(A)]^c \subseteq L$. That is $Mic\text{-}cl(A^c) \subseteq$ L. Therefore A^c is Micro g^* -closed. Hence A is Micro g^*-open in U.

Theorem 2.5.11

If A and B are Micro g^*-open sets in $(U, \tau_R(X), \mu_R(X))$, then $A \cap B$ is Micro g^*-open in $(U, \tau_R(X), \mu_R(X))$.

Proof : Let A and B be Micro g^*-open sets in $(U, \tau_R(X), \mu_R(X))$ Then $U - A$ and $U - B$ are Micro g^* -closed sets and $(U - A) \cup (U - B) = U - (A \cap B)$ is Micro g^* -closed. Hence $A \cap B$ is Micro g^*-open in U.

Theorem 2.5.12

Let A be Micro g^*-open in $(U, \tau_R(X), \mu_R(X))$. If G is Micro open and $Mic\text{-}int(A) \cup A^c \subseteq G$, then $G = U$.

Proof: Let A be a Micro g^*-open set in $(U, \tau_R(X), \mu_R(X))$ and G be Micro open and $Mic\text{-}int(A) \cup A^c \subseteq G$. This gives $G^c \subseteq [Mic\text{-}int(A) \cup A^c]^c = [Mic\text{-}int(A)]^c \cap A = [Mic\text{-}int(A)]^c - A^c = [Mic\text{-}cl(A^c) - A^c$. Since A^c is Micro g^*-closed and G^c is Micro closed by Theorem 4.6, it follows that $G^c = \phi$. Therefore $G = U$.

2.6 Micro g^*-interior operator

In this section the concept of Micro g^*-interior operator is introduced and its basic properties are studied.

Definition 2.6.1

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space and $A \subseteq U$. Then the Micro g^*-interior of a set A is denoted by $Mic\text{-}g^*\text{-}int(A)$ and is defined as $Mic\text{-}g^*\text{-}int(A) = \cup\{$B: B is Micro g^*-open in U and $A \supseteq B\}$.

Remark 2.6.2

For a subset A of a Micro topological space $(U, \tau_R(X), \mu_R(X))$, $Mic\text{-}int(A) \subseteq Mic\text{-}g^*\text{-}int(A) \subseteq A$.

Proposition 2.6.3

Let A and B be any two subsets of $(U, \tau_R(X), \mu_R(X))$. Then the following statements are true

(a) $Mic\text{-}g^*\text{-}int(\phi) = \phi$ and $Mic\text{-}g^*\text{-}int(U) = U$.

(b) $Mic\text{-}g^*\text{-}int(A) \subseteq A$.

(c) If B is any Micro g^*-open set contained in A, then $B \subseteq Mic\text{-}g^*\text{-}int(A)$.

(d) If $A \subseteq B$ then $Mic\text{-}g^*\text{-}int(A) \subseteq Mic\text{-}g^*\text{-}int(B)$.

(e) $Mic\text{-}g^*\text{-}int(A \cap B) \subseteq Mic\text{-}g^*\text{-}int(A) \cap Mic\text{-}g^*\text{-}int(B)$.

(f) $Mic\text{-}g^*\text{-}int(A \cup B) \supseteq Mic\text{-}g^*\text{-}int(A) \cup Mic\text{-}g^*\text{-}int(B)$.

(g) $Mic\text{-}g^*\text{-}int[Mic\text{-}g^*\text{-}int(A)] = Mic\text{-}g^*\text{-}int(A)$.

Proposition 2.6.4

Let A be any subset of $(U, \tau_R(X), \mu_R(X))$. If A is Mic- g^* -open in $(U, \tau_R(X), \mu_R(X))$ then $Mic\text{-}g^*\text{-}int(A) = A$.

Remark 2.6.5

Converse part of Proposition 2.6.4 need not be true as seen from the following example.

Example 2.6.6

Let $U = \{a, b, c, d\}$, $U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{c\} \subseteq U$. Then $\tau_R(X) = \{\phi, \{b\}, \{c\}, \{b, c\}, U\}$. $Mic\text{-}g^*\text{-}int(\{a, b\}) = \cup$ {all Micro g^*-open sets contained in $\{a, b\}\} = \{a\} \cup \{b\} \cup \phi = \{a, b\}$. But $\{a, b\}$ is not Micro g^* -open in $(U, \tau_R(X), \mu_R(X))$.

2.7 Application of Micro g^*-closed Sets

As an application of Micro g^*-closed sets four new spaces namely, Micro T_b- space, Micro $T^*_{1/2}$- space, Micro ${}_{\alpha}T_c$- space and Micro ${}^*T_{1/2}$- space are introduced and their properties and interrelations between newly defined Micro spaces and already existing Micro spaces are obtained.

Definition 2.7.1

A Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be a

(i) Micro T_b- space (briefly Mic-T_b- space) if every Micro gs-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro closed in $(U, \tau_R(X), \mu_R(X))$.

(ii) Micro $T^*_{1/2}$- space (briefly Mic-$T^*_{1/2}$- space) if every Micro g^*-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro closed in $(U, \tau_R(X), \mu_R(X))$.

(iii) Micro ${}_\alpha T_c$- space (briefly Mic-${}_\alpha T_c$- space) if every Micro αg-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$.

(iv) Micro ${}^*T_{1/2}$- space (briefly Mic-${}^*T_{1/2}$-space) if every Micro g-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$.

Proposition 2.7.2

Every Micro $T_{1/2}$- space is a Micro $T^*_{1/2}$- space but not conversely.

Proof: Assume that a set A is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$. Since every Micro g^*-closed set is Micro g-closed and $(U, \tau_R(X), \mu_R(X))$ is a Micro $T_{1/2}$- space, A is Micro-closed in $(U, \tau_R(X), \mu_R(X))$. Hence $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$- space.

Example 2.7.3

Let $U = \{a, b, c\}$, $U/R = \{\{a, b, c\}\}$. Let $X = \{b, c\} \subseteq U$. Then $\tau_R(X) = \{U, \phi\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a\}, U\}$. Micro-closed sets are $\phi, \{b, c\}, U$. Micro g-closed sets are $\phi, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, U$. Micro g^*-closed sets are $\phi, \{b, c\}, U$. Here the space $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$- space but not a Micro $T_{1/2}$- space, since the subset $\{b\}$ is Micro g-closed but not Micro-closed.

Proposition 2.7.4

Every Micro ${}_\alpha T_b$- space is a Micro $T^*_{1/2}$- space but not conversely.

Proof: Assume that a set A is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$. Since every Micro g^*-closed set is Micro αg-closed and $(U, \tau_R(X), \mu_R(X))$ is a Micro ${}_\alpha T_b$- space, A is Micro-closed in $(U, \tau_R(X), \mu_R(X))$. Hence $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$-space.

Example 2.7.5

Let $U = \{a, b, c\}$, $U/R = \{\{a, b, c\}\}$. Let $X = \{b, c\} \subseteq U$. Then $\tau_R(X) = \{U, \phi\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a\}, U\}$. Micro-closed sets are $\phi, \{b, c\}, U$. Micro g^*-closed sets are $\phi, \{b, c\}, U$. Micro αg-closed sets are $\phi, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, U$. Here the space $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$- space but not a Micro ${}_\alpha T_b$- space, since the subset $\{a, b\}$ is Micro αg-closed but not Micro closed.

Proposition 2.7.6

Every Micro T_b- space is a Micro $T^*_{1/2}$- space but not conversely.

Proof: Assume that a set A is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$. Since every Micro g^*-closed set is Micro gs-closed and $(U, \tau_R(X), \mu_R(X))$ is a Micro T_b- space, A is Micro-closed in $(U, \tau_R(X), \mu_R(X))$. Hence $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$-space.

Example 2.7.7

Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b, c\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{\phi, \{a\}, U\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{U, \phi, \{a\}, \{b\}, \{a, b\}\}$. Micro-closed sets are $\phi, \{c\}, \{a, c\}, \{b, c\}, U$. Micro gs-closed sets are $\phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, U$. Micro g^*-closed sets are $\phi, \{c\}, \{a, c\}, \{b, c\}, U$. Here the space $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$- space but not a Micro T_b- space, since the subset $\{a\}$ is Micro gs-closed but not Micro-closed.

Proposition 2.7.8

Every Micro T_b- space is a Micro ${}_{\alpha}T_b$- space but not conversely.

Proof: Let A be a Micro αg-closed set in $(U, \tau_R(X), \mu_R(X))$. Since every Micro αg-closed set is Micro gs-closed and $(U, \tau_R(X), \mu_R(X))$ is a Micro T_b- space, A is Micro-closed in $(U, \tau_R(X), \mu_R(X))$. Hence $(U, \tau_R(X), \mu_R(X))$ is a Micro ${}_{\alpha}T_b$- space.

Example 2.7.9

Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b, c\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{\phi, \{a\}, U\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{U, \phi, \{a\}, \{b\}, \{a, b\}\}$. Micro closed sets are $\phi, \{c\}, \{a, c\}, \{b, c\}, U$. Micro gs-closed sets are $\phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, U$. Micro αg-closed sets are $\phi, \{c\}, \{a, c\}, \{b, c\}, U$. Here the space $(U, \tau_R(X), \mu_R(X))$ is a Micro ${}_{\alpha}T_b$- space but not a Micro T_b- space, since the subset $\{a\}$ is Micro gs-closed but not Micro closed.

Proposition 2.7.10

If $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$-space, then Mic-$g^*cl(B)$= Mic-$cl(B)$ for each subset B of $(U, \tau_R(X), \mu_R(X))$.

Proof: Since $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$-space and every Micro closed set is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$, Mic- $g^*C(U, \tau_R(X), \mu_R(X))$ = Mic-$C(U, \tau_R(X), \mu_R(X))$. Hence Mic-$g^*cl(B)$= Mic-$cl(B)$ for each subset B of $(U, \tau_R(X), \mu_R(X))$.

Theorem 2.7.11

If $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$ -space, then for each $x \in U$ either $\{x\}$ is Micro g-closed or Micro open.

Proof: Let $x \in U$ and suppose $\{x\}$ is not Micro g-closed in $(U, \tau_R(X), \mu_R(X))$. Then $U - \{x\}$ is not Micro open. Hence U is the only Micro g-open set containing U–$\{x\}$. So $U - \{x\}$ is a Micro g^*-closed set in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$-space, $U - \{x\}$ is a Micro closed or $\{x\}$ is Micro open in $(U, \tau_R(X), \mu_R(X))$.

Theorem 2.7.12

For a space $(U, \tau_R(X), \mu_R(X))$, the following conditions are equivalent

(i) $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$-space

(ii) For each $x \in U$, $\{x\}$ is either Micro g-closed or Micro open.

Proof: (i) $\Rightarrow$ (ii) Let $x \in U$ and suppose $\{x\}$ is not a Micro g-closed set of $(U, \tau_R(X), \mu_R(X))$. Then $U - \{x\}$ is not Micro g- open. Hence U is the only Micro g - open set containing $U - \{x\}$. So $U - \{x\}$ is a Micro g^*-closed set in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$-space, $U - \{x\}$ is Micro closed or equivalently $\{x\}$ is Micro open in $(U, \tau_R(X), \mu_R(X))$.

(ii) $\Rightarrow$ (i) Let A be a Micro g^*-closed set in $(U, \tau_R(X), \mu_R(X))$ and $x \in$ Mic-$cl(A)$. We show that $x \in A$ for the following two cases.

Case 1: Assume that $\{x\}$ is Micro open. Then $U - \{x\}$ is Micro closed. If $x \notin A$, then $A \subseteq U - \{x\}$. Since $x \in$ Mic- $cl(A)$, we have $x \in U - \{x\}$, which is a contradiction. Hence $x \in A$.

Case 2: Assume that $\{x\}$ is Micro g-closed and $x \notin A$. Then Mic- $cl(A) - A$ contains a non-empty Micro g-closed set $\{x\}$. This contradicts Theorem 2.3.6 as A is a Micro g^*-closed set. Therefore $x \in A$.

Proposition 2.7.13

Every Micro ${}_\alpha T_b$- space is a Micro ${}_\alpha T_c$- space but not conversely.

Proof: Let A be a Micro αg-closed set in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro ${}_\alpha T_b$ -space, A is Micro closed. Since every Micro closed set is Micro g^*-closed, A is Micro g^*-closed. Therefore $(U, \tau_R(X), \mu_R(X))$ is a Micro ${}_\alpha T_c$- space.

Example 2.7.14

Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b, c\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{\phi, U\}$. Let $\mu = \{a, b\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a, b\}, U\}$. Micro closed sets are $\phi, \{c\}, U$. Micro g^*-closed sets are $\phi, \{c\}, \{a, c\}, \{b, c\}, U$. Micro αg-closed sets are $\phi, \{c\}, \{a, c\}, \{b, c\}, U$. Here the space $(U, \tau_R(X), \mu_R(X))$ is a Micro ${}_\alpha T_c$- space but not a Micro ${}_\alpha T_b$- space, since the subset $\{a, c\}$ is Micro αg-closed but not Micro closed.

Proposition 2.7.15

Every Micro ${}_\alpha T_c$- space is a Micro ${}_\alpha T_d$- space but not conversely.

Proof: Let A be a Micro αg-closed set in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro ${}_\alpha T_c$- space, A is Micro g^*-closed. Since every Micro g^*-closed set is Micro g-closed, A is Micro g-closed. Therefore $(U, \tau_R(X), \mu_R(X))$ is a Micro ${}_\alpha T_d$- space.

Example 2.7.16

Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b, c\}\}$. Let $X = \{b, c\} \subseteq U$. Then $\tau_R(X) = \{U, \phi\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a\}, U\}$. Micro g-closed sets are $\phi, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, U$. Micro g^*-closed sets are $\phi, \{b, c\}, U$. Micro αg-closed sets are $\phi, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, U$. Here the space $(U, \tau_R(X), \mu_R(X))$ is a Micro ${}_\alpha T_d$- space but not a Micro ${}_\alpha T_c$- space, since the subset $\{c\}$ is Micro αg-closed but not Micro g^*-closed.

Proposition 2.7.17

Every Micro $T_{1/2}$- space is a Micro ${}^*T_{1/2}$- space but not conversely.

Proof: Let A be a Micro g-closed set in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro $T_{1/2}$- space, A is Micro closed. Since every Micro closed set is Micro g^*-closed, A is Micro g^*-closed. Therefore $(U, \tau_R(X), \mu_R(X))$ is a Micro ${}^*T_{1/2}$- space.

Example 2.7.18

Let $U = \{a, b, c\}$, $U/R = \{\{c\}, \{a, b\}\}$. Let $X = \{a, b\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a\}, \{a, b\}, U\}$. Micro closed sets are $\phi, \{c\}, \{b, c\}, U$. Micro g-closed sets are $\phi, \{c\}, \{a, c\}, \{b, c\}, U$. Micro g^*-closed sets are $\phi, \{c\}, \{a, c\}, \{b, c\}, U$. Here the space $(U, \tau_R(X), \mu_R(X))$

is a ${}^*T_{1/2}$- space but not a Micro $T_{1/2}$- space, since the subset $\{a,c\}$ is Micro g-closed but not Micro closed.

Proposition 2.7.19

Every Micro ${}_{\alpha}T_b$- space is a Micro $T_{1/2}$- space.

Proof: Let A be a Micro g-closed set in $(U,\tau_R(X),\ \mu_R(X))$. Since every Micro g-closed set is Micro αg-closed, A is Micro αg-closed and since $(U,\tau_R(X),\ \mu_R(X))$ is a Micro ${}_{\alpha}T_b$ -space, A is Micro closed in $(U,\tau_R(X),\ \mu_R(X))$. Hence $(U,\tau_R(X),\ \mu_R(X))$ is a Micro $T_{1/2}$- space.

Theorem 2.7.20

If $(U,\tau_R(X),\ \mu_R(X))$ is a Micro ${}_{\alpha}T_c$- space, then for each $x\in U$, $\{x\}$ is either Micro αg-closed or Micro g^*-open.

Proof: Let $x\in U$ and suppose $\{x\}$ is not Mic αg-closed set of $(U,\tau_R(X),\ \mu_R(X))$. Then $\{x\}$ is not a Micro closed set. Since every Micro-closed set is Mic-αg-closed set. So $U-\{x\}$ is not a Micro open set. Therefore $U-\{x\}$ is a Mic-αg-closed set. Since U is the only Micro open set which contains $U-\{x\}$. Since $(U,\tau_R(X),\ \mu_R(X))$ is a Micro ${}_{\alpha}T_c$- space, then $U-\{x\}$ is a Micro g^*-closed set or equivalently $\{x\}$ is Micro g^*-open.

Proposition 2.7.21

Micro ${}_{\alpha}T_c$- space and Micro $T^*_{1/2}$- space are independent as seen from the following example.

Example 2.7.22

Let $U=\{a,b,c\}$, $U/R=\{\{a,b,c\}\}$. Let $X=\{a,b\}\subseteq U$. Then $\tau_R(X)=\{U,\phi\}$. Let $\mu=\{a,b\}\notin\tau_R(X)$. Then, $\mu_R(X)=\{\phi,\{a,b\},U\}$. Micro closed sets are $\phi,\{c\},U$. Micro g^* -closed sets are $\phi,\{c\},\{a,c\},\{b,c\},U$. Micro αg-closed sets are $\phi,\{c\},\{a,c\},\{b,c\},U$. Here $(U,\tau_R(X),\ \mu_R(X))$ is a Micro ${}_{\alpha}T_c$- space but not a Micro $T^*_{1/2}$- space, since the subset $\{a,c\}$ is Micro g^*-closed but not Micro closed.

Example 2.7.23

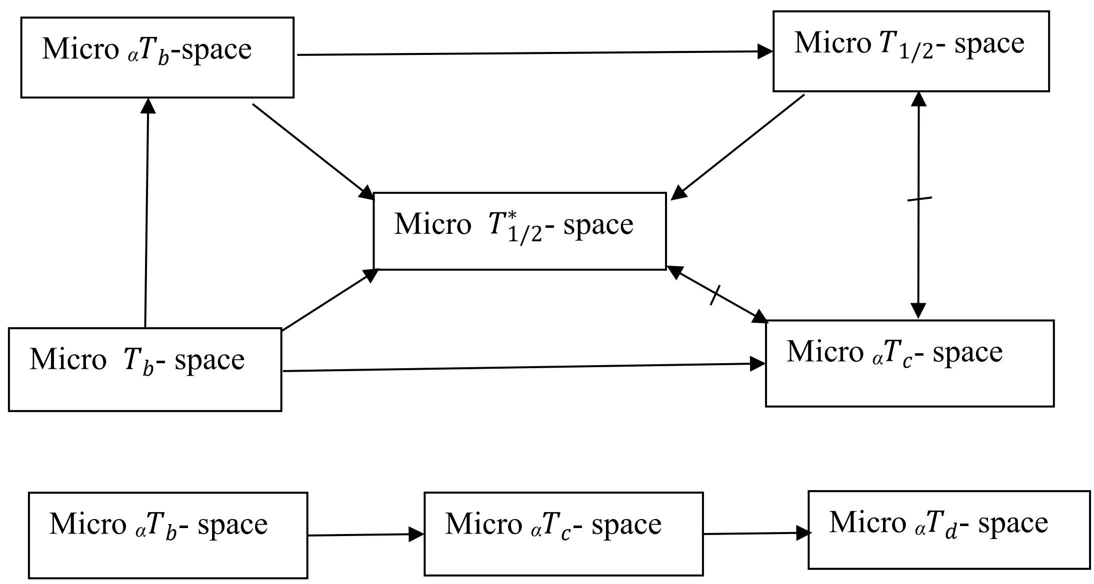
Let $U=\{a,b,c\}$, $U/R=\{\{a,b,c\}\}$. Let $X=\{b,c\}\subseteq U$. Then $\tau_R(X)=\{U,\phi\}$. Let $\mu=\{a\}\notin\tau_R(X)$. Then, $\mu_R(X)=\{\phi,\{a\},U\}$. Micro closed sets are $\phi,\{b,c\},U$. Micro g^*-closed sets are $\phi,\{b,c\},U$. Micro αg-closed sets are $\phi,\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},U$. Here $(U,\tau_R(X),\ \mu_R(X))$ is a Micro $T^*_{1/2}$- space but not Micro ${}_{\alpha}T_c$- space. Since the subset $\{b\}$ is Micro αg-closed but not Micro closed.

Remark 2.7.24

Micro ${}_{\alpha}T_c$- space and Micro $T_{1/2}$- space are independent.

Remark 2.7.25

The following diagram shows the dependency and independency relations of newly defined Micro spaces with already existing Micro spaces.



CHAPTER 3

CHAPTER 3

Micro g^*-Continuous Maps and Micro g^*-Irresolute Maps in Micro Topological Spaces

3.1 Introduction

Continuous and irresolute maps are important notions in the field of Mathematics. The concept of Micro continuous map was introduced by Sakkraiveeranan Chandrasekar [2019]. He also defined Micro pre-continuous and Micro semi-continuous maps in Micro topological spaces. Chandrasekar and Swathi [2018] introduced Micro α-continuity in Micro topological spaces. Taha et.al. [2020] initiated the concept of Micro g-continuous maps. Anandhi and Balamani [2022 c] initiated the concept of Micro αg-continuous map in Micro topological spaces.

In this chapter Micro g^*-continuous maps using Micro g^*-closed sets is introduced and obtained some of its properties. Moreover, Micro g^*-irresolute maps are also analyzed.

3.2 Micro g^*-Continuous Maps

In this section, Micro g^*-continuous maps in Micro topological spaces are introduced and its properties are derived. It is shown that the composition of two Micro g^*-continuous maps need not be Micro g^*-continuous.

Definition 3.3.1

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ is called a Micro g^*-continuous map if $f^{-1}(P)$ is Micro g^*-closed in U for every Micro closed set P in V.

Example 3.3.2

Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b, c\}\}$. Let $X = \{a, b\} \subseteq U$. Then, $\tau_R(X) = \{\phi, \{a\}, U\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{U, \phi, \{a\}, \{b\}, \{a, b\}\}$. Micro g^*-closed sets in U are $\phi, \{c\}, \{a, c\}, \{b, c\}, U$. Let $V = \{a, b, c\}$, $V/R = \{\{a\}, \{b\}, \{c\}\}$. Let $Y = \{b, c\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{a\}, V\}$. Micro closed sets in $V$are $\phi, \{b, c\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a$, $f(b) =$

$c, f(c) = b, f(d) = d$. Therefore for every Micro closed set P in $V, f^{-1}(P)$ is Micro g^*-closed in U. Hence f is Micro g^*-continuous.

Proposition 3.3.3

Every Micro continuous map is Micro g^*-continuous but not conversely.

Proof: Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be Micro continuous. Let P be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro continuous, $f^{-1}(P)$ is Micro closed. Since every Micro closed set is Micro g^*-closed, $f^{-1}(P)$ is Micro g^*-closed. Hence f is Micro g^*-continuous.

Example 3.3.4

Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{c, d\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a\}, \{c, d\}, \{a, c, d\}, U\}$. Micro closed sets in U are $\phi, \{b\}, \{a, b\}, \{b, c, d\}, U$. Micro g^*-closed sets in $U$are $\phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, U$. Let $V = \{a, b, c, d\}$, $V/R = \{\{a, b\}, \{c\}, \{d\}\}$. Let $Y = \{a, b\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi, \{a, b\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{a\}, \{a, b\}, V\}$. Micro closed sets in V are $\phi, \{c, d\}, \{b, c, d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a$, $f(b) = c, f(c) = b, f(d) = d$. Then f is Micro g^*-continuous but not Micro continuous, since for the Micro closed set $\{c, d\}$ in $V, f^{-1}(\{c, d\}) = \{b, d\}$ is not Micro closed in U.

Proposition 3.3.5

Every Micro g^*-continuous map is Micro g-continuous but not conversely.

Proof: Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be Micro g^*-continuous. Let P be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro g^*-continuous, $f^{-1}(P)$ is Micro g^*-closed. Since every Micro g^*-closed set is Micro g-closed, $f^{-1}(P)$ is Micro g-closed. Hence f is Micro g-continuous.

Example 3.3.6

Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$. Let $X = \{b, d\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{b, d\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{b\}, \{b, d\}, U\}$. Micro g^*-closed sets in U are $\phi, \{a, c\}, \{a, b, c\}, \{a, b, d\}, U$. Micro g-closed sets in U are $\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, U$. Let $V = \{a, b, c, d\}$, $V/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $Y = \{c, d\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then , $\mu_R(Y) =$

$\{\phi, \{a\}, \{c, d\}, \{a, c, d\}, V\}$. Micro closed sets in V are $\phi, \{b\}, \{a, b\}, \{b, c, d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a$, $f(b) = c, f(c) = b, f(d) = d$. Then f is Micro g-continuous but not Micro g^*-continuous, since for the Micro closed set $\{b, c, d\}$ in $V, f^{-1}(\{b, c, d\}) = \{b, c, d\}$ is not Micro g^*-closed in U.

Proposition 3.3.7

Every Micro g^*-continuous map is Micro αg-continuous but not conversely.

Proof: Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be Micro g^*-continuous. Let P be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro g^*-continuous, $f^{-1}(P)$ is Micro g^*-closed. Since every Micro g^*-closed set is Micro αg-closed, $f^{-1}(P)$ is Micro αg-closed. Hence f is Micro αg-continuous.

Example 3.3.8

Let $U = \{a, b, c, d\}$, $U/R = \{\{c\}, \{d\}\{a, b\}\}$. Let $X = \{a, b\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a\}, \{a, b\}, U\}$. Micro g^*-closed sets in U are $\phi, \{c, d\}, \{a, c, d\}, \{b, c, d\}, U$. Micro αg-closed sets in U are $\phi, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, U$. Let $V = \{a, b, c, d\}$, $V/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $Y = \{c, d\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{a\}, \{c, d\}, \{a, c, d\}, V\}$. Micro closed sets in V are $\phi, \{b\}, \{a, b\}, \{b, c, d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a$, $f(b) = c, f(c) = b, f(d) = d$. Then f is Micro αg-continuous but not Micro g^*-continuous, since for the Micro closed set $\{b\}$ in $V, f^{-1}(\{b\}) = \{c\}$ is not Micro g^*-closed in U.

Remark 3.3.9

Micro g^*-continuous maps and Micro semi continuous maps are independent as observed from the following examples.

Example 3.3.10

Let $U = \{a, b, c, d\}$, $U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{a, b\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a\}, \{a, b\}, U\}$. Micro semi closed sets in U are $\phi, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}, U$. Micro g^*-closed sets in U are $\phi, \{c, d\}, \{a, c, d\}, \{b, c, d\}, U$. Let $V = \{a, b, c, d\}$, $V/R = \{\{a, b\}, \{c\}, \{d\}\}$. Let

$Y = \{a,b,c\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi, \{a,b,c\}\}$. Let $\mu = \{a,b\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{a,b\}, \{a,b,c\}, V\}$. Micro closed sets in V are $\phi, \{d\}, \{c,d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a$, $f(b) = c, f(c) = b, f(d) = d$. Then f is Micro semi continuous but not Micro g^*-continuous, since for the Micro closed set $\{d\}$ in $V, f^{-1}(\{d\}) = \{d\}$ is not Micro g^*-closed in U.

Example 3.3.11

Let $U = \{a,b,c,d\}$, $U/R = \{\{c\},\{d\},\{a,b\}\}$. Let $X = \{c,d\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{c,d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then , $\mu_R(X) = \{\phi, \{a\}, \{c,d\}, \{a,c,d\}, U\}$. Micro g^* -closed sets in U are $\phi, \{b\}, \{a,b\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}, U$. Micro semi closed sets in U are $\phi, \{b\}, \{a,b\}, \{b,c,d\}, U$. Let $V = \{a,b,c,d\}$, $V/R = \{\{c\},\{d\},\{a,b\}\}$. Let $Y = \{a,b\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi, \{a,b\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{a\}, \{a,b\}, V\}$. Micro closed sets in V are $\phi, \{c,d\}, \{b,c,d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a$, $f(b) = c, f(c) = b, f(d) = d$. Then f is Micro g^*-continuous but not Micro semi continuous, since for the Micro closed set $\{c,d\}$ in $V, f^{-1}(\{c,d\}) = \{b,d\}$ is not Micro semi closed in U.

Remark 3.3.12

Micro g^* -continuous maps and Micro pre-continuous maps are independent as observed from the following examples.

Example 3.3.13

Let $U = \{a,b,c\}$, $U/R = \{\{a\},\{b\},\{c\}\}$. Let $X = \{b,c\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a\}, U\}$. Micro g^*-closed sets in U are $\phi, \{b,c\}, U$. Micro pre-closed sets in U are $\phi, \{b\}, \{c\}, \{b,c\}, U$. Let $V = \{a,b,c\}$, $V/R = \{\{a\},\{b,c\}\}$. Let $Y = \{a,b\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi, \}$. Let $\mu = \{a,b\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{a,b\}, V\}$. Micro closed sets in V are $\phi, \{c\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be the identity map. Then f is Micro pre continuous but not Micro g^*-continuous, since for the Micro closed set $\{c\}$ in $V, f^{-1}(\{c\}) = \{c\}$ is not Micro g^*-closed in U.

Example 3.3.14

Let $U = \{a,b,c,d\}$, $U/R = \{\{c\},\{d\},\{a,b\}$. Let $X = \{a,b\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{a,b\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then , $\mu_R(X) =$

$\{\phi, \{a\}, \{a,b\}, U\}$. Micro pre-closed sets in U are $\phi, \{\{b\}, \{c\}, \{d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{b,c,d\}\}, U$. Micro g^*-closed sets in U are $\phi, \{c,d\}, \{a,c,d\}, \{b,c,d\}, U$. Let $V = \{a,b,c,d\}$, $V/R = \{\{c\}, \{d\}, \{a,b\}\}$. Let $Y = \{c\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi, \{c\}\}$. Let $\mu = \{b\} \notin \tau_R(Y)$. Then , $\mu_R(Y) = \{\phi, \{b\}, \{c\}, \{b,c\}, V\}$. Micro closed sets in V are $\phi, \{a,d\}, \{a,b,d\}, \{a,c,d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = c$, $f(b) = b, f(c) = a, f(d) = d$. Then f is Micro g^*-continuous but not Micro pre-continuous, since for the Micro closed set $\{a,c,d\}$ in $V, f^{-1}(\{a,c,d\}) = \{a,c,d\}$ is not Micro pre-closed in U.

Remark 3.3.15

Micro g^*-continuous maps and Micro α-continuous maps are independent as observed from the following examples .

Example 3.3.16

Let $U = \{a,b,c,d\}$, $U/R = \{\{a\}, \{c\}, \{b,d\}\}$. Let $X = \{b,d\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{b,d\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then , $\mu_R(X) = \{\phi, \{b\}, \{b,d\}, U\}$. Micro α-closed sets in U are $\phi, \{a\}, \{c\}, \{d\}, \{a,c\}, \{c,d\}, \{a,c,d\}, U$. Micro g^*-closed sets in U are $\phi, \{a,c\}, \{a,b,c\}, \{a,c,d\}, U$. Let $V = \{a,b,c,d\}$, $V/R = \{\{a,b\}, \{c\}, \{d\}\}$. Let $Y = \{a,b\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi, \{a,b\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{a\}, \{a,b\}, V\}$. Micro closed sets in V are $\phi, \{c,d\}, \{b,c,d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = b$, $f(b) = a, f(c) = c, f(d) = d$. Then f is Micro α-continuous but not Micro g^*-continuous, since for the Micro closed set $\{c,d\}$ in $V, f^{-1}(\{c,d\}) = \{c,d\}$ is not Micro g^*-closed in U.

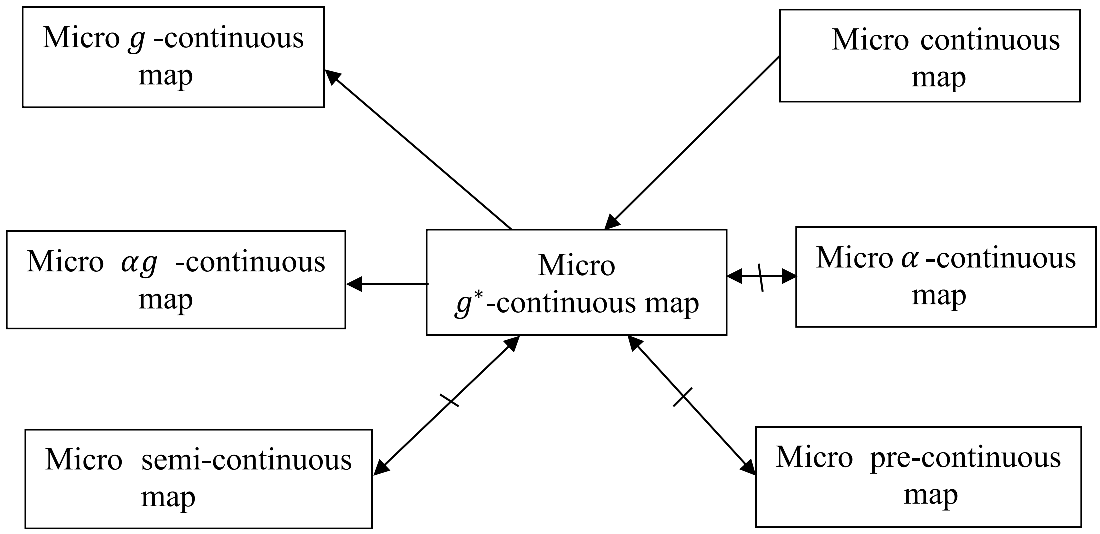
Example 3.3.17

Let $U = \{a,b,c\}$, $U/R = \{\{a\}, \{b,c\}\}$. Let $X = \{a,b\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi\}$. Let $\mu = \{a,b\} \notin \tau_R(X)$. Then , $\mu_R(X) = \{\phi, \{a,b\}, U\}$. Micro α-closed sets in U are $\phi, \{c\}, U$. Micro g^*-closed sets in U are $\phi, \{c\}, \{a,c\}, \{b,c\}, U$. Let $V = \{a,b,c\}$, $V/R = \{\{c\}, \{a,b\}\}$. Let $Y = \{b\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi, \{b\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then , $\mu_R(Y) = \{\phi, \{a\}, \{b\}, \{a,b\}, V\}$. Micro closed sets in V are $\phi, \{c\}, \{a,c\}, \{b,c\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be the identity map . Then f is Micro

g^*-continuous but not Micro α-continuous, since for the Micro closed set $\{a,c\}$ in $V, f^{-1}(\{a,c\}) = \{a,c\}$ is not Micro α-closed in U.

Remark 3.3.18

The following diagram shows the dependency and independency relations of Micro g^*-continuous maps with already existing Micro continuous maps.



Theorem 3.3.19

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y)$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ is Micro g^*-continuous if and only if $f^{-1}(M)$ is Micro g^*-open in $(U, \tau_R(X), \mu_R(X))$ whenever M is Micro open in $(V, \tau_R(Y), \mu_R(Y))$.

Proof: Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro g^*-continuous map and M be Micro open in $(V, \tau_R(Y), \mu_R(Y))$. Then M^c is Micro closed in $(V, \tau_R(Y), \mu_R(Y))$. By hypothesis $f^{-1}(M^c)$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$. i.e. $[f^{-1}(M)]^c$ is Microg^*-closed in $(U, \tau_R(X), \mu_R(X))$. Hence $f^{-1}(M)$ is Micro g^*-open in U. Conversely, suppose$f^{-1}(M)$ is Micro g^*-open in $(U, \tau_R(X), \mu_R(X))$ whenever M is Micro open in $(V, \tau_R(Y), \mu_R(Y))$. Let P be Micro closed in$(V, \tau_R(Y), \mu_R(Y))$, then P^c is Micro open in $(V, \tau_R(Y), \mu_R(Y))$. By assumption $f^{-1}(P^c)$ is Micro g^*-open in $(U, \tau_R(X), \mu_R(X))$. i.e. $[f^{-1}(P)]^c$ is Micro g^*-open in $(U, \tau_R(X), \mu_R(X))$. Then $f^{-1}(P)$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$. Hence f is Micro g^*-continuous map.

Remark 3.3.20

The composition of two Micro g^*-continuous maps need not be a Micro g^*-continuous map as seen from the following example.

Example 3.3.21

Let $U = \{a, b, c\}$, $U/R = \{\{c\}, \{a, b\}\}$. Let $X = \{a, b\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{c\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{c\}, \{a, b\}, U\}$. Micro g^*-closed sets in U are $\phi, \{c\}, \{a, b\}, U$. Let $V = \{a, b, c\}$, $V/R = \{\{a\}, \{b, c\}\}$. Let $Y = \{a, b\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi\}$. Let $\mu = \{a, b\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{a, b\}, V\}$. Micro closed sets in V are $\phi, \{c\}, V$. Micro g^*-closed sets in V are $\phi, \{c\}, \{a, c\}, \{b, c\}, V$. Let $W = \{a, b, c\}$, $W/R = \{\{c\}, \{a, b\}\}$. Let $Z = \{b\} \subseteq W$. Then, $\tau_R(Z) = \{W, \phi, \{b\}\}$. Let $\mu = \{a\} \notin \tau_R(Z)$. Then, $\mu_R(Z) = \{\phi, \{a\}, \{b\}, \{a, b\}, W\}$. Micro closed sets in W are $\phi, \{c\}, \{a, c\}, \{b, c\}, W$. Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ and $g: (V, \tau_R(Y), \mu_R(Y)) \rightarrow (W, \tau_R(Z), \mu_R(Z))$ be the maps defined by $f(a) = b$, $f(b) = a, f(c) = c, g(a) = b$, $g(b) = a, g(c) = c$. Then the maps f and g are Micro g^*-continuous but their composition $g \circ f: (U, \tau_R(X), \mu_R(X)) \rightarrow (W, \tau_R(Z), \mu_R(Z))$ is not a Micro g^*-continuous map, since for the closed set $\{a, c\}$ in $(W, \tau_R(Z), \mu_R(Z))$, $(g \circ f)^{-1}(\{a, c\}) = \{a, c\}$ is not Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$.

Theorem 3.3.22

Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a Micro g^*-continuous map and $g: (V, \tau_R(Y), \mu_R(Y)) \rightarrow (W, \tau_R(Z), \mu_R(Z))$ be a Micro continuous map. Then $g \circ f: (U, \tau_R(X), \mu_R(X)) \rightarrow (W, \tau_R(Z), \mu_R(Z))$ is a Micro g^*-continuous map.

Proof: Let P be a Micro closed set in $(W, \tau_R(Z), \mu_R(Z))$. Since g is Micro continuous, $g^{-1}(P)$ is Micro closed in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro g^*-continuous, $(g \circ f)^{-1}(P) = f^{-1}(g^{-1}(P))$ is Micro g^*-closed. Hence $g \circ f$ is a Micro g^*-continuous map.

Definition 3.3.23

A Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be Micro $T^*_{1/2}$- space (briefly Mic- $T^*_{1/2}$- space) if every Micro g^*-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro closed in $(U, \tau_R(X), \mu_R(X))$.

Definition 3.3.24

A Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be Micro ${}_\alpha T_c$-space (briefly Mic- ${}_\alpha T_c$ - space) if every Micro αg -closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$.

Definition 3.3.25

A Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be Micro* $T_{1/2}$-space (briefly Mic-* $T_{1/2}$ -space) if every Micro g -closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$.

Theorem 3.3.26

Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro g^*-continuous map and if $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$- space then f is a Micro continuous map.

Proof: Let P be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is a Micro g^*-continuous map, $f^{-1}(P)$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$ - space, $f^{-1}(P)$ is Micro closed in $(U, \tau_R(X), \mu_R(X))$. Hence f is a Micro continuous map.

Theorem 3.3.27

Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro g^*-continuous map and if $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$- space then f is a Micro g continuous map.

Proof: Let P be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is a Micro g^*-continuous map, $f^{-1}(P)$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$ - space, $f^{-1}(P)$ is Micro closed. Since every Micro closed set is Micro g-closed, $f^{-1}(P)$ is Micro g-closed. Hence f is a Micro g-continuous map in $(U, \tau_R(X), \mu_R(X))$.

Theorem 3.3.28

Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro αg-continuous map and if $(U, \tau_R(X), \mu_R(X))$ is a Micro ${}_\alpha T_c$- space then f is a Micro g^*-continuous map.

Proof: Let P be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since every Micro g^*-closed set is Micro αg-closed and f is Micro αg-continuous, $f^{-1}(P)$ is Micro αg-closed in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro ${}_\alpha T_c$- space, $f^{-1}(P)$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$. Hence f is a Micro g^*-continuous map.

Theorem 3.3.29

Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a Micro g-continuous map and if $(U, \tau_R(X), \mu_R(X))$ is a Micro $^*T_{1/2}$- space, then f is Micro g^*- continuous.

Proof: Let P be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro g-continuous, $f^{-1}(P)$ is Micro g -closed in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro $^*T_{1/2}$-space, $f^{-1}(P)$ is Micro g^*-closed. Hence f is Micro g^*-continuous.

3.3 Micro g^*-Irresolute Maps

In this section, the strong form Micro g^* -continuous maps, namely Micro g^*-irresolute maps is introduced and its properties are analyzed. It is shown that composition of two Micro g^*-irresolute maps is also a Micro g^*-irresolute map.

Definition 3.3.1

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ is called a Micro g^*-irresolute map if $f^{-1}(P)$ is Micro g^*-closed in U for every Micro g^*-closed set P in V.

Example 3.3.2

Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{c, d\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{c, d\}\}$ .Let $\mu = \{a\} \notin \tau_R(X)$. Then , $\mu_R(X) = \{\phi, \{a\}, \{c, d\}, \{a, c, d\}, U\}$. Micro g^* -closed sets in U are $\phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, U$. Let $V = \{a, b, c, d\}$, $V/R = \{\{a, b\}, \{c\}, \{d\}\}$. Let $Y = \{a, b\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi, \{a, b\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{a\}, \{a, b\}, V\}$. Micro g^*-closed sets in V are $\phi, \{c, d\}, \{a, c, d\}, \{b, c, d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a$, $f(b) = c, f(c) = b, f(d) = d$. Therefore for every Micro g^*-closed P in $V, f^{-1}(P)$is Micro g^*-closed in U. Hence f is a Micro g^*-irresolute map.

Proposition 3.3.3

Every Micro g^*-irresolute map is a Micro g^*-continuous map but not conversely.

Proof: Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a Micro g^*-irresolute map. Let P be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since every Micro closed set is Micro g^*-closed and f is Micro g^*-irresolute, $f^{-1}(P)$ is Micro g^*-closed. Hence f is a Micro g^*-continuous map.

Example 3.3.4

Let $U = \{a,b,c\}$, $U/R = \{\{a,b\},\{c\}\}$. Let $X = \{a,b\} \subseteq U$. Then $,\tau_R(X) = \{U,\phi,\{a,b\}\}$. Let $\mu = \{c\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi,\{c\},\{a,b\},U\}$. Micro g^*-closed sets in U are $\phi,\{c\},\{a,b\},U$. Let $V = \{a,b,c\}$, $V/R = \{\{a\},\{b,c\}\}$. Let $Y = \{a,b\} \subseteq V$. Then, $\tau_R(Y) = \{V,\phi\}$. Let $\mu = \{a,b\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi,\{a,b\},V\}$. Micro closed sets in V are $\phi,\{c\},V$. Micro g^*-closed sets in V are $\{\phi,\{c\},\{a,c\},\{b,c\},V\}$. Let $f: (U,\tau_R(X), \mu_R(X)) \to (V,\tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = b$, $f(b) = a, f(c) = c$. Then f is Micro g^*-continuous but not Micro g^*-irresolute, since for the Micro closed set $\{a,c\}$ in $V, f^{-1}(\{a,c\}) = \{b,c\}$ is not Micro g^*-closed in U.

Theorem 3.3.5

Let $f: (U,\tau_R(X), \mu_R(X)) \to (V,\tau_R(Y), \mu_R(Y))$ be a Micro g^*-irresolute map and $g: (V,\tau_R(Y), \mu_R(Y)) \to (W,\tau_R(Z), \mu_R(Z))$ be a Micro g^*-irresolute map then $g \circ f: (U,\tau_R(X), \mu_R(X)) \to (W,\tau_R(Z), \mu_R(Z))$ is a Micro g^*-irresolute map.

Proof: Let P be a Micro g^*-closed set in $(W,\tau_R(Z), \mu_R(Z))$. Since g is Micro g^*-irresolute,$g^{-1}(P)$ is Micro g^*-closed in $(V,\tau_R(Y), \mu_R(Y))$. Since f is Micro g^*-irresolute, $(g \circ f)^{-1}(P) = f^{-1}(g^{-1}(P))$ is Micro g^*-closed. Hence $g \circ f$ is a Micro g^*- irresolute map.

Theorem 3.3.6

Let $f: (U,\tau_R(X), \mu_R(X)) \to (V,\tau_R(Y), \mu_R(Y))$ be a Micro g^*-irresolute map and $g: (V,\tau_R(Y), \mu_R(Y)) \to (W,\tau_R(Z), \mu_R(Z))$ be a Micro g^*-continuous map then $g \circ f: (U,\tau_R(X), \mu_R(X)) \to (W,\tau_R(Z), \mu_R(Z))$ is a Micro g^*- continuous map.

Proof: Let P be a Micro closed set in $(W,\tau_R(Z), \mu_R(Z))$. Since g is Micro g^*-continuous, $g^{-1}(P)$ is Micro g^*-closed in $(V,\tau_R(Y), \mu_R(Y))$. Since f is Micro g^*-irresolute, $(g \circ f)^{-1}(P) = f^{-1}(g^{-1}(P))$is Micro g^*-closed. Hence $g \circ f$ is a Micro g^*- continuous map.

Theorem 3.3.7

Let $f: (U,\tau_R(X), \mu_R(X)) \to (V,\tau_R(Y), \mu_R(Y))$ be a Micro g-irresolute map and if $(U,\tau_R(X), \mu_R(X))$ is a Micro $^*T_{1/2}$- space, then f is Micro g^*-irresolute.

Proof: Let P be a Microg^*-closed set in $(V,\tau_R(Y), \mu_R(Y))$. Since every Micro g^*-closed set is Micro g-closed and f is Micro g-irresolute,$f^{-1}(P)$ is Micro g-closed in $(U,\tau_R(X), \mu_R(X))$. Since $(U,\tau_R(X), \mu_R(X))$ is a Micro* $T_{1/2}$-space, $f^{-1}(P)$ is Micro g^*-closed. Hence f is Micro g^*-irresolute.

Theorem 3.3.8

Let $f: (U, \tau_R(X),\ \mu_R(X)) \rightarrow (V, \tau_R(Y),\ \mu_R(Y))$ be a Micro αg-irresolute map and if $(U, \tau_R(X),\ \mu_R(X))$ is Micro ${}_{\alpha}T_c$- space, then f is Micro g^*-irresolute.

Proof: Let P be a Micro g^*-closed set in $(V, \tau_R(Y),\ \mu_R(Y))$. Since every Micro g^*-closed set is Micro αg-closed and f is Micro αg- irresolute, $f^{-1}(P)$ is Micro αg-closed in $(U, \tau_R(X),\ \mu_R(X))$. Since $(U, \tau_R(X),\ \mu_R(X))$ is a Micro ${}_{\alpha}T_c$-space, $f^{-1}(P)$ is Micro g^*-closed. Hence f is Micro g^*-irresolute.

SUMMARY AND CONCLUSION

Summary and Conclusion

Preliminary definitions which are required in the course of study are presented in **Chapter 1.**

In **Chapter 2**, Micro g^*-closed sets and Micro g^*-open sets are introduced in Micro topological spaces. Properties of the defined sets and interrelations between Micro g^*-closed sets with other existing Micro generalized closed sets are derived. Further, Micro g^*-closure and Micro g^*-interior operators are defined and their properties are analyzed. As an application of Micro g^*-closed sets four new Micro spaces namely, Micro T_b- space, Micro $T^*_{1/2}$- space, Micro ${}_\alpha T_c$- space and Micro ${}^*T_{1/2}$- space are introduced and their properties and interrelations are studied.

In **Chapter 3**, Micro g^*- continuous maps and Micro g^*- irresolute maps in Micro topological spaces are introduced. Properties and characterizations are derived with respect to the defined maps. Also we have analyzed composition of mappings relevant to the defined maps.

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