Journal of Advanced Studies in Topology
ISSN: 2090-388X online
Vol. 1, No. 2, 2010, 1–8

RESEARCH ARTICLE

Contra Generalized Semi-Preopen Mappings in Intuitionistic Fuzzy Topological Spaces

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(*Received: 5 August 2010 , Accepted: 6 October 2010*)

In this paper we introduce intuitionistic fuzzy contra generalized semi-preopen mappings, intuitionistic fuzzy almost contra generalized semi-preopen mappings and intuitionistic fuzzy contra M-generalized semi-preopen mappings. We investigate some of their properties. Also we provide the relation between the intuitionistic fuzzy contra generalized semi-preopen mappings and intuitionistic fuzzy almost contra generalized semi-preopen mappings.

Keywords: Intuitionistic fuzzy topology; intuitionistic fuzzy contra generalized semi-preopen mappings, intuitionistic fuzzy contra M-generalized semi-preopen mappings and intuitionistic fuzzy almost contra generalized semi-preopen mappings.

AMS Subject Classification: 03F55.

1. Introduction

After the introduction of fuzzy sets by Zadeh [1],there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [2] is one among them. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological spaces. The notion of intuitionistic fuzzy generalized semi-pre closed mappings and intuitionistic fuzzy almost generalized semi-pre closed mappings are introduced by R. Santhi and D. Jayanthi [4, 5]. In this paper we introduce intuitionistic fuzzy contra generalized semi-pre open mappings, intuitionistic fuzzy contra M-generalized semi-pre open mappings and intuitionistic fuzzy almost contra generalized semi-pre open mappings. We investigate some of their properties. Also we provide the relation between the intuitionistic fuzzy contra generalized semi-pre open mappings and intuitionistic fuzzy almost contra generalized semi-pre open mappings.

2. Preliminaries

Definition 2.1 [2] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x)+\nu_A(x) \le 1$ for each $x \in X$. Denote by $IFS(X)$, the set of all intuitionistic fuzzy sets in X.

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Definition 2.2 [2] Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in X\}$. Then

(1) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \leq \nu_B(x)$ for all $x \in X$
(2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(3) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in X\}$
(4) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle | x \in X\}$
(5) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle | x \in X\}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$.

The intuitionistic fuzzy sets $\tilde{0} = \{\langle x, 0, 1\rangle | x \in X\}$ and $\tilde{1} = \{\langle x, 1, 0\rangle | x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3 [6] The IFS $c(\alpha, \beta) = \langle x, c_\alpha, c_{1-\beta}\rangle$ where $\alpha \in (0, 1]$, $\beta \in [0, 1)$ and $\alpha + \beta \leq 1$ is called an intuitionistic fuzzy point (IFP for short) in X.

Definition 2.4 [6]Two IFSs are said to be q-coincident (AqB in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.5 [3] An intuitionistic fuzzy topology (IFT for short) on X is a family τ of IFSs in X satisfying the following axioms.

(1) $\tilde{0}, \tilde{1} \in \tau$
(2) $G_1 \cap G_2 \in \tau$ for any G_1, $G_2 \in \tau$
(3) $\cup G_i \in \tau$ for any family $\{G_i | i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.6 [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$$Int(A) = \cup\{G | G \text{is an IFOS in } X \text{ and } G \subseteq A\},$$

$$Cl(A) = \cap\{K | K \text{is an IFCS in } X \text{ and } A \subseteq K\}.$$

Note that for any IFS A in (X, τ), we have $Cl(A^c) = [Int(A)]^c$ and $Int(A^c) = [Cl(A)]^c$ [6].

Definition 2.7 [7] An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an

(1) intuitionistic fuzzy semi closed set (IFSCS in short) if $Int(Cl(A)) \subseteq A$.
(2) intuitionistic fuzzy pre closed set (IFPCS in short) if $Cl(Int(A)) \subseteq A$.
(3) intuitionistic fuzzy α closed set (IFαCS in short) if $Cl(Int(Cl(A)) \subseteq A$.

The respective complements of the above IFCSs are called their respective IFOSs.

The family of all IFSCSs, IFPCSs, and IFαCSs (respectively IFSOSs, IFPOSs and IFαOSs) of an IFTS (X,τ) are respectively denoted by IFSC(X), IFPC(X) and IFαC(X) (respectively IFSO(X), IFPO(X) and IFαO(X)).

Definition 2.8 [8] An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an:

(1) intuitionistic fuzzy semi-preclosed set (IFSPCS for short) if there exists an IFPCS B such that $Int(B) \subseteq A \subseteq B$,
(2) intuitionistic fuzzy semi-preopen set (IFSPOS for short) if there exists an intuitionistic fuzzy preopen set (IFPOS for short) B such that $B \subseteq A \subseteq Cl(B)$.

The family of all IFSPCSs (respectively IFSPOSs) of an IFTS (X, τ) is denoted by IFSPC(X) (respectively IFSPO(X)).

Every IFSCS (respectively IFSOS) and every IFPCS (respectively IFPOS) is an IFSPCS (respectively IFSPOS). But the separate converses need not be true in general [8].

Note that an IFS A is an IFSPCS if and only if $Int(Cl(Int(A))) \subseteq A$ [9].

Definition 2.9 [9]Let A be an IFS in an IFTS (X, τ). Then the intuitionistic fuzzy semi-preinterior and intuitionistic fuzzy semi-pre closure are defined by:

$$spInt(A) = \cup\{G|G \text{ is an IFSPOS in } X \text{ and } G \subseteq A\},$$

$$spCl(A) = \cap\{K|K \text{ is an IFSPCS in } X \text{ and } A \subseteq K\}.$$

Note that for any IFS A in (X, τ), we have $spCl(A^c) = [spInt(A)]^c$ and $spInt(A^c) = [spCl(A)]^c$ [9].

Definition 2.10 [6] An IFS A is an:

(1) intuitionistic fuzzy regular closed set (IFRCS for short) if $A = Cl(Int(A))$.
(2) intuitionistic fuzzy regular open set (IFROS for short) if $A = Int(Cl(A))$.

Definition 2.11 [9]An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi-preclosed set (IFGSPCS for short) if $spCl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ).

Every IFCS, IFSCS, IFPCS, IFRCS, IFαCS and IFSPCS is an IFGSPCS but the separate converses may not be true in general [9]. The family of all IFGSPCSs of an IFTS (X, τ) is denoted by IFGSPC(X).

Definition 2.12 Let A be an IFS in an IFTS (X, τ). Then the generalized semi-preinterior and the generalized semi- preclosure of A are defined by:

$$gspInt(A) = \cup\{G|G \text{ is an IFGSPOS in } X \text{ and } G \subseteq A\},$$

$$gspCl(A) = \cap\{K|K \text{ is an IFGSPCS in } X \text{ and } A \subseteq K\}.$$

Note that for any IFS A in (X, τ), we have $gspCl(A^c) = [gspInt(A)]^c$ and $gspInt(A^c) = [gspCl(A)]^c$.

Remark 2.13 If an IFS A in an IFTS (X, τ) is an IFGSPCS in X, then $gspCl(A) = A$. But the converse may not be true in general, since intersection does not exist in IFGSPCSs [9].

Remark 2.14 If an IFS A in an IFTS (X, τ) is an IFGSPOS in X, then $gspInt(A) = A$. But the converse may not be true in general, since union does not exist in IFGSPOSs [9].

Definition 2.15 [9] The complement A^c of an IFGSPCS A in an IFTS (X, τ) is called an intuitionistic fuzzy generalized semi-preopen set (IFGSPOS for short) in X.

Every IFOS, IFSOS, IFPOS, IFROS, IFαOS and IFSPOS is an IFGSPOS but the separate converses may not be true in general [9]. The family of all IFGSPOSs of an IFTS (X, τ) is denoted by IFGSPO(X).

Definition 2.16 [9] If every IFGSPCS in (X, τ) is an IFSPCS in (X, τ), then the space can be called as an intuitionistic fuzzy semi- pre $T_{1/2}$ space (IFSP$T1/2$ space for short).

Definition 2.17 [10] A map $f : X \to Y$ is called an intuitionistic fuzzy open mapping (IFOM for short) if $f(A)$ is an IFOS in Y for each IFOS A in X.

Definition 2.18 [5] A map $f : X \to Y$ is called an intuitionistic fuzzy almost generalized semi-preclosed mapping (IFaGSPCM for short) if $f(A)$ is an IFGSPCS in Y for each IFRCS A in X.

For any IFS A in an IFTS X, if A is an IFSPOS, then $Cl(A)$ is an IFRCS in X.

Definition 2.19 [4] A map $f : X \to Y$ is called an intuitionistic fuzzy generalized semi-preclosed mapping (IFGSPCM for short) if $f(A)$ is an IFGSPCS in Y for each IFCS A in X.

3. Intuitionistic fuzzy contra generalized semi-preopen mappings

In this section we introduce intuitionistic fuzzy contra generalized semi-preopen mappings. We investigate some of its properties.

Definition 3.1 A mapping $f : X \to Y$ is said to be an intuitionistic fuzzy contra generalized semi-preopen mapping (IFCGSPOM) if $f(A)$ is an IFGSPCS in Y for every IFOS A in X.

For the sake of simplicity, we shall use the notation $A = \langle x, (\mu_a, \mu_b), (\nu_a, \nu_b)\rangle$ instead of $A = \langle x, (a/\mu_a, b/\mu_b), (a/\nu_a, b/\nu_b)\rangle$ in the following example. Similarly we shall use the notation $B = \langle y, (\mu_u, \mu_v), (\nu_u, \nu_v)\rangle$ instead of $B = \langle y, (u/\mu_u, v/\mu_v), (u/\nu_u, v/\nu_v)\rangle$ in the following example:

Example 3.2 Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{\tilde{0}, G_1, \tilde{1}\}$ and $\sigma = \{\tilde{0}, G_2, \tilde{1}\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFCGSPOM.

Theorem 3.3 Let $f : X \to Y$ be a surjective mapping. Then the following are equivalent:

(1) f is an IFCGSPOM.
(2) $f(A)$ is an IFGSPOS in Y for every IFCS A in X.

Proof (1) $\Rightarrow$ (2): Let A be an IFCS in X. Then A^c is an IFOS in X. By hypothesis, $f(A^c)$ is an IFGSPCS in Y. That is $f(A)^c$ is an IFGSPCS in Y. Hence $f(A)$ is an IFGSPOS in Y.

(2)$\Rightarrow$ (1): Let A be an IFOS in X. Then A^c is an IFCS in X. By hypothesis, $f(A^c) = [f(A)]^c$ is an IFGSPOS in Y. Hence $f(A)$ is an IFGSPCS in Y. Thus f is an IFCGSPOM. ■

Theorem 3.4 For a bijective mapping $f : X \to Y$, where Y is an IFSP$T_{1/2}$ space, the following are equivalent:

(1) f is an IFCGSPOM.
(2) for every IFCS A in X, $f(A)$ is an IFGSPOS in Y.
(3) for every IFOS B in X, $f(B)$ is an IFGSPCS in Y.
(4) for any IFCS A in X and for any IFP $c(\alpha, \beta) \in Y$, if $f^{-1}(c(\alpha, \beta))qA$, then $c(\alpha, \beta)qspInt(f(A))$.
(5) For any IFCS A in X and for any $c(\alpha, \beta) \in Y$, if $f^{-1}(c(\alpha, \beta))qA$, then there exists an IFGSPOS B such that $c(\alpha, \beta)qB$ and $f^{-1}(B) \subseteq A$.

Proof (1) $\Leftrightarrow$ (2) and (2) $\Leftrightarrow$ (3) are obvious.

(2) $\Rightarrow$ (4): Let $A \subseteq X$ be an IFCS and let $c(\alpha, \beta) \in Y$. Let $f^{-1}(c(\alpha, \beta))qA$. Then $c(\alpha, \beta)qf(A)$. By hypothesis, $f(A)$ is an IFGSPOS in Y. Since Y is an IFSP$T_{1/2}$ space, $f(A)$ is an IFSPOS in Y . This implies $spInt(f(A)) = f(A)$. Hence $c(\alpha, \beta)qspInt(f(A))$.

(4) $\Rightarrow$ (2): Let $A \subseteq X$ be an IFCS and let $c(\alpha, \beta) \in Y$ Let $f^{-1}(c(\alpha, \beta))qA$. Then $c(\alpha, \beta)qf(A)$. By hypothesis this implies $c(\alpha, \beta)qspInt(f(A))$. That is $f(A) \subseteq spInt(f(A))$. But $spInt(f(A)) \subseteq f(A)$. Therefore $f(A) = spInt(f(A))$. Thus $f(A)$ is an IFSPOS in Y and hence an IFGSPOS in Y.

(4) $\Rightarrow$ (5): Let $A \subseteq X$ be an IFCS and let $c(\alpha, \beta) \in Y$ Let $f^{-1}(c(\alpha, \beta))qA$. Then $c(\alpha, \beta)qf(A)$. By hypothesis this implies $c(\alpha, \beta)qspInt(f(A))$. Thus $f(A)$ is an IFSPOS in Y and hence an IFGSPOS in Y . Let $f(A) = B$. Therefore $c(\alpha, \beta)qB$ and $f^{-1}(B) = f^{-1}(f(A)) \subseteq A$.

(5) $\Rightarrow$ (4): Let $A \subseteq X$ be an IFCS and let $c(\alpha, \beta) \in Y$ Let $f^{-1}(c(\alpha, \beta))qA$. Then $c(\alpha, \beta)qf(A)$. By hypothesis there exists an IFGSPOS B in Y such that $c(\alpha, \beta)qB$ and $f^{-1}(B) \subseteq A$. Let $B = f(A)$. Then $c(\alpha, \beta)qf(A)$. Since Y is an IFSP$T_{1/2}$ space, $f(A)$ is an IFSPOS in Y . Therefore $c(\alpha, \beta)qspInt(f(A))$.
■

Theorem 3.5 Let $f : X \to Y$ be a bijective mapping. Suppose that one of the following properties hold:

(1) $f(Cl(B)) \subseteq Int(spCl(f(B)))$ for each IFS B in X.
(2) $Cl(spInt(f(B))) \subseteq f(Int(B))$ for each IFS B in X.
(3) $f^{-1}(Cl(spInt(A))) \subseteq Int(f^{-1}(A))$ for each IFS A in Y .
(4) $f^{-1}(Cl(A)) \subseteq Int(f^{-1}(A))$ for each IFSPOS A in Y .

Then f is an IFCGSPOM.

Proof (1) $\Rightarrow$ (2) is obvious by taking the complement in (1).

(2) $\Rightarrow$ (3): Let $A \subseteq Y$. Put $B = f^{-1}(A)$ in X. This implies $A \subseteq f(B)$ in Y . Now $Cl(spInt(A)) \subseteq Cl(spInt(f(B))) \subseteq f(Int(B))$ by (2). Therefore $f^{-1}(Cl(spInt(A))) \subseteq f^{-1}(f(Int(B))) = Int(B) = Int(f^{-1}(A))$.

(3) $\Rightarrow$ (4): Let $A \subseteq Y$ be an IFSPOS. Then $spInt(A) = A$. By hypothesis, $f^{-1}(Cl(spInt(A))) \subseteq Int(f^{-1}(A))$. Therefore $f^{-1}(Cl(A)) \subseteq Int(f^{-1}(A))$.

Suppose (4) holds: Let A be an IFOS in X. Then $f(A)$ is an IFS in Y and $spInt(f(A))$ is an IFSPOS in Y . Hence by hypothesis, $f^{-1}(Cl(spInt(f(A)))) \subseteq Int(f^{-1}(spInt(f(A)))) \subseteq Int(f^{-1}(f(A))) \subseteq Int(A) \subseteq A$. Therefore $Cl(spInt(f(A))) \subseteq f(A)$. Now $Cl(Int(f(A))) \subseteq Cl(spInt(f(A))) \subseteq f(A)$. This implies $f(A)$ is an IFPCS in Y and hence an IFGSPCS in Y . Thus f is an IFCGSPOM. ■

Theorem 3.6 Let $f : X \to Y$ be a surjective map. Suppose that one of the following properties hold

(1) $f^{-1}(spCl(A)) \subseteq Int(f^{-1}(A))$ for each IFS A in Y.
(2) $spCl(f(B)) \subseteq f(Int(B))$ for each IFS B in X.
(3) $f(Cl(B)) \subseteq spInt(f(B))$ for each IFS B in X.

Then f is an IFCGSPOM.

Proof (1) $\Rightarrow$ (2): Let $B \subseteq X$. Then $f(B)$ is an IFS in Y. By hypothesis, $f^{-1}(spCl(f(B))) \subseteq Int(f^{-1}(f(B))) \subseteq Int(B)$. Now $spCl(f(B)) \subseteq f(f^{-1}(spCl(f(B)))) \subseteq f(Int(B))$.

(2) $\Rightarrow$ (3): is obvious by taking complement in (2).

Suppose (3) holds: Let A be an IFCS in X. Then $Cl(A) = A$ and $f(A)$ is an IFS in Y . Now $f(A) = f(Cl(A)) \subseteq spInt(f(A)) \subseteq f(A)$, by hypothesis. This implies $f(A)$ is an IFSPOS in Y and hence an IFGSPOS in Y. Thus f is an IFCGSPOM by Theorem 3.3. ■

Theorem 3.7 Let $f : X \to Y$ be a bijective mapping. Then f is an IFCGSPOM if $Cl(f^{-1}(A)) \subseteq f^{-1}(spInt(A))$ for every IFS A in Y.

Proof Let A be an IFCS in X. Then $Cl(A) = A$ and $f(A)$ is an IFS in Y. By hypothesis $Cl(f^{-1}(f(A))) \subseteq f^{-1}(spInt(f(A)))$. Since f is one to one $f^{-1}(f(A)) = A$. Therefore $A = Cl(A) = Cl(f^{-1}(f(A))) \subseteq f^{-1}(spInt(f(A)))$. Now $f(A) \subseteq f(f^{-1}(spInt(f(A))) = spInt(f(A)) \subseteq f(A)$. Hence $f(A)$ is an IFSPOS in Y and hence an IFGSPOS in Y. Thus f is an IFCGSPOM by Theorem 3.3. ■

Theorem 3.8 If $f : X \to Y$ is a surjective IFCGSPOM, where Y is an IFSP$T_{1/2}$ space, then the following conditions hold:

(1) $spCl(f(B)) \subseteq f(Int(spCl(B)))$ for every IFOS B in X.
(2) $f(Cl(spInt(B))) \subseteq spInt(f(B))$ for every IFCS B in X.

Proof (1) Let $B \subseteq X$ be an IFOS. Then $Int(B) = B$. By hypothesis $f(B)$ is an IFGSPCS in Y . Since Y is an IFSP$T_{1/2}$ space, $f(B)$ is an IFSPCS in Y. This implies $spCl(f(B)) = f(B) = f(Int(B)) \subseteq f(Int(spCl(B)))$.

(2) can be proved easily by taking complement in (1). ■

Theorem 3.9 Let $f : X \to Y$ be a one-to-one mapping where Y is an IFSP$T_{1/2}$ space. Then the following are equivalent:

(1) f is an IFCGSPOM.
(2) for each IFP $c(\alpha,\beta) \in Y$ and for each IFCS B containing $f^{-1}(c(\alpha,\beta))$, there exists an IFSPOS $A \subseteq Y$ and $c(\alpha,\beta) \in A$ such that $A \subseteq f(B)$.
(3) for each IFP $c(\alpha,\beta) \in Y$ and for each IFCS B containing $f^{-1}(c(\alpha,\beta))$, there exists an IFSPOS $A \subseteq Y$ and $c(\alpha,\beta) \in A$ such that $f^{-1}(A) \subseteq B$.

Proof (1) $\Rightarrow$ (2): Let B be an IFCS in X. Let $c(\alpha,\beta)$ be an IFP in Y such that $f^{-1}(c(\alpha,\beta)) \in B$. Then $c(\alpha,\beta) \in f(B)$. By hypothesis $f(B)$ is an IFGSPOS in Y. Since Y is an IFSP$T_{1/2}$ space, $f(B)$ is an IFSPOS in Y. Now $A = spInt(f(B)) \subseteq f(B)$.

(2) $\Rightarrow$ (3): Let B be an IFCS in X. Let $c(\alpha,\beta)$ be an IFP in Y such that $f^{-1}(c(\alpha,\beta)) \in B$. Then $c(\alpha,\beta) \in f(B)$. By hypothesis $f(B)$ is an IFGSPOS in Y. Since Y is an IFSP$T_{1/2}$ space, $f(B)$ is an IFSPOS in Y and $A \subseteq f(B)$. This implies $f^{-1}(A) \subseteq f^{-1}(f(B)) \subseteq B$.

(3) $\Rightarrow$ (1): Let B be any IFCS in X and let $c(\alpha,\beta) \in Y$. Let $f^{-1}(c(\alpha,\beta)) \in B$. By hypothesis there exists an IFSPOS A in Y such that $c(\alpha,\beta) \in A$ and $f^{-1}(A) \subseteq B$. This implies $c(\alpha,\beta) \in A \subseteq f(f^{-1}(A)) \subseteq f(B)$. That is $c(\alpha,\beta) \in f(B)$. Since A is an IFSPOS, $A = spInt(A) \subseteq spInt(f(B))$. Therefore $c(\alpha,\beta) \in spInt(f(B))$. But $f(B) = \cup c(\alpha,\beta) \in f(B) c(\alpha,\beta) \subseteq spInt(f(B)) \subseteq f(B)$. Hence $f(B)$ is an IFSPOS in Y and hence an IFGSPOS in Y. Thus f is an IFCGSPOM. ■

Theorem 3.10 A mapping $f : X \to Y$ is an IFCGSPOM if $f(spCl(B)) \subseteq Int(f(B))$ for every IFS B in X.

Proof Let $B \subseteq X$ be an IFCS. Then $Cl(B) = B$. Since every IFCS is an IFSPCS, $spCl(B) = B$. Now by hypothesis, $f(B) = f(spCl(B)) \subseteq Int(f(B)) \subseteq f(B)$. This implies $f(B)$ is an IFOS in Y. Therefore $f(B)$ is an IFGSPOS in Y. Hence f is an IFCGSPOM. ■

Theorem 3.11 A mapping $f : X \to Y$ is an IFCGSPOM, where Y is an IFSP$T_{1/2}$ space if and only if $f(spCl(B)) \subseteq spInt(f(Cl(B)))$ for every IFS B in X.

Proof **Necessity:** Let $B \subseteq X$ be an IFS. Then $Cl(B)$ is an IFCS in X. By hypothesis $f(Cl(B))$ is an IFGSPOS in Y. Since Y is an IFSP$T_{1/2}$ space, $f(Cl(B))$ is an IFSPOS in Y. Therefore $f(spCl(B)) \subseteq f(Cl(B)) = spInt(f(Cl(B)))$.

Sufficiency: Let $B \subseteq X$ be an IFCS. Then $Cl(B) = B$. By hypothesis, $f(spCl(B)) \subseteq spInt(f(Cl(B))) = spInt(f(B))$. Now $f(B) \subseteq f(spCl(B)) \subseteq spInt(f(B) \subseteq f(B)$. This implies $f(B)$ is an IFSPOS in Y and hence an IFGSPOS in Y. Hence f is an IFCGSPOM. ■

Theorem 3.12 An IFOM $f : X \to Y$ is an IFCGSPOM if $IFGSPO(Y) = IFGSPC(Y)$.

Proof Let $A \subseteq X$ be an IFOS. By hypothesis, $f(A)$ is an IFOS in Y and hence is an IFGSPOS in Y. Thus $f(A)$ is an IFGSPCS in Y. Therefore f is an IFCGSPOM. ■

Definition 3.13 A mapping $f : X \to Y$ is said to be an intuitionistic fuzzy almost contra generalized semi-pre open mapping (IFaCGSPOM for short) if $f(A)$ is an IFGSPCS in Y for every IFROS A in X.

Theorem 3.14 Every IFCGSPOM is an IFaCGSPOM but not conversely.

Proof Let $f : X \to Y$ be an IFCGSPOM. Let $A \subseteq X$ be an IFROS. Then A is an IFOS in X. By hypothesis, $f(A)$ is an IFGSPCS in Y. Hence f is an IFaCGSPOM. ■

Example 3.15 Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.7_b), (0.2_a, 0.3_b)\rangle$, $G_3 = \langle y, (0.9_u, 0.8_v), (0.1_u, 0.2_v)\rangle$ and $G_4 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{\tilde{0}, G_1, G_2, \tilde{1}\}$ and $\sigma = \{\tilde{0}, G_3, G_4, \tilde{1}\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFaCGSPOM but not an IFCGSPOM, since $f(G_2) = \langle y, (0.8_u, 0.7_v), (0.2_u, 0.3_v)\rangle \subseteq G_3$ but $spCl(f(G_2)) = 1 \nsubseteq G_3$, is not an IFGSPCS in Y .

Theorem 3.16 If $f : X \to Y$ is a mapping, where Y is an IFSP$T_{1/2}$ space, then the following are equivalent:

(1) f is an IFaCGSPOM.
(2) $f(A) \in IFGSPO(Y)$ for every $A \in IFRC(X)$.
(3) for each IFP $c(\alpha, \beta) \in Y$ and for each IFRCS A in X containing $f^{-1}(c(\alpha, \beta))$, there exists an IFGSPOS B in Y containing $c(\alpha, \beta)$ such that $f^{-1}(B) \subseteq A$.
(4) $f(Int(Cl(G))) \in IFGSPC(Y)$ for every IFOS $G \subseteq X$.
(5) $f(Cl(Int(H))) \in IFGSPO(Y)$ for every IFCS $H \subseteq X$.

Proof (1) $\Leftrightarrow$ (2): is obvious.

(2) $\Rightarrow$ (3): Let $c(\alpha, \beta) \in Y$ and let $A \subseteq X$ be any IFRCS. Let $f^{-1}(c(\alpha, \beta)) \in A$. Then $c(\alpha, \beta) \in f(A)$. By hypothesis $f(A)$ is an IFGSPOS in Y. Let $B = f(A)$. Then $f^{-1}(B) = f^{-1}(f(A)) \subseteq A$.

(3) $\Rightarrow$ (2): Let $A \subseteq X$ be an IFRCS. Let $c(\alpha, \beta) \in Y$ and $f^{-1}(c(\alpha, \beta)) \in A$. Then $c(\alpha, \beta) \in f(A)$. By hypothesis there exists an IFGSPOS B in Y and hence an IFSPOS in Y, since Y is an IFSP$T_{1/2}$ space, such that $c(\alpha, \beta) \in B$ and $f^{-1}(B) \subseteq A$. Therefore $c(\alpha, \beta) \in B \subseteq f(A)$. This implies $f(A) = \cup_{c(\alpha,\beta)\in f(A)} B$. Since each of B is an IFSPOS, $f(A)$ is also an IFSPOS and hence an IFGSPOS in Y.

(1) $\Rightarrow$ (4): Let G be any IFOS in X. Then $Int(Cl(G))$ is an IFROS in X . By hypothesis, $f(Int(Cl(G)))$ is an IFGSPCS in Y. Hence $f(Int(Cl(G))) \in IFGSPC(Y)$.

(5) $\Rightarrow$ (1): Let A be any IFROS in X. Then A is an IFOS in X. By hypothesis, $f(Int(Cl(A))) \in IFGSPC(Y)$. That is $f(A) \in IFGSPC(Y)$, since $Int(Cl(A)) = A$. Hence f is an IFaCGSPOM.

(2) $\Leftrightarrow$ (5): is similar as (1) $\Leftrightarrow$ (4). ■

Definition 3.17 A mapping $f : X \to Y$ is said to be an intuitionistic fuzzy contra M-generalized semi-preopen mapping (IFCMGSPOM) if $f(A)$is an IFGSPCS in Y for every IFGSPOS A in X .

Example 3.18 Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle y, (0.2_u, 0.3_v), (0.8_u, 0.7_v)\rangle$. Then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFCMGSPOM.

Theorem 3.19 Let $f : X \to Y$ be a surjective mapping. Then the following are equivalent:

(1) f is an IFCMGSPOM.
(2) $f(A)$ is an IFGSPOS in Y for every IFGSPCS A in X .

Proof (1) $\Rightarrow$ (2): Let A be an IFGSPCS in X. Then A^c is an IFGSPOS in X. By hypothesis, $f(A^c)$ is an IFGSPCS in Y. That is $f(A)^c$ is an IFGSPCS in Y . Hence $f(A)$ is an IFGSPOS in Y .

(2)$\Rightarrow$ (1): Let A be an IFGSPOS in X . Then A^c is an IFGSPCS in X. By hypothesis, $f(A^c) = [f(A)]^c$ is an IFGSPOS in Y. Hence $f(A)$ is an IFGSPCS in Y. Thus f is an IFCMGSPOM. ■

Theorem 3.20 Every IFCMGSPOM is an IFCGSPOM but not conversely.

Proof Let $f : X \to Y$ be an IFCMGSPOM. Let $A \subseteq X$ be an IFOS. Then A is an IFGSPOS in X. By hypothesis, $f(A)$ is an IFGSPCS in Y. Hence f is an IFCGSPOM. ■

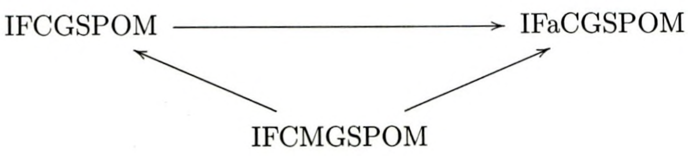
Example 3.21 Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$ and $G_3 = \langle y, (0.4_u, 0.7_v), (0.6_u, 0.3_v)\rangle$. Then $\tau = \{\tilde{0}, G_1, \tilde{1}\}$ and $\sigma = \{\tilde{0}, G_2, G_3, \tilde{1}\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFCGSPOM but not an IFCMGSPOM, since $A = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)\rangle$ is an IFGSPOS in X but $f(A) = \langle y, (0.5_u, 0.7_v), (0.5_u, 0.3_v)\rangle$ is not an IFGSPCS in Y , since $f(A) \subseteq G_1$ but $spCl(f(A)) = \tilde{1} \nsubseteq G_1$.

Theorem 3.22 Every IFCMGSPOM is an IFaCGSPOM but not conversely.

Proof Let $f : X \to Y$ be an IFCMGSPOM. Let $A \subseteq X$ be an IFROS. Then A is an IFGSPOS in X . By hypothesis, $f(A)$ is an IFGSPCS in Y. Hence f is an IFCMGSPOM. ■

Example 3.23 In Example 3.21, f is an IFaCGSPOM but not an IFCMGSPOM.

The relation between various types of intuitionistic fuzzy contra openness is given in the following diagram.



The reverse implications are not true in general in the above diagram.

Theorem 3.24

(1) If $f : X \to Y$ is an IFOM and $g : Y \to Z$ be an IFCGSPOM, then $g \circ f$ is an IFCGSPOM.
(2) If $f : X \to Y$ is an IFCGSPOM and $g : Y \to Z$ be an IFMGSPCM, then $g \circ f$ is an IFCGSPOM.
(3) If $f : X \to Y$ be an IFGSPOM and $g : Y \to Z$ be an IFCMGSPOM, then $g \circ f$ is an IFCGSPOM.
(4) If $f : X \to Y$ be an IFCGSPOM and $g : Y \to Z$ is an IFCMGSPOM, then $g \circ f : X \to Z$ is an IFGSPCM.
(5) If $f : X \to Y$ be an IFaCGSPOM and $g : Y \to Z$ is an IFCMGSPOM, then $g \circ f : X \to Z$ is an IFaGSPOM.

Proof (1) Let A be an IFOS in X. Then $f(A)$ is an IFOS in Y. Therefore $g(f(A))$ is an IFGSPCS in Z. Hence $g \circ f$ is an IFCGSPOM.

(2) Let A be an IFOS in X. Then $f(A)$ is an IFGSPCS in Y. Therefore $g(f(A))$ is an IFGSPCS in Z. Hence $g \circ f$ is an IFCGSPOM.

(3) Let A be an IFOS in X. Then $f(A)$ is an IFGSPOS in Y. Therefore $g(f(A))$ is an IFGSPCS in Z. Hence $g \circ f$ is an IFCGSPOM.

(4) Let A be an IFOS in X. Then $f(A)$is an IFGSPCS in Y, since f is an IFCGSPOM. Since f is an IFCMGSPOM, $g(f(A))$ is an IFGSPOS in Z, by Theorem 3.20. Therefore $g \circ f$ is an IFGSPOM.

(5) Let A be an IFROS in X. Then $f(A)$is an IFGSPCS in Y, since f is an IFaCGSPOM. Since g is an IFCMGSPOM, $g(f(A))$ is an IFGSPOS in Z. Therefore $g \circ f$ is an IFaGSPOM. ■

Theorem 3.25 If $f : X \to Y$ is an IFCMGSPOM, then for any IFGSPCS A in X and for any IFP $c(\alpha, \beta) \in Y$, if $f^{-1}(c(\alpha, \beta))qA$, then $c(\alpha, \beta)q\ gspInt(f(A))$.

Proof Let $A \subseteq X$ be an IFGSPCS and let $c(\alpha, \beta) \in Y$. Let $f^{-1}(c(\alpha, \beta))qA$. Then $c(\alpha, \beta)qf(A)$. By hypothesis, $f(A)$ is an IFGSPOS in Y. This implies $gspInt(f(A)) = f(A)$. Hence $c(\alpha, \beta)q\ gspInt(f(A))$. ■

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