Micro ψ-Closed Sets in Micro Topological Spaces

By

Sowmiya T

(20PMA016)

Supervisor

Dr. N.Balamani

Thesis Submitted to

Avinashilingam Institute for Home Science and Higher Education for Women

Coimbatore-641 043

In Partial Fulfilment of the Requirement for the Degree of

Master of Science in Mathematics

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N.Balamani
17.05.2022
Signature of the Head of the Department

N.Balamani
17.05.2022
Signature of the Supervisor

DECLARATION

I declare that the dissertation entitled **"Micro ψ-Closed Sets in Micro Topological Spaces"** submitted by me for the degree of **Master of Science** is a record of research work carried out by me during the period from December 2021 to May 2022 under the guidance of **Dr. (Tmt). N. Balamani,** Assistant Professor (SS) and Head (i/c), Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore and has not formed he basis for the award of any Degree, Diploma, Associateship, Fellowship or other titles in this University or any other University or Institution of Higher Learning.

T. Sowmiya

Signature of the Candidate

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ACKNOWLEDGEMENT

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CONTENTS

INTRODUCTION

INTRODUCTION

Topology is a area of Mathematics. Topology means the study of surfaces or the science of position. The subject, topology can be defined as the study of all topological properties of topological spaces. A topological space is a general type of geometric configuration.

The theory of Nano topology proposed by Carmel Richard [2013] is an extension of set theory for the study of intelligent systems characterized by insufficient and incomplete information. It originates from the Greek word 'Nanos' which means 'dwarf' in its modern scientific sense, an order to magnitude-one billionth. The topology is named as Nano topology because of its size, since it has at most five elements. The author has defined Nano topological space in terms of Lower and Upper approximations.

Sakkraiveeranan Chandrasekar [2019] introduced the concepts of Micro topology which is a simple extension of nano topology and he also studied the relations between Micro open sets, Micro pre-open sets and Micro semi-open sets.

In the study of Micro topological space many concepts of Micro topology have been generalized. Chandrasekar and Swathi [2018] introduced Micro α-open sets and Micro α-continuous maps in Micro topological spaces. Sakkraiveeranan Chandrasekar [2019] introduced Micro continuous, Micro semi-continuous, Micro pre-continuous maps in Micro topological spaces.

The concept of Micro generalized closed sets was introduced by Ibrahim [2020 b]. Ibrahim [2020 a] introduced Micro β-open sets. Bhavani [2021] introduced the Micro semi-generalized closed sets and Micro generalized semi-closed set in Micro topological spaces.

Micro generalized continuous map was introduced by Taha et al. [2021]. Anandhi and Balamani [2022 a] introduced Micro α-generalized closed sets and Micro generalized α-closed sets in Micro topological spaces.

The present study focuses on the following concepts:

(i) Micro ψ-closed sets in Micro topological spaces

(ii) Micro ψ-continuous maps and Micro ψ-irresolute maps in Micro topological spaces

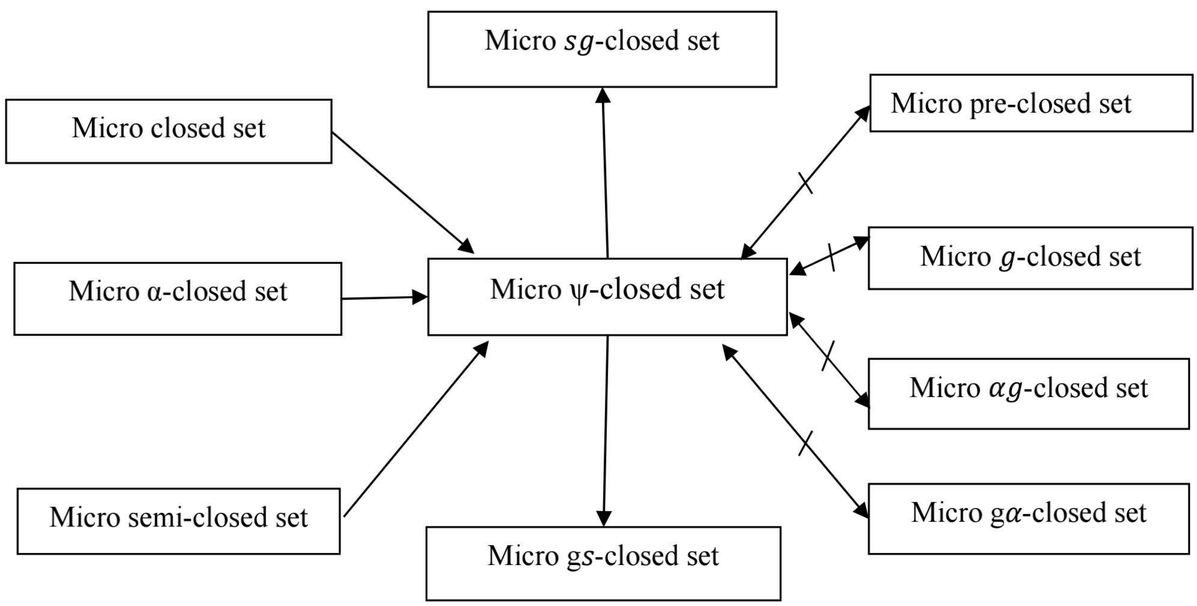
Chapter 1 deals with the preliminary definitions in Micro topological spaces.

In **Chapter 2**, a new class of Micro closed sets called Micro ψ-closed sets in Micro topological spaces is introduced. The interrelations are obtained by comparing various existing Micro closed sets with newly defined Micro ψ-closed sets. The properties of proposed sets are derived. Further Micro ψ-open sets are introduced and its properties are also analyzed. Micro ψ-closure and Micro ψ-interior operators are analyzed and derived their properties. Four new Micro spaces namely Micro semi $T_{1/3}$-space, Micro semi $T_{1//2}$-space, Micro T_b-space and Micro ${}_{\psi}T_c$-space are introduced and their interrelations are obtained.

Important definitions and results:

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be a **Micro ψ-closed set** if $Mic\text{-}scl(A) \subseteq L$ whenever $A \subseteq L$ and L is Micro sg-open in U.

The following diagram exhibits the relations between Micro ψ-closed sets with other existing Micro closed sets.



where A→B represents A implies B and A↮B represents A and B are independent.

A Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be a

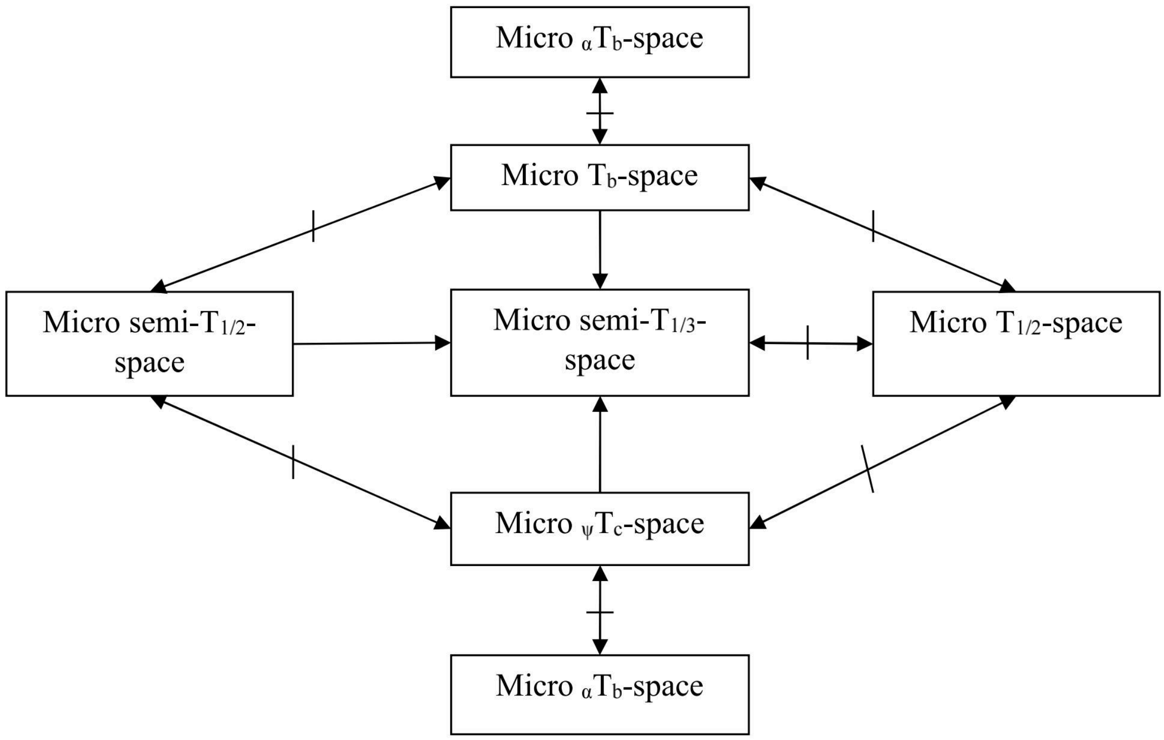
(i) Micro semi-$T_{1/3}$-space (briefly Mic-semi-$T_{1/3}$-space) if every Micro ψ-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro semi-closed in $(U, \tau_R(X), \mu_R(X))$.

(ii) Micro semi-$T_{1/2}$-space (briefly Mic-semi-$T_{1/2}$-space) if every Micro sg-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro semi-closed in $(U, \tau_R(X), \mu_R(X))$.

(iii) Micro T_b-space (briefly Mic-T_b-space) if every Micro gs-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro closed in $(U, \tau_R(X), \mu_R(X))$.

(iv) Micro ${}_{\psi}T_c$-space (briefly Mic-${}_{\psi}T_c$-space) if every Micro ψ-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro closed in $(U, \tau_R(X), \mu_R(X))$.

The following diagram shows the interrelations between the newly defined Micro spaces with already existing Micro spaces.



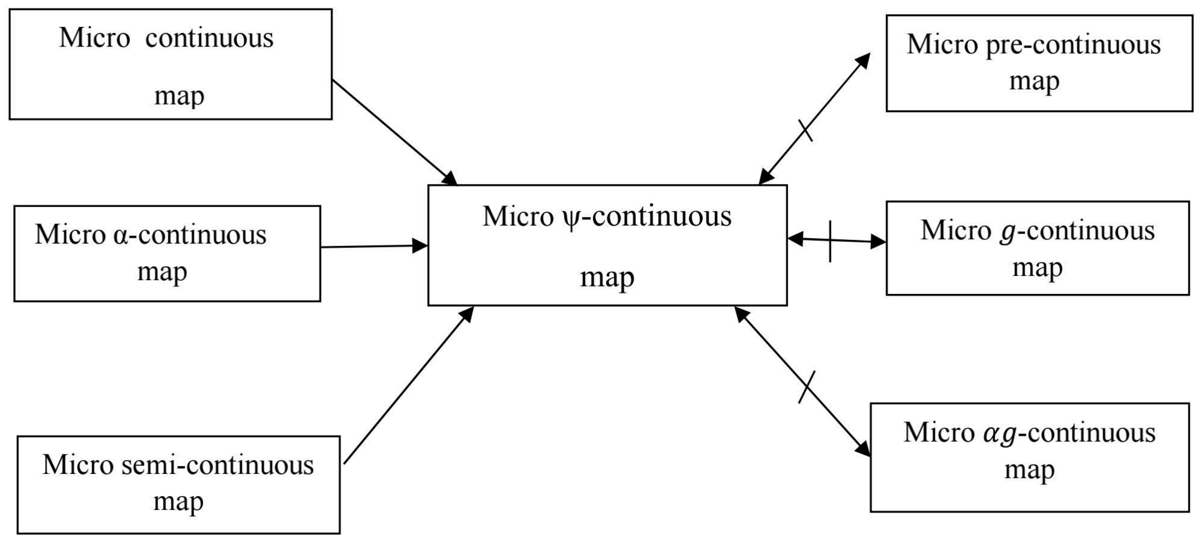
In **Chapter 3**, By using Micro ψ-closed sets we have introduced the concept of Micro ψ-continuous maps and Micro ψ-irresolute maps. The properties of Micro ψ-continuous and Micro ψ-irresolute maps are analyzed in Micro topological spaces.

Important definitions and results:

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topologcal spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ is called a **Micro ψ-continuous map** if $f^{-1}(K)$ is Micro ψ-closed in U for every Micro closed set K in V.

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topologcal spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ is called a **Micro ψ-irresolute map** if $f^{-1}(K)$ is Micro ψ-closed in U for every Micro ψ-closed set K in V.

The following diagram is the output of the comparative study of Micro ψ-continuous maps with already existing Micro continuous maps.



where A→B represents A implies B and A↮B represents A and B are independent.

REVIEW OF LITERATURE

REVIEW OF LITERATURE

Topology is a branch of mathematics which is good at extracting global qualitative features from complicated geometric structures. The word topology literally means the study of position or location. The branch of topology dealing with the basic set theoretic definitions and constructions used in topology. Recently, topology has also become an important component of applied mathematics, with many mathematical and scientists employing concepts of topology to model and understand real-world structures and phenomena. The fundamental concepts of point set topology are continuity, compactness and connectedness. The topological structures are modeled suitably in the fields of computer graphics, data mining, information systems, rough set theory, etc.

The word “Nano” comes from the Greek letter which means “dwarf” in this modern scientific sense. The topology is named as Nano topology because of its size, i.e., it has at most five elements.

The concept of Nano topology was proposed by Carmel Richard [2013] which is an extension of set theory for the study of intelligent systems characterized by insufficient and incomplete information. Nano topology is based on the concept of lower approximation, upper approximation and boundary region. He introduced weak form of Nano open sets such as Nano α-open sets, Nano semi-open sets and Nano pre-open sets. He also established strong form of Nano open sets called Nano θ -open sets. As a generalization, Nano generalized closed sets, Nano semi-generalized closed sets and Nano θ-generalized closed sets were introduced by him. A brief study of Nano regular open sets was also made by him.

Micro topology is an extension of Nano topology. Micro topology was introduced by Sakkraiveeranan Chandrasekar [2019]. The notion of rough set theory was proposed by Pawlak [2004]. Rough set theory is a new mathematical approach to imperfect knowledge. Rough set theory overlaps with many other theories. Rough set concept can be defined by means of topological operations, interior and closure called approximations. The definitions of set approximations like lower, upper approximation and boundary region can be expressed in terms of granules of knowledge.

Every Nano topology is Micro topology. Sakkraiveeranan Chandrasekar [2019] in the article "On Micro Topological Spaces" introduced Micro topology and also he analyzed Micro pre-open sets, Micro semi-open sets and derived some of their properties. He also introduced the concept of Micro continuous maps, Micro pre-continuous and Micro semi-continuous maps in Micro topological spaces.

Chandrasekar and Swathi [2018] initiated the concept of Micro α-open sets in the article entitled "Micro α-open sets in Micro Topological Spaces". They also studied Micro α-continuity in Micro topological spaces and investigated some of its important properties. In the article "On Micro α-open sets and Micro α-continuous functions in Micro Topological Spaces" by Reem O. Rasheed and Taha H. Jasim [2020], Micro α-open sets and Micro α-continuous functions are introduced and their properties are analyzed.

In the article "Micro β-open sets in Micro Topological Spaces" by Ibrahim [2020 a] the concepts of Micro β-open sets, Micro β-continuous maps and Micro β-irresolute maps are introduced and some of their properties are investigated.

Ibrahim [2020 b], in the article "On Micro $T_{1/2}$ -Space" introduced and studied the concepts of Micro generalized closed sets and obtain some of its properties and also he introduced and analyzed a space called Micro $T_{1/2}$-space. Also, the notions of Micro difference and Micro kernel of sets are investigated. θ-Continuity in Micro topological spaces was introduced by Rana H. Jassim et al. [2020]. The concepts of θ-Micro open sets and θ-Micro continuous maps and some of their properties are also studied by them.

Taha H. Jasim et al. [2021] presented an article "On Micro generalized closed sets and Micro generalized continuity in Micro Topological Spaces" and studied the important properties of Micro generalized closed sets, Micro generalized continuous maps and Micro generalized irresolute maps.

The article entitled "Micro $s\omega$ -open sets and $s\omega p$ -open sets in Micro Topological Spaces" by Jamil Mahmoud Jamil [2021] introduced a new type of open sets called Micro $s\omega$-open set in Micro topological spaces and basic characterizations and properties are obtained by him. He has also introduced Micro $s\omega$-continuous and Micro $s\omega p$-continuous functions and some of the interrelations are also analyzed.

Saja S. Mohsen [2021] introduced Micro-generalized pre-minimal closed sets in Micro topological spaces and examined its properties.

In the paper "On Strong Form of Generalized Closed Sets in Micro Topological Spaces" Bhavani [2021] introduced and studied different types of Micro generalized closed sets and their interrelations are obtained sequentially.

The paper " $m\omega$-Closed Sets in Micro Topological Spaces" was presented by Selvaraj Ganesan [2021]. He introduced a new class of sets called $m\omega$-closed sets in Micro topological spaces. Later, the definition $T m\omega$-spaces, $_{g}T m\omega$-spaces in Micro topological spaces are introduced and some of their basic properties are studied. Further he also introduced $m\omega$-continuous maps and examined some of its properties.

Anandhi and Balamani [2022 a] initiated and studied the concept of Micro α-generalized closed sets in Micro topological spaces. They have derived the interralations and properties of Micro αg-closed sets in the paper "Micro α-Generalized Closed Sets in Micro Topological Spaces". Further, they [2022 b] have also studied separation axioms on Micro αg-closed sets. They [2022 c] have also analyzed Micro αg-continuous maps and its properties in Micro topological spaces.

Subramanian Jeyashri and Selvaraj Ganesan [2020] initiated the study of Micro generalized locally closed sets in Micro topological spaces, by the article entitled "Micro Generalized Locally Closed Sets in Micro Topological Spaces". They have also studied mg-closure and mg-interior operators and studied some of their properties. Finally, they have introduced MLC-continuous, MGLC-continuous maps and MGLC-irresolute maps in Micro topological spaces.

As an extension of Micro topology, Ekram Abdul Kadir saleh and Taha Hameed Jasim [2020] in the paper "N-Micro Topological Spaces" studied the new concept of the N-Micro topology. They have generalized some definitions in general topology to Micro topological spaces. They have also studied Micro i-open set, N-Micro open set, N-Micro semi open set and N-Micro α-open set. Further, they have also defined and studied N-Micro continuous functions.

Micro Ideal Generalized Closed Sets in Micro ideal topological spaces was introduced by Selvaraj Ganesan [2020]. He introduced a new type of generalized closed sets and open sets called $m\mathfrak{I}_g$-closed set and $m\mathfrak{I}_g$-open set in Micro ideal topological spaces and investigated some of their characterizations.

CHAPTER 1

PRELIMINARIES

Definition 1.1 [Pawlak, 2004]

Let U be a nonempty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U} \{R(x): R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U} \{R(x): R(x) \cap X \neq \phi\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 1.2 [Carmel Richard, 2013]

Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$.
2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets and the complement of a Nano open set is called a Nano closed set.

Definition 1.3 [Sakkraiveeranan Chandrasekar, 2019]

Let $(U, \tau_R(X))$ be a Nano topological space. Then $\mu_R(X) = \{N \cup (N' \cap \mu): N, N' \in \tau_R(X)$ and $\mu \notin \tau_R(X)\}$ and $\mu_R(X)$ satisfies the following axioms:

(i) $U, \phi \in \mu_R(X)$.

(ii) The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.

(iii)The intersection of the elements of any finite sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then, $\mu_R(X)$ is called the Micro Topology on U with respect to X. The triplet $(U, \tau_R(X), \mu_R(X))$ is called Micro topological space and the elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 1.4 [Sakkraiveeranan Chandrasekar, 2019]

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. The intersection of all Micro closed sets containing A is called Micro closure of A and is denoted by Mic-cl(A). The union of all Micro open sets contained in A is called Micro interior of A and is denoted by Mic-int(A).

Definition 1.5 [Sakkraiveeranan Chandrasekar, 2019]

A subset A of Micro topological space $(U, \tau_R(X), \mu_R(X))$ is called Micro semi-closed (briefly Mic-semi-closed) if Mic-int(Mic-cl(A)) ⊆ A and Micro semi-open (briefly Mic-semi-open) if A ⊆ Mic-cl(Mic-int(A)).

Definition 1.6 [Sakkraiveeranan Chandrasekar, 2019]

A subset A of Micro topological space $(U, \tau_R(X), \mu_R(X))$ is called Micro pre-closed (briefly Mic-pre-closed) if Mic - cl(Mic - int(A)) ⊆ A and Micro pre-open (briefly Mic-pre-open) if A ⊆ Mic-int(Mic-cl(A)).

Definition 1.7 [Chandrasekar and Swathi, 2018]

A subset A of Micro topological space $(U, \tau_R(X), \mu_R(X))$ is called Micro α-closed (briefly Mic-α-closed) if Mic-cl(Mic-int(Mic-cl(A))) ⊆ A and Micro α-open (briefly Mic-α-open) if A ⊆ Mic-int(Mic-cl(Mic-int(A))).

Definition 1.8 [Ibrahim, 2020 a]

A subset A of Micro topological space $(U, \tau_R(X), \mu_R(X))$ is called Micro β-closed (briefly Mic-β-closed) if Mic-int(Mic-cl(Mic-int(A))) ⊆ A and Micro β-open (briefly Mic-β-open) if A ⊆ Mic-cl(Mic-int(Mic-cl(A))).

Definition 1.9 [Ibrahim, 2020 b]

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro generalized closed (briefly Mic-g-closed) if $Mic\text{-}cl(A) \subseteq L$ whenever $A \subseteq L$ and L is Micro open in U.

Definition 1.10 [Ibrahim, 2020 b]

A Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be a Micro $T_{1/2}$-space (briefly Mic-$T_{1/2}$-space) if every Micro g-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro closed in $(U, \tau_R(X), \mu_R(X))$.

Definition 1.11 [Bhavani, 2021]

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro semi generalized closed (briefly Mic-sg-closed) if $Mic\text{ - }scl(A) \subseteq L$, whenever $A \subseteq L$ and L is Mic-semi-open in U.

Definition 1.12 [Bhavani, 2021]

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro generalized semi closed (briefly Mic-gs-closed) if $Mic\text{ - }scl(A) \subseteq L$, whenever $A \subseteq L$ and L is Micro open in U.

Definition 1.13 [Anandhi and Balamani, 2022 a]

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro α-generalized closed (briefly $Mic\text{-}\alpha g$-closed) if $Mic\text{-}\alpha cl(A) \subseteq L$, whenever $A \subseteq$L and L is Micro open in U.

Definition 1.14 [Anandhi and Balamani, 2022 a]

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro generalized α-closed (briefly Mic-gα-closed) if $Mic\text{-}\alpha$cl(A)$\subseteq L$, whenever $A \subseteq$L and L is Micro α-open in U.

Definition 1.15 [Anandhi and Balamani, 2022 b]

A Micro topological space $\left(U, \tau_R(X), \mu_R(X)\right)$ is said to be a Micro $_{\alpha}T_{b}$ -space (briefly Mic-$_{\alpha}T_{b}$-space) if every Micro αg-closed subset of $\left(U, \tau_R(X), \mu_R(X)\right)$ is Micro closed in $\left(U, \tau_R(X), \mu_R(X)\right)$.

Definition 1.16 [Sakkraiveeranan Chandrasekar, 2019]

Let $\left(U, \tau_R(X), \mu_R(X)\right)$ and $\left(V, \tau_R(Y), \mu_R(Y)\right)$ be two Micro topological spaces. A map $f: \left(U, \tau_R(X), \mu_R(X)\right) \to \left(V, \tau_R(Y), \mu_R(Y)\right)$ is called Micro-continuous if $f^{-1}(K)$ is Micro-closed in U for every Micro closed set K in V.

Definition 1.17 [Sakkraiveeranan Chandrasekar, 2019]

Let $\left(U, \tau_R(X), \mu_R(X)\right)$ and $\left(V, \tau_R(Y), \mu_R(Y)\right)$ be two Micro topological spaces. A map $f: \left(U, \tau_R(X), \mu_R(X)\right) \to \left(V, \tau_R(Y), \mu_R(Y)\right)$ is called Micro semi-continuous if $f^{-1}(K)$ is Micro semi-closed in U for every Micro closed set K in V.

Definition 1.18 [Sakkraiveeranan Chandrasekar, 2019]

Let $\left(U, \tau_R(X), \mu_R(X)\right)$ and $\left(V, \tau_R(Y), \mu_R(Y)\right)$ be two Micro topological spaces. A map $f: \left(U, \tau_R(X), \mu_R(X)\right) \to \left(V, \tau_R(Y), \mu_R(Y)\right)$ is called Micro pre-continuous if $f^{-1}(K)$ is Micro pre-closed in U for every Micro closed set K in V.

Definition 1.19 [Ibrahim, 2020 b]

Let $\left(U, \tau_R(X), \mu_R(X)\right)$ and $\left(V, \tau_R(Y), \mu_R(Y)\right)$ be two Micro topological spaces. A map $f: \left(U, \tau_R(X), \mu_R(X)\right) \to \left(V, \tau_R(Y), \mu_R(Y)\right)$ is called Micro β-continuous if $f^{-1}(K)$ is Micro β-closed in U for every Micro closed set K in V.

Definition 1.20 [Reem O. Rasheed and Taha H. Jasim, 2020]

Let $\left(U, \tau_R(X), \mu_R(X)\right)$ and $\left(V, \tau_R(Y), \mu_R(Y)\right)$ be two Micro topological spaces. A map $f: \left(U, \tau_R(X), \mu_R(X)\right) \to \left(V, \tau_R(Y), \mu_R(Y)\right)$ is called Micro α-continuous if $f^{-1}(K)$ is Micro α-closed in U for every Micro closed set K in V.

Definition 1.21 [Taha et al., 2021]

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ is called Micro g-continuous if $f^{-1}(K)$ is Micro g-closed in U for every Micro closed set K in V.

Definition 1.22 [Taha et al., 2021]

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ is called Micro g-irresolute if $f^{-1}(K)$ is Micro g-closed in U for every Micro g-closed set K in V.

Definition 1.23 [Anandhi and Balamani, 2022 c]

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ is called Micro αg-continuous if $f^{-1}(K)$ is Micro αg-closed in U for every Micro closed set K in V.

Result 1.24

- **(i)** Every Micro closed set is Micro semi closed. **[Chandrasekar and Swathi, 2018]**
- **(ii)** Every Micro α-closed set is Micro semi closed. **[Chandrasekar and Swathi, 2018]**
- **(iii)** A is Micro closed if and only if $A = Mic\text{-}cl(A)$. **[Sakkraiveeranan Chandrasekar, 2019]**

CHAPTER 2

Micro ψ-Closed Sets in Micro Topological Spaces

2.1 Introduction

Chandrasekar and Swathi [2018] introduced Micro α-open sets in Micro topological spaces. The concept of Micro semi-open and Micro pre-open sets was introduced by Sakkraiveeranan Chandrasekar [2019]. Anandhi and Balamani [2022 a] introduced Micro αg-closed sets and studied its basic properties.

In this chapter we have introduced a new class of Micro closed sets called Micro ψ-closed sets which settled properly between the class of Micro semi-closed sets and the class of Micro sg-closed sets in Micro topological spaces. The interrelations are derived by comparing various Micro closed sets with newly defined Micro ψ-closed sets. Micro ψ-closure operator and Micro ψ-interior operator are introduced and their fundamental properties are examined. New Micro spaces namely Micro semi-$T_{1/3}$-space, Micro semi-$T_{1/2}$-space, Micro T_b-space and Micro ${}_{\psi}T_c$-space are introduced and their properties are analyzed.

2.2 Micro ψ-closed sets

In this section we have introduced Micro ψ-closed sets in Micro topological spaces and derived dependency and independency relations of newly defined sets with already existing Micro closed sets.

Definition 2.2.1

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro ψ-closed if $Mic\text{-}scl(A) \subseteq L$ whenever $A \subseteq L$ and L is Mic-sg-open in U.

Example 2.2.2

Let $U = \{a, b, c, d\}, U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{a, b\} \subseteq U$. Then Nano topology $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{d\} \notin \tau_R(X)$. Then Micro topology $\mu_R(X) = \{U, \phi, \{d\}, \{a, b\}, \{a, b, d\}\}$. Then $\phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, U$ are Micro ψ-closed sets.

Proposition 2.2.3

Every Micro closed set in $(U, \tau_R(X), \mu_R(X))$ is Micro ψ-closed but not conversely.

Proof: Let A be a Micro closed set then $Mic\text{-}cl(A) = A$. Let $A \subseteq L$ where L is Micro sg-open in U. Since every Micro closed set is Micro semi closed, $Mic\text{-}scl(A) \subseteq Mic\text{-}cl(A)$. Thus $Mic\text{-}scl(A) \subseteq A \subseteq L$. Hence $Mic\text{-}scl(A) \subseteq L$. Therefore A is Micro ψ-closed**.**

Example 2.2.4

Let $U = \{a, b, c, d\}$, U/R=$\{\{a\}, \{c\}, \{b, d\}\}$. Let $X = \{b, d\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{b, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Micro closed sets are $\phi, \{c\}, \{a, c\}, \{b, c, d\}, U$. Micro ψ**-**closed sets are $\phi, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, (a, c, d\}, \{b, c, d\}, U$. Here the subset $\{a, b, c\}$ is Micro ψ**-**closed, but not Micro closed.

Proposition 2.2.5

Every Micro semi closed set in $(U, \tau_R(X), \mu_R(X))$ is Micro ψ-closed but not conversely.

Proof: Let A be a Micro semi closed set and L be any Micro sg-open set containing A in U. Since A is Micro semi closed, $Mic\text{-}scl(A) = A \subseteq L, Mic\text{-}scl(A) \subseteq L$. Therefore A is Micro ψ-closed.

Example 2.2.6

Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b, c\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi\}$. Let $\mu = \{a, b\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a, b\}\}$. Micro semi closed sets are $\phi, \{c\}, \{d\}, \{c, d\}, U$. Micro ψ-closed sets are $\phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, U$. Here the subset $\{a, c, d\}$ is Micro ψ-closed, but not Micro semi closed.

Proposition 2.2.7

Every Micro α-closed set in $(U, \tau_R(X), \mu_R(X))$ is Micro ψ-closed but not conversely.

Proof: Let A be a Micro α-closed set and L be any Micro sg-open set containing A in U. Since A is Micro α-closed, $Mic\text{-}\alpha cl(A) = A$. Since every Micro α-closed set is Micro semi closed, $Mic\text{-}scl(A) \subseteq Mic\text{-}\alpha cl(A)$, $Mic\text{-}scl(A) \subseteq A \subseteq L$. Therefore A is Micro ψ-closed.

Example 2.2.8

Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$. Let $X = \{b, d\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{b, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Micro α-closed sets are $\phi, \{c\}, \{a, c\}, \{b, c, d\}, U$. Micro ψ-closed sets are $\phi, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, U$. Here the subset $\{a, b, c\}$ is Micro ψ-closed, but not Micro α-closed.

Proposition 2.2.9

Every Micro semi closed set is Micro sg-closed.

Proof: Let A be a Micro semi closed set and L be any Micro semi-open set containing A in U. Since A is Micro semi closed, $Mic\text{-}scl(A) = A, Mic\text{-}scl(A) = A \subseteq L$. Therefore A is Micro sg- closed.

Proposition 2.2.10

Every Micro ψ-closed set in $(U, \tau_R(X), \mu_R(X))$ is Micro sg-closed but not conversely.

Proof: Let A be a Micro ψ-closed set and L be any Micro semi open set containing A in U. Since every semi open set is Micro sg-open and A is Micro ψ-closed, $Mic\text{-}scl(A) \subseteq L$. Therefore A is Micro sg-closed.

Example 2.2.11

Let $U = \{a, b, c, d\}$, U/R=$\{\{a, b\}, \{c, d\}\}$. Let $X = \{a, b, c\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{c\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{c\}, \{a, b\}, \{a, b, c\}\}$. Micro ψ-closed sets are $\phi, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, U$. Micro sg-closed sets are $\phi, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}, U$. Here the subset $\{a, c, d\}$ is Micro sg-closed, but not Micro ψ-closed.

Proposition 2.2.12

Every Micro ψ-closed set in $(U, \tau_R(X), \mu_R(X))$ is Micro gs-closed but not conversely.

Proof: Let A be a Micro ψ-closed set and L be any Micro open set containing A in U. Since every Micro open set is Micro sg-open and A is Micro ψ-closed, Mic-$scl(A) \subseteq L$. Therefore A is Micro gs-closed.

Example 2.2.13

Let $U = \{a, b, c, d\}$, U/R=$\{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{d\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{d\}, \{a, b\}, \{a, b, d\}\}$. Micro ψ-closed sets are $\phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, U$. Micro gs-closed sets are $\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{b, c,\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, U$. Here the subset $\{b, c, d\}$ is Micro gs-closed, but not Micro ψ-closed.

Remark 2.2.14

The following example shows that Micro ψ-closed set is independent from Micro pre-closed set.

Example 2.2.15

Let $U = \{a, b, c, d\}, U/R$=$\{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{c, d\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$. Micro pre-closed sets are $\phi, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, U$. Micro ψ-closed sets are $\phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, U$. Here the subset $\{c\}$ is Micro pre-closed, but not Micro ψ-closed and the subset $\{a, b, d\}$ is Micro ψ-closed, but not Micro pre-closed.

Remark 2.2.16

The following example shows that Micro ψ-closed set is independent from Micro αg-closed set.

Example 2.2.17

Let $U = \{a, b, c, d\}$, U/R=$\{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{d\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{d\}, \{a, b\}, \{a, b, d\}\}$.

Micro αg -closed sets are $\phi, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, U$. Micro ψ-closed sets are $\phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, U$. Here the subset $\{a, c\}$ is Micro αg-closed, but not Micro ψ-closed and the subset $\{d\}$ is Micro ψ-closed, but not Micro αg-closed.

Remark 2.2.18

The following example shows that Micro ψ-closed set is independent from Micro g-closed set.

Example 2.2.19

Let $U = \{a, b, c\}$, $U/R = \{\{c\}, \{a, b\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{a, b\}\}$. Micro g-closed sets are $\phi, \{c\}, \{a, c\}, \{b, c\}, U$. Micro ψ-closed sets are $\phi, \{b\}, \{c\}, \{b, c\}, U$. Here the subset $\{a, c\}$ is Micro g-closed, but not Micro ψ-closed and the subset $\{b\}$ is Micro ψ-closed, but not Micro g-closed.

Remark 2.2.20

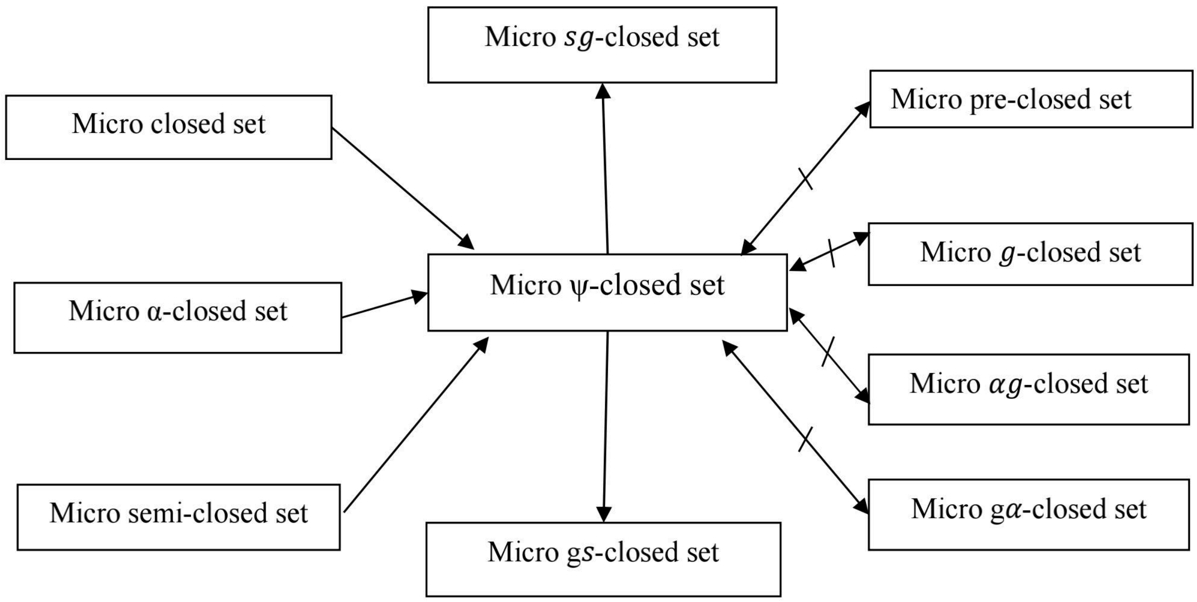
The following example shows that Micro ψ-closed set is independent from Micro $g\alpha$-closed set.

Example 2.2.21

Let $U = \{a, b, c, d\}$, $U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{d\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{d\}, \{a, b\}, \{a, b, d\}\}$. Micro gα-closed sets are $\phi, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, U$. Micro ψ-closed sets are $\phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, U$. Here the subset $\{b, c\}$ is Micro $g\alpha$-closed, but not Micro ψ-closed and the subset $\{a, b\}$ is Micro ψ -closed but not Micro $g\alpha$-closed.

Remark 2.2.22

The following diagram shows the dependency and independency relations of Micro ψ-closed sets with already existing Micro closed sets.



2.3 Properties of Micro ψ-closed sets

In this section the fundamental properties of Micro ψ-closed sets are derived.

Theorem 2.3.1

If A is a Micro ψ-closed set of $(U, \tau_R(X), \mu_R(X))$ such that $A \subseteq B \subseteq Mic$-$scl(A)$, then B is also a Micro ψ-closed set of $(U, \tau_R(X), \mu_R(X))$.

Proof: Suppose that A is a Micro ψ-closed set. Let L be a Micro-sg-open set such that $B \subseteq L$. Then $A \subseteq L$. Since A is Micro ψ-closed, Mic-$scl(A) \subseteq L$. Now Mic-$scl(B) \subseteq Mic$-$scl(Mic$-$scl(A)) = Mic$-$scl(A) \subseteq L$. Therefore B is also a Micro ψ-closed set of $(U, \tau_R(X), \mu_R(X))$.

Theorem 2.3.2

If A is both Micro sg-open and Micro ψ-closed set in $\left(U, \tau_R(X), \mu_R(X)\right)$, then A is Micro semi closed in $(U, \tau_R(X), \mu_R(X))$.

Proof: Let A be Micro sg-open and Micro ψ-closed set in $(U, \tau_R(X), \mu_R(X))$ then by the definition of Micro ψ-closed set, Mic-$scl(A) \subseteq A$. Always $A \subseteq Mic$-$scl(A)$. Therefore Mic-$scl(A) = A$. Hence A is Micro semi closed.

Remark 2.3.3

The intersection of any two Micro ψ-closed sets in $(U, \tau_R(X), \mu_R(X))$ need not to be a Micro ψ-closed set in $(U, \tau_R(X), \mu_R(X))$ as seen from the following example.

Example 2.3.4

Let $U = \{a, b, c, d\}$, $U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{c\} \subseteq U$. Then $\tau_R(X)=\{U, \phi, \{c\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{b\}, \{c\}, \{b, c\}, U\}$. Micro ψ-closed sets are $\phi, \{a\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, U$. Here the subsets $\{a, c\}$ and $\{c, d\}$ are Micro ψ-closed sets but their intersection $\{a, c\} \cap \{c, d\} = \{c\}$ is not Micro ψ-closed.

Theorem 2.3.5

Let A be a Micro ψ-closed set of $(U, \tau_R(X), \mu_R(X))$, then $Mic\text{-}scl(A) - A$ contains no non empty Micro closed set.

Proof: Suppose that A is a Micro ψ-closed set of $(U, \tau_R(X), \mu_R(X))$. Let F be a Micro closed subset of $Mic\text{-}scl(A) - A$. Then F^c is Micro open and hence Micro sg-open such that $A \subseteq F^c$. Since A is a Micro ψ-closed set, $Mic\text{-}scl(A) \subseteq F^c$. Thus $F \subseteq$ (Mic-$scl(A)$)^c. Therefore $F \subseteq$(Mic-$scl(A)$)$^c \cap Mic\text{-}scl(A) = \phi$. Hence $F = \phi$.

Theorem 2.3.6

Let A be a Micro ψ-closed set of $(U, \tau_R(X), \mu_R(X))$ if and only if $Mic\text{-}scl(A) - A$ does not contain any non empty Micro sg-closed set.

Proof: (**Necessity**) Suppose that A is a Micro ψ-closed set of $(U, \tau_R(X), \mu_R(X))$. Let F be a Micro sg-closed set such that $F \subseteq Mic\text{-}scl(A) - A$. Then $A \subseteq F^c$. Since A is a Micro ψ-closed set and F^c is Micro sg-open, then $Mic\text{-}scl(A) \subseteq F^c$. This implies $F \subseteq (Mic\text{-}scl(A))^c$. So $F \subseteq (Mic\text{-}scl(A))^c \cap (Mic\text{-}scl(A) - A) \subseteq (Mic\text{-}scl(A))^c \cap Mic\text{-}scl(A) = \phi$. Therefore $F = \phi$.

Sufficiency: Suppose that $Mic\text{-}scl(A) - A$ contains no non empty Micro sg-closed set. Let $A \subseteq H$ and H be Micro sg-open. If $Mic\text{-}scl(A)$ is not a subset of H then $Mic\text{-}scl(A) \cap H^C$ is a non empty Micro sg-closed subset of $Mic\text{-}scl(A) - A$, which is a contradiction. Therefore $Mic\text{-}scl(A) \subseteq H$ and hence A is Micro ψ-closed.

Theorem 2.3.7

If Mic-scl$(\{x\}) \cap A \neq \phi$ holds for every $x \in Mic\text{-}scl(A)$, then $Mic\text{-}scl(A) - A$ does not contain a non-empty Micro semi closed set.

Proof: Suppose there exists a non-empty Micro semi closed set F such that $F \subseteq Mic\text{-}scl(A) - A$. Let $x \in F$, then $x \in Mic\text{-}scl(A)$. It follows that $F \cap A = [Mic\text{-}scl(A) - A] \cap A \supseteq Mic$-scl$(\{x\}) \cap A \neq \phi$. Hence $F \cap A \neq \phi$, which is a contradiction. Thus $F = \phi$.

Theorem 2.3.8

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. Then, for each $x \in U$, either $\{x\}$ is Micro sg-open or $U - \{x\}$ is Micro ψ-closed.

Proof: Let $x \in U$ and $\{x\}$ is not Micro sg-closed in U. Then $U - \{x\}$ is not Micro sg-open in U. Hence U is the only Micro sg-open set containing $U - \{x\}$. i.e.,$U - \{x\} \subseteq U$. Hence Mic-scl$(U - \{x\}) \subseteq U$. Thus $U - \{x\}$ is Micro ψ-closed.

Theorem 2.3.9

Let A be a Micro ψ-closed set of $(U, \tau_R(X), \mu_R(X))$, then A is Micro semi-closed if and only if $Mic\text{-}scl(A) - A$ is Micro sg-closed.

Proof:(Necessity) Let A be a Micro semi closed subset of $(U, \tau_R(X), \mu_R(X))$, then $Mic\text{-}scl(A) = A$ and therefore $Mic\text{-}scl(A) - A = \phi$ which is a Micro sg-closed set.

Sufficiency: Let $Mic\text{-}scl(A) - A$ be a Micro sg-closed set. Since A is Micro ψ-closed by Theorem 2.3.6, $Mic\text{-}scl(A) - A$ does not contain any non empty Micro sg-closed set which implies $Mic\text{-}scl(A) - A = \phi$.(i.e) $Mic\text{-}scl(A) = A$. Hence A is Micro semi-closed.

2.4 Micro ψ-closure operator

In this section the notion of Micro ψ-closure operator is introduced and its properties are derived.

Definition 2.4.1

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space and $K \subseteq U$. Then the Micro ψ-closure of a set K is denoted by Mic-ψ-$cl(K)$ and is defined as Mic-ψ-$cl(K)$= ∩ {T: T is a Micro ψ-closed set in U and $K \subseteq T$}.

Remark 2.4.2

For a subset K of a Micro topological space $(U, \tau_R(X), \mu_R(X))$, $K \subseteq Mic$- ψ-$cl(K) \subseteq Mic$-$scl(K)$.

Proof: Follows from the Proposition 2.2.3 and Definition 2.4.1.

Remark 2.4.3

Both inclusion relations in Remark 2.4.2 may be proper as seen from the following example.

Example 2.4.4

Let $U = \{a, b, c, d\}$, $U/R = \{\{a, b\}, \{c, d\}\}$. Let $X = \{a, b, c\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{c\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{c\}, \{a, b\}, \{a, b, c\}, U\}$ and $K = \{a, d\}$. Then Mic- ψ-$cl(K) = \{a, b, d\}$ and Mic-$scl(K) = U$ and so $K \subseteq Mic$-ψ-$cl(K) \subseteq Mic$-$scl(K)$.

Proposition 2.4.5

Let K and T be any two subsets of $(U, \tau_R(X), \mu_R(X))$. Then the following statements are true

(a) Mic-ψ-$cl(\phi) = \phi$ and Mic-ψ-$cl(U) = U$.

(b) $K \subseteq Mic$-ψ-$cl(K)$.

(c) If T is any Micro ψ-closed set containing in K, then Mic-ψ-$cl(K) \subseteq T$.

(d) If $K \subseteq T$, then Mic-ψ-$cl(K) \subseteq Mic$-ψ-$cl(T)$.

(e) Mic-ψ-$cl(K) \cup Mic$-ψ-$cl(T) \subseteq Mic$-ψ-$cl(K \cup T)$.

(f) Mic-ψ-$cl(K \cap T) \subseteq Mic$-ψ-$cl(K) \cap Mic$-ψ-$cl(T)$.

(g) Mic-ψ-$cl(Mic$-ψ-$cl(K)) = Mic$-ψ-$cl(K)$.

Proof:

(a) Follows from the Definition 2.4.1.

(b) By the definition of Mic-ψ-$cl(K)$, it is obvious that $K \subseteq Mic$-ψ-$cl(K)$.

(c) Let T be any Micro ψ-closed set containing K. Since Mic-ψ-$cl(K)$ is the intersection of all Micro ψ-closed sets containing K, Mic-ψ-$cl(K)$ is contained in every Micro ψ-closed set containing K. Hence Mic-ψ-$cl(K) \subseteq T$.

(d) Follows from the Definition 2.4.1.

(e) Since $K \subseteq K \cup T$ and $T \subseteq K \cup T$ by (d) Mic-ψ-$cl(K) \subseteq Mic$-ψ-$cl(K \cup T)$ and Mic-ψ-$cl(T) \subseteq Mic$-ψ-$cl(K \cup T)$. Hence Mic-ψ-$cl(K) \cup Mic$-ψ-$cl(T) \subseteq Mic$-ψ-$cl(K \cup T)$.

(f) Since $K \cap T \subseteq K$ and $K \cap T \subseteq T$ by (d) Mic-ψ-$cl(K \cap T) \subseteq Mic$-ψ-$cl(K)$ and Mic-ψ-$cl(K \cap T) \subseteq Mic$-ψ-$cl(T)$. Hence Mic-ψ-$cl(K \cap T) \subseteq Mic$-ψ-$cl(K) \cap Mic$-ψ-$cl(T)$.

(g) By (c), Mic-ψ-$cl(K) \subseteq T$. Let K be any subset of U. By the definition of Mic-ψ-$cl(K)$, Mic-ψ-$cl(K)= \cap \{T: T$ is a Micro ψ-closed set in U and $K \subseteq T\}$. If $K \subseteq T$ then Mic -ψ- $cl(K) \subseteq T$. Since T is a Micro-ψ-closed set containing Mic-ψ-$cl(K)$. By (c) Mic-ψ-$cl(Mic$-ψ-$cl(K)) \subseteq T$. Hence Mic-ψ-$cl(Mic$-ψ-$cl(K)) \subseteq \cap \{T: T$ is a Micro ψ-closed set in U and $K \subseteq T\}$. That is Mic-ψ-$cl(Mic$-ψ-$cl(K)) = Mic$-ψ-$cl(K)$.

Remark 2.4.6

Since Mic-ψ-$cl(K) \cup Mic$-ψ-$cl(T) \neq Mic$-ψ-$cl(K \cup T)$, Micro-ψ-closure is not a Kuratowski closure operator on $(U, \tau_R(X), \mu_R(X))$.

Remark 2.4.7

In general, the reverse inclusion of (f) in Proposition 2.4.5 is not true.

Example 2.4.8

Let $U = \{a, b, c, d\}, U/R = \{\{a, b\}, \{c, d\}\}$. Let $X = \{a, b, c\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{c\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{c\}, \{a, b\}, \{a, b, c\}, U\}$ and $K = \{a\}$ and $T = \{d\}$ Then Mic-ψ-$cl(K) = \{a, d\}$ and Mic-ψ-$cl(T) = \{d\}$, $K \cap T = \phi$, Mic-ψ-$cl(K \cap T) = \phi$. But Mic-ψ-$cl(K) \cap Mic$-ψ-$cl(T) = \{d\}$.

Proposition 2.4.9

Let K be any subset of $(U, \tau_R(X), \mu_R(X))$. If K is Micro ψ-closed in $(U, \tau_R(X), \mu_R(X))$ then Mic-ψ-$cl(K) = K$.

Proof: Assume that a set K is Micro ψ-closed in $(U, \tau_R(X), \mu_R(X))$. By Definition 2.4.1, $K \subseteq Mic$-ψ-$cl(K)$. Also K is a Micro ψ-closed set containing K, From Proposition 2.4.5(c), Mic-ψ-$cl(K) \subseteq K$. Hence Mic-ψ-$cl(K) = K$.

Remark 2.4.10

Converse part of Proposition 2.4.9 need not be true as seen from the following example.

Example 2.4.11

Let $U = \{a, b, c, d\}, U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{c\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{c\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{b\}, \{c\}, \{b, c\}, U\}$ and $K = \{b, c\}$ then Mic-ψ-$cl(\{b, c\}) = \cap\{$ all Micro ψ-closed sets containing $\{b, c\}\} = \{a, b, c\} \cap \{b, c, d\} \cap U = \{b, c\}$ but K is not Micro ψ-closed.

Remark 2.4.12

Let $K \subseteq U$. Then Mic-ψ-$cl(K)$ need not be Micro ψ-closed.

Example 2.4.13

Let $U = \{a, b, c, d\}, U/R = \{\{c\}, \{d\}, \{a, b\}\}$, Let $X = \{c\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{c\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{b\}, \{c\}, \{b, c\}, U\}$ and $K = \{b, c\}$ then Mic-ψ-$cl(\{c\}) = \cap\{$ all Micro ψ-closed sets containing $\{c\}\} = \{a, c\} \cap \{c, d\} \cap \{a, b, c\} \cap \{a, c, d\} \cap \{b, c, d\} \cap U = \{c\}$. But $\{c\}$ is not a Micro ψ-closed set.

2.5 Micro ψ-open sets

In this section the concept of Micro ψ-open sets is introduced and its properties are studied in Micro topological spaces.

Definition 2.5.1

A subset A of a Micro topological space $(U, \tau_R(X), \mu_R(X))$ is called Micro ψ-open (briefly Mic-ψ-open) if its complement A^c is Micro ψ-closed in $(U, \tau_R(X), \mu_R(X))$.

Example 2.5.2

Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$. Let $X = \{b, d\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{b, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{a\}, \{b, d\}, \{a, b, d\}, U\}$. Then $\phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}, U$ are Micro ψ-open sets.

Proposition 2.5.3

Every Micro open set is Micro ψ-open but not conversely.

Proof: Let A be a Micro open set in U. Then A^c is Micro closed in U. By Proposition 2.2.3, A^c is Micro-ψ-closed in U. Hence A is Micro ψ-open in U.

Example 2.5.4

Let $= \{a, b, c, d\}$, $U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{\phi, \{a, b\}, U\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{d\}, \{a, b\}, \{a, b, d\}, U\}$. Micro ψ-open sets are $\phi, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, U$. Here the subset $\{c, d\}$ is Micro ψ-open but not Micro open.

Proposition 2.5.5

Every Micro α-open set is Micro ψ-open but not conversely.

Proof: Let A be a Micro α-open set in U. Then A^c is Micro α-closed in U. By Proposition 2.2.7, A^c is Micro ψ-closed in U. Hence A is Micro ψ-open in U.

Example 2.5.6

Let $= \{a, b, c, d\}, U/R = \{\{a\}, \{c\}, \{b, d\}\}$. Let $X = \{b, d\} \subseteq U$. Then $\tau_R(X) = \{\phi, \{b, d\}, U\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{a\}, \{b, d\}, \{a, b, d\}, U\}$. Micro α-open sets are $\phi, \{a\}, \{b, d\}, \{a, b, d\}, U$. Micro ψ-open sets are $\phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}, U$. Then the subset $\{d\}$ is Micro ψ-open but not Micro α-open.

Proposition 2.5.7

Every Micro semi-open set is Micro ψ-open.

Proposition 2.5.8

Every Micro ψ-open set is Micro sg-open.

Lemma 2.5.9

For a subset A of $(U, \tau_R(X), \mu_R(X))$, $Mic\text{-}scl(U - A) = U - Mic\text{-}sint(A)$.

Theorem 2.5.10

A subset A of a Micro topological space $(U, \tau_R(X), \mu_R(X))$ is Micro ψ-open if and only if $L \subseteq Mic\text{-}sint(A)$ whenever $L \subseteq A$ and L is Micro sg-closed.

Proof: Assume that A is Micro ψ-open. Then A^c is Micro ψ-closed. Let L be a Micro sg-closed set in U contained in A. Then L^c is a Micro sg-open set in U containing A^c. Since A^c is Micro ψ-closed, $Mic\text{-}scl(A^c) \subseteq L^c$ equivalently $L \subseteq Mic\text{-}sint(A)$. Conversely assume that L is contained in $Mic\text{-}sint(A)$ whenever L is contained in A and L is Micro sg-closed in U. Let A^c be contained in L, where L is Micro sg-open. Then L^c is contained in A. By hypothesis $L^c \subseteq Mic\text{-}sint(A)$. This implies $[Mic\text{-}sint(A)]^c \subseteq L$. That is $Mic\text{-}scl(A^c) \subseteq$ L. Therefore A^c is Micro ψ-closed. Hence A is Micro ψ-open in U.

Theorem 2.5.11

If A and B are Micro ψ-open sets in $(U, \tau_R(X), \mu_R(X))$, then $A \cap B$ is Micro ψ-open in $(U, \tau_R(X), \mu_R(X))$.

Proof : Let A and B be Micro ψ-open sets in $(U, \tau_R(X), \mu_R(X))$. Then $U - A$ and $U - B$ are Micro ψ-closed sets and $(U - A) \cup (U - B) = U - (A \cap B)$ is Micro ψ-closed. Hence $A \cap B$ is Micro ψ-open in U.

Theorem 2.5.12

Let A be Micro ψ-open in $(U, \tau_R(X), \mu_R(X))$. If K is Micro sg-open and $Mic\text{-}sint(A) \cup A^c \subseteq K$, then $K = U$.

Proof: Let A be a Micro ψ-open set in $(U, \tau_R(X), \mu_R(X))$ and K be Micro sg-open and $Mic\text{-}sint(A) \cup A^c \subseteq K$. This gives $K^c \subseteq [Mic\text{-}sint(A) \cup A^c]^c = [Mic\text{-}sint(A)]^c \cap A = [Mic\text{-}sint(A)]^c - A^c = Mic\text{-}scl(A^c) - A^c$. Since A^c is Micro ψ-closed and K^c is Micro sg-closed by Theorem 2.3.6, it follows that $K^c = \phi$. Therefore $K = U$.

2.6 Micro ψ- interior operator

In this section Micro ψ-interior operator is introduced and its properties are examined.

Definition 2.6.1

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space and $K \subseteq U$. Then the Micro ψ-interior of a set K is denoted by $Mic\text{-}\psi\text{-}int(K)$ and is defined as $Mic\text{-}\psi\text{-}int(K) = \cup \{T : T$ is a Micro ψ-open set in U and $K \supseteq T\}$.

Remark 2.6.2

For a subset K of a Micro topological space $(U, \tau_R(X), \mu_R(X))$, $Mic\text{-}sint(K) \subseteq Mic\text{-}\psi\text{-}int(K) \subseteq K$.

Proposition 2.6.3

Let K and T be any two subsets of $(U, \tau_R(X), \mu_R(X))$. Then the following statements are true.

(a) Mic-ψ-$int(\phi) = \phi$ and Mic-ψ-$int(U) = U$.

(b) Mic-ψ-$int(K) \subseteq K$.

(c) If T is any Micro ψ-closed set contained in K, then $T \subseteq Mic$-ψ-$int(K)$.

(d) If $K \subseteq T$, then Mic-ψ-$int(K) \subseteq Mic$-ψ-$int(T)$.

(e) Mic-ψ-$int(K \cap T) \subseteq Mic$-ψ-$int(K) \cap Mic$-ψ-$int(T)$.

(f) Mic-ψ-$int(K \cup T) \supseteq Mic$-ψ-$int(K) \cup Mic$-ψ-$int(T)$.

(g) Mic-ψ-$int(Mic$-ψ-$int(K)) = Mic$-ψ-$int(K)$.

Proof: Follows from the Proposition 2.4.5.

Proposition 2.6.4

Let K be any subset of $(U, \tau_R(X), \mu_R(X))$. If K is Micro ψ-open in $(U, \tau_R(X), \mu_R(X))$, then Mic-ψ-$int(K) = K$.

Remark 2.6.5

Converse part of Proposition 2.6.4 need not be true as observed from the following example.

Example 2.6.6

Let $U = \{a, b, c, d\}$, $U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{c\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{c\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{b\}, \{c\}, \{b, c\}, U\}$. Mic-ψ-int $(\{a, d\}) = \cup\{$ all Micro ψ-open sets contained in $\{a, d\}\} = \{a\} \cup \{d\} \cup \phi = \{a, d\}$. But $\{a, d\}$ is not Micro ψ-open in $(U, \tau_R(X), \mu_R(X))$.

2.7 Application of Micro ψ-closed sets

As an application of Micro ψ-closed sets four new spaces namely, Micro semi-$T_{1/3}$-space, Micro semi-$T_{1/2}$-space, Micro T_b-space and Micro $_{\psi}T_c$-space are introduced and their properties and interrelations are studied.

Definition 2.7.1

A Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be a

(i) Micro semi-$T_{1/3}$-space (briefly Mic-semi-$T_{1/3}$-space) if every Micro ψ-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro semi-closed in $(U, \tau_R(X), \mu_R(X))$.

(ii) Micro semi-$T_{1/2}$-space (briefly Mic-semi-$T_{1/2}$-space) if every Micro sg-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro semi-closed in $(U, \tau_R(X), \mu_R(X))$.

(iii) Micro T_b-space (briefly Mic-T_b-space) if every Micro gs-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro closed in $(U, \tau_R(X), \mu_R(X))$.

(iv) Micro ${}_\psi T_c$-space (briefly Mic-${}_\psi T_c$-space) if every Micro ψ-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro closed in $(U, \tau_R(X), \mu_R(X))$.

Proposition 2.7.2

Every Micro semi-$T_{1/2}$-space is a Micro semi-$T_{1/3}$-space but not conversely.

Proof: Assume that a set A is Micro ψ-closed in $(U, \tau_R(X), \mu_R(X))$. Since every Micro ψ-closed set is Micro sg-closed and $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/2}$-space, A is Micro semi-closed in $(U, \tau_R(X), \mu_R(X))$. Hence $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/3}$-space.

Example 2.7.3

Let $U = \{a, b, c\}$, $U/R = \{\{a, b, c\}\}$. Let $X = \{b, c\} \subseteq U$. Then $\tau_R(X)$=$\{U, \phi, \{b, c\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{b, c\}\}$. Micro semi-closed sets are $\phi, \{a\}, \{b, c\}, U$. Micro sg-closed sets are $\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, U$. Micro ψ-closed sets are $\phi, \{a\}, \{b, c\}, U$. Here $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/3}$-space but not a Micro semi-$T_{1/2}$-space, since the subset $\{a, b\}$ is Micro sg-closed, but not Micro semi-closed in $(U, \tau_R(X), \mu_R(X))$.

Proposition 2.7.4

Every Micro T_b-space is a Micro semi-$T_{1/3}$ -space but not conversely.

Proof: Assume that a set A is Micro ψ-closed in $(U, \tau_R(X), \mu_R(X))$. Since every Micro ψ-closed set is Micro gs-closed and $(U, \tau_R(X), \mu_R(X))$ is a Micro T_b- space, A

is Micro closed and hence Micro semi-closed in $(U, \tau_R(X), \mu_R(X))$. Hence $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/3}$ -space.

Example 2.7.5

Let $U = \{a, b, c, d\}$, $U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{a, b\}\}$. Micro closed sets are $\phi, \{c, d\}, \{b, c, d\}, U$. Micro semi-closed sets are $\phi, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}, U$. Micro gs-closed sets are $\phi, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, U$. Micro ψ-closed sets are $\phi, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}, U$. Here $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/3}$- space but not a Micro T_b-space, since the subset $\{a, c\}$ is Micro gs-closed, but not Micro closed in $(U, \tau_R(X), \mu_R(X))$.

Proposition 2.7.6

Every Micro $_{\psi}T_c$ - space is a Micro semi-$T_{1/3}$-space but not conversely.

Proof: Assume that a set A is Micro ψ-closed in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro $_{\psi}T_c$-space, A is Micro closed and hence Micro semi-closed in $(U, \tau_R(X), \mu_R(X))$. Hence $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/3}$-space.

Example 2.7.7

Let $U = \{a, b, c, d\}$, $U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{d\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{d\}, \{a, b\}, \{a, b, d\}\}$. Micro closed sets are $\phi, \{c\}, \{c, d\}, \{a, b, c\}, U$. Micro semi-closed sets are $\phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, U$. Micro ψ-closed sets are $\phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c,\}, U$. Here $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/3}$-space but not a Micro $_{\psi}T_c$-space, since the subset $\{a, b\}$ is Micro ψ-closed, but not Micro closed in $(U, \tau_R(X), \mu_R(X))$.

Proposition 2.7.8

If $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/3}$-space then Mic-ψ-$scl(B) = Mic$-$scl(B)$ for each subset B of $(U, \tau_R(X), \mu_R(X))$.

Proof: Since $(U, \tau_R(X), \mu_R(X))$ is Micro semi-$T_{1/3}$-space and every Micro semi closed set is Micro ψ-closed in $(U, \tau_R(X), \mu_R(X))$, Mic-ψ-$C(U, \tau_R(X), \mu_R(X)) = Mic$ - $sC(U, \tau_R(X), \mu_R(X))$. Hence Mic-ψ-$cl((B)) = Mic$-$scl((B))$ for each subset B of $(U, \tau_R(X), \mu_R(X))$.

Theorem 2.7.9

If $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/3}$-space, then for each $x \in U$ either $\{x\}$ is Micro sg-closed or Micro semi-open.

Proof: Let $x \in U$ and suppose $\{x\}$ is not a Micro sg-closed set in $(U, \tau_R(X), \mu_R(X))$. Then $U - \{x\}$ is not a Micro sg-open set. Hence U is only Micro sg-open set containing $U - \{x\}$. So $U - \{x\}$ is a Micro ψ-closed set in U. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/3}$-space, $U - \{x\}$ is Micro semi-closed in U or equivalently $\{x\}$ is Micro semi-open in $(U, \tau_R(X), \mu_R(X))$.

Theorem 2.7.10

For a Micro topological space $(U, \tau_R(X), \mu_R(X))$, the following conditions are equivalent:

1. $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/3}$ -space.
2. Every $\{x\}$ is either Micro sg-closed or Micro semi-open.

Proof: 1⇒2 Let $x \in U$. Suppose that $\{x\}$ is a not Micro sg-closed set of U. Then $U - \{x\}$ is not a Micro sg-open set. Hence U is only Micro sg-open set containing $U - \{x\}$. So $U - \{x\}$ is a Micro ψ-closed set in U. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/3}$-space, $U - \{x\}$ is Micro semi-closed in U. Hence $\{x\}$ is Micro semi open in $(U, \tau_R(X), \mu_R(X))$.

2⇒1 Let A be a Micro ψ-closed set of $(U, \tau_R(X), \mu_R(X))$ and $x \in Mic$-$scl(A)$. We show that $x \in A$ for the following cases.

Case(1): Assume that $\{x\}$ is a Micro sg-closed and $x \notin A$. Then Mic-$scl(A) - A$ contains a non-empty Micro sg-closed $\{x\}$. This contradicts Theorem 2.3.6 as A is a Micro ψ-closed set. Therefore $x \in A$.

Case(2): Assume that $\{x\}$ is a Micro semi-open set. Then $U-\{x\}$ is Micro semi-closed. If $x \notin A$, then $A \subseteq U-\{x\}$. Since $x \in Mic\text{-}scl(A)$, we have $x \in U-\{x\}$, which is a contradiction. Hence $x \in A$.

Remark 2.7.11

The following examples show that Micro semi-$T_{1/3}$-space is independent from Micro $T_{1/2}$- space.

Example 2.7.12

Let $U=\{a,b,c\}$, $U/R=\{\{a,b\},\{c\}\}$. Let $X=\{a,b\} \subseteq U$. Then $\tau_R(X)=\{U,\phi,\{a,b\}\}$. Let $\mu=\{c\} \notin \tau_R(X)$. Then $\mu_R(X)=\{U,\phi,\{c\},\{a,b\}\}$. Micro closed sets are $\phi,\{c\},\{a,b\},U$. Micro semi-closed sets are $\phi,\{c\},\{a,b\},U$. Micro g-closed sets are $\phi,\{a\},\{b\},\{c\},\{a,b\},U$. Micro ψ-closed sets are $\phi,\{c\},\{a,b\},U$. Here $(U,\tau_R(X),\mu_R(X))$ is a Micro semi-$T_{1/3}$-space but not a Micro $T_{1/2}$-space, since the subset $\{a\}$ is Micro g-closed, but not Micro closed in $(U,\tau_R(X),\mu_R(X))$.

Example 2.7.13

Let $U=\{a,b,c,d\}$, $U/R=\{\{a\},\{c\},\{b,d\}\}$. Let $X=\{b,d\} \subseteq U$. Then $\tau_R(X)=\{U,\phi,\{b,d\}\}$. Let $\mu=\{b\} \notin \tau_R(X)$. Then $\mu_R(X)=\{U,\phi,\{b\},\{b,d\}\}$. Micro closed sets are $\phi,\{a,c\},\{a,c,d\},U$. Micro semi-closed sets are $\phi,\{a\},\{c\},\{d\},\{a,c\},\{a,d\},\{c,d\},\{a,c,d\},U$. Micro g-closed sets are $\phi,\{a,c\},\{a,c,d\},U$. Micro ψ-closed set are $\phi,\{a\},\{c\},\{d\},\{a,c\},\{a,d\},\{c,d\},\{a,c,d\},\{a,b,d\},\{b,c,d\},U$. Here $(U,\tau_R(X),\mu_R(X))$ is Micro $T_{1/2}$-space but not a Micro semi-$T_{1/3}$-space, since the subset $\{a,b,d\}$ is Micro ψ-closed, but not Micro semi closed in $(U,\tau_R(X),\mu_R(X))$.

Remark 2.7.14

The following examples show that Micro $_{\psi}T_c$-space is independent from Micro $T_{1/2}$-space.

Example 2.7.15

Let $U=\{a,b,c\}$, $U/R=\{\{c\},\{a,b\}\}$. Let $X=\{a,b\} \subseteq U$. Then $\tau_R(X)=\{U,\phi,\{a,b\}\}$. Let $\mu=\{c\} \notin \tau_R(X)$. Then $\mu_R(X)=\{U,\phi,\{c\},\{a,b\}\}$. Micro closed sets are $\phi,\{c\},\{a,b\},U$. Micro g-closed sets are $\phi,\{a\},\{b\},\{c\},\{a,b\},U$. Micro ψ-closed sets are $\phi, \{c\}, \{a,b\}, U$. Here $(U, \tau_R(X), \mu_R(X))$ is a Micro ${}_{\psi}T_c$-space but not a Micro $T_{1/2}$-space, since the subset $\{b\}$ is Micro g-closed, but not Micro closed in $(U, \tau_R(X), \mu_R(X))$.

Example 2.7.16

Let $U = \{a,b,c\}$, $U/R = \{\{a,b,c\}\}$. Let $X = \{a,b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a,b\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{a,b\}\}$. Micro closed sets are $\phi, \{c\}, \{b,c\}, U$. Micro ψ-closed sets are $\phi, \{b\}, \{c\}, \{b,c\}, U$. Micro g-closed sets are $\phi, \{c\}, \{b,c\}, U$. Here $(U, \tau_R(X), \mu_R(X))$ is a Micro $T_{1/2}$-space but not a Micro ${}_{\psi}T_c$-space, since the subset $\{a,c\}$ is Micro ψ-closed, but not Micro closed in $(U, \tau_R(X), \mu_R(X))$.

Remark 2.7.17

The following examples show that Micro semi-$T_{1/2}$-space is independent from Micro ${}_{\psi}T_c$- space.

Example 2.7.18

Let $U = \{a,b,c,d\}$, $U/R = \{\{a\}, \{c\}, \{b,d\}\}$. Let $X = \{a,b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{b,d\}, \{a,b,d\}\}$. Let $\mu = \{c\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{c\}, \{a,c\}, \{b,d\}, \{a,b,d\}, \{b,c,d\}\}$. Micro closed sets are $\phi, \{a\}, \{c\}\{a,c\}, \{b,d\}, \{a,b,d\}, \{b,c,d\}, U$. Micro semi-closed sets are $\phi, \{a\}, \{c\}, \{a,c\}, \{b,d\}, \{a,b,d\}, \{b,c,d\}, U$. Micro sg-closed sets are $\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, U$. Micro ψ-closed sets are $\phi, \{a\}, \{c\}\{a,c\}, \{b,d\}, \{a,b,d\}, \{b,c,d\}, U$. Here $(U, \tau_R(X), \mu_R(X))$ is a Micro ${}_{\psi}T_c$-space but not a Micro semi-$T_{1/2}$-space, since the subset $\{a,b,c\}$ is Micro sg-closed, but not Micro semi closed in $(U, \tau_R(X), \mu_R(X))$.

Example 2.7.19

Let $U = \{a,b,c,d\}$, $U/R = \{\{c\}, \{d\}, \{a,b\}\}$. Let $X = \{a,b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a,b\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{a,b\}\}$. Micro closed sets are $\phi, \{c,d\}, \{b,c,d\}, U$. Micro semi-closed sets are $\phi, \{b\}, \{c\}, \{d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{b,c,d\}, U$. Micro sg-closed sets are $\phi, \{b\}, \{c\}, \{d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{b,c,d\}, U$. Micro ψ-closed sets are

$\phi, \{b\}, \{c\}, \{d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{b,c,d\}, U$. Here $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/2}$-space but not a Micro ${}_{\psi}T_c$-space, since the subset $\{b,c\}$ is Micro ψ-closed, but not Micro closed in $(U, \tau_R(X), \mu_R(X))$.

Remark 2.7.20

(i) Micro semi-$T_{1/2}$-space is independent from Micro T_b- space.

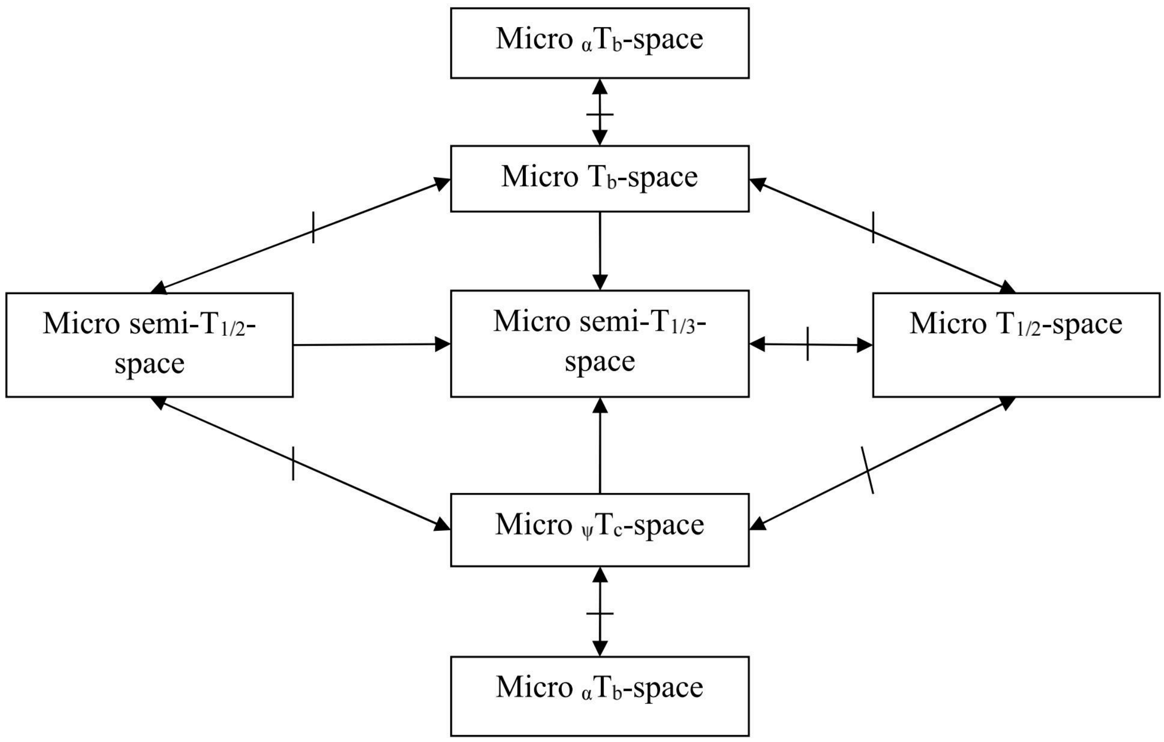
(ii) Micro T_b-space is independent from Micro ${}_{\alpha}T_b$-space.

(iii) Micro T_b-space is independent from Micro $T_{1/2}$-space.

(iv) Micro ${}_{\psi}T_c$-space is independent from Micro ${}_{\alpha}T_b$-space.

Remark 2.7.21

The following diagram shows the dependency and independency relations of newly defined Micro spaces with already existing Micro spaces.



CHAPTER 3

Micro ψ-Continuous Maps and Micro ψ-Irresolute Maps in Micro Topological Spaces

3.1 Introduction

Sakkraiveeranan Chandrasekar [2019] introduced the concepts of Micro continuous maps in Micro topological spaces and he also studied Micro semi-continuous maps and Micro pre-continuous maps and some important properties. Chandrasekar and Swathi [2018] studied the concept of Micro α-continuous maps and its properties. The concept of Micro g-continuous map was introduced by Taha H. Jasim et al. [2021]. Anandhi and Balamani [2022 c] analyzed the concept of Micro αg-continuous maps in Micro topological spaces.

In this chapter we have introduced a new class of Micro continuous maps and Micro irresolute maps called Micro ψ-continuous maps and Micro ψ-irresolute maps in Micro topological spaces. Dependency and independency relations are obtained by comparing the Micro ψ-continuous maps with already existing Micro continuous maps. Various theorems in Micro ψ-continuous maps and Micro ψ-irresolute maps are obtained by using the newly defined Micro spaces namely Micro semi-$T_{1/3}$-space, Micro semi-$T_{1/2}$-space, Micro T_b-space and Micro ${}_{\psi}T_c$-space.

3.2 Micro ψ-Continuous Maps and its Properties

In this section we have introduced Micro ψ-continuous maps in Micro topological spaces and derived the dependency and independency relations of newly defined map with already existing Micro continuous maps.

Definition 3.2.1

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ is called Micro ψ-continuous if $f^{-1}(K)$ is Micro ψ-closed in U for every Micro closed set K in V.

Example 3.2.2

Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$. Let $X = \{b, d\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{b, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) =$

$\{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Micro ψ-closed sets in U are $\phi, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, U$. Let $V = \{a, b, c, d\}, V/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $Y = \{c, d\} \subseteq V$. Then $\tau_R(Y) = \{V, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$. Micro closed sets in V are $\phi, \{b\}, \{a, b\}, \{b, c, d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a, f(b) = c, f(c) = b, f(d) = d$. Therefore for every Micro closed set K in V, $f^{-1}(K)$ is Micro ψ-closed in U. Hence f is a Micro ψ-continuous map.

Proposition 3.2.3

Every Micro continuous map is Micro ψ-continuous but not conversely.

Proof: Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro continuous map. Let K be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro continuous, $f^{-1}(K)$ is Micro closed in U. Since every Micro closed set is Micro ψ-closed, $f^{-1}(K)$ is Micro ψ-closed. Hence f is Micro ψ-continuous.

Example 3.2.4

Let $U = \{a, b, c, d\}, U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{c, d\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$. Micro closed sets in U are $\phi, \{b\}, \{a, b\}, \{b, c, d\}, U$. Micro ψ-closed sets in U are $\phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, U$. Let $V = \{a, b, c, d\}, V/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $Y = \{a, b\} \subseteq V$. Then $\tau_R(Y) = \{V, \phi, \{a, b\}\}$. Let $\mu = \{d\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V, \phi, \{d\}, \{a, b\}, \{a, b, d\}\}$. Micro closed sets in V are $\phi, \{c\}, \{c, d\}, \{a, b, c\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a, f(b) = c, f(c) = b, f(d) = d$. Then f is Micro ψ-continuous but not Micro continuous, since for the Micro closed set $\{c, d\}$ in V, $f^{-1}(\{c, d\}) = \{b, d\}$ is not Micro closed in U.

Proposition 3.2.5

Every Micro semi-continuous map is Micro ψ-continuous but not conversely.

Proof: Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro semi-continuous map. Let K be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro semi continuous, $f^{-1}(K)$ is Micro semi closed in U. Since every Micro semi-closed set is Micro ψ-closed, $f^{-1}(K)$ is Micro ψ-closed. Therefore f is Micro ψ-continuous.

Example 3.2.6

Let $U = \{a, b, c, d\}, U/R = \{\{a\}, \{c\}, \{b, d\}\}$. Let $X = \{b, d\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{b, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Micro semi-closed sets in U are $\phi, \{a\}, \{c\}, \{d, \{a, c\}, \{a, d\}, \{a, c, d\}, U$. Micro ψ-closed sets in U are $\phi, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, U$. Let $V = \{a, b, c, d\}$, $V/R = \{\{c, d\}, \{a, b\}\}$. Let Y=$\{a, b, c\} \subseteq V$. Then $\tau_R(Y)$=$\{V, \phi, \{a, b\}\}$. Let $\mu = \{c\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V, \phi, \{c\}, \{a, b\}, \{a, b, c\}\}$. Micro closed sets in V are $\phi, \{d\}, \{c, d\}, \{a, b, d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a, f(b) = b, f(c) = d, f(d) = c$. Then f is Micro ψ-continuous but not Micro semi continuous, since for the Micro closed set $\{a, b, d\}$ in V, $f^{-1}(\{a, b, d\}) = \{a, b, c\}$ is not Micro semi-closed in U.

Proposition 3.2.7

Every Micro α-continuous map is Micro ψ-continuous but not conversely.

Proof: Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro α-continuous map. Let K be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro α-continuous, $f^{-1}(K)$ is Micro α-closed in U. Since every Micro α-closed set is Micro ψ-closed, $f^{-1}(K)$ is Micro ψ-closed. Therefore f is Micro ψ-continuous.

Example 3.2.8

Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}$. Let $X = \{b, d\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{b, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Micro α-closed sets in U are $\phi, \{c\}, \{a, c\}, \{b, c, d\}, U$. Micro ψ-closed sets in U are $\phi, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, U$. Let $V = \{a, b, c, d\}, V/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let Y=$\{c, d\} \subseteq V$. Then $\tau_R(Y)$=$\{V, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$. Micro closed sets in V are $\phi, \{b\}, \{a, b\}, \{b, c, d\}, V$.Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = d, f(b) = c, f(c) = b, f(d) = a$. Then f is Micro ψ-continuous but not Micro α-continuous, since for the Micro closed set $\{a, b\}$ in V, $f^{-1}(\{a, b\}) = \{c, d\}$ is not Micro α-closed in U.

Remark 3.2.9

Micro ψ-continuous maps and Micro pre-continuous maps are independent as observed from the following examples.

Example 3.2.10

Let $U = \{a, b, c, d\}, U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{c, d\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$. Micro pre closed sets in U are $\phi, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, U$. Micro ψ-closed sets in U are $\phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, U$. Let $V = \{a, b, c, d\}, V/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $Y = \{c\} \subseteq V$. Then $\tau_R(Y) = \{V, \phi, \{c\}\}$. Let $\mu = \{b\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V, \phi, \{b\}, \{c\}, \{b, c\}\}$. Micro closed sets in V are $\phi, \{a, d\}, \{a, b, d\}, \{a, c, d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a, f(b) = d, f(c) = b, f(d) = c$. Then f is Micro ψ-continuous but not Micro pre-continuous, since for the Micro closed set $\{a, c, d\}$ in V, $f^{-1}(\{a, c, d\}) = \{a, b, d\}$ is not Micro pre-closed in U.

Example 3.2.11

Let $U = \{a, b, c, d\}, U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{d\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{d\}, \{a, b\}, \{a, b, d\}\}$. Micro pre closed sets in U are $\phi, \{b\}, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, U$. Micro ψ-closed sets in U are $\phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, U$. Let $V = \{a, b, c, d\}, V/R = \{\{a\}, \{c\}, \{b, d\}$. Let $Y = \{b, d\} \subseteq V$. Then $\tau_R(Y) = \{V, \phi, \{b, d\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Micro closed sets in V are $\phi, \{c\}, \{a, c\}, \{b, c, d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be the identity map. Then f is Micro pre-continuous but not Micro ψ-continuous, since for the Micro closed set $\{a, c\}$ in V, $f^{-1}(\{a, c\}) = \{a, c\}$ is not Micro ψ-closed in U.

Remark 3.2.12

Micro ψ-continuous maps and Micro g-continuous maps are independent as observed from the following examples.

Example 3.2.13

Let $U = \{a, b, c, d\}, U/R = \{\{a, b\}, \{c, d\}$.Let $X = \{a, b, c\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{c\} \notin \tau_R(X)$.Then $\mu_R(X) = \{U, \phi, \{a\}, \{a, b\}, \{a, b, c\}\}$. Micro g-closed sets in U are $\phi, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, U$. Micro ψ-closed sets in U are $\phi, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, U$. Let $V = \{a, b, c, d\}$, $V/R = \{\{a\}, \{c\}, \{b, d\}\}$. Let $Y = \{b, d\} \subseteq V$. Then $\tau_R(Y) = \{V, \phi, \{b, d\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Micro closed sets in V are $\phi, \{c\}, \{a, c\}, \{b, c, d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a, f(b) = b, f(c) = d, f(d) = c$. Then f is Micro g-continuous but not Micro ψ-continuous, since for the Micro closed set $\{b, c, d\}$ in V, $f^{-1}(\{b, c, d\}) = \{b, c, d\}$ is not Micro ψ-closed in U.

Example 3.2.14

Let $U = \{a, b, c, d\}, U/R = \{\{a\}, \{c\}, \{b, d\}$. Let $X = \{b, d\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{b, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Micro g-closed sets in U are $\phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, U$. Micro ψ-closed sets in U are $\phi, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, U$. Let $V = \{a, b, c, d\}, V/R = \{\{a, b\}, \{c, d\}$. Let $Y = \{a, b, c\} \subseteq V$. Then $\tau_R(Y) = \{V, \phi, \{a, b\}\}$. Let $\mu = \{c\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V, \phi, \{c\}, \{a, b\}, \{a, b, c\}$. Micro closed sets in V are $\phi, \{d\}, \{c, d\}, \{a, b, d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = b, f(b) = a, f(c) = d, f(d) = c$. Then f is Micro ψ-continuous but not Micro g-continuous, since for the Micro closed set $\{d\}$ in V, $f^{-1}(\{d\}) = \{c\}$ is not Micro g-closed in U.

Remark 3.2.15

Micro ψ-continuous maps and Micro αg-continuous maps are independent as observed from the following examples.

Example 3.2.16

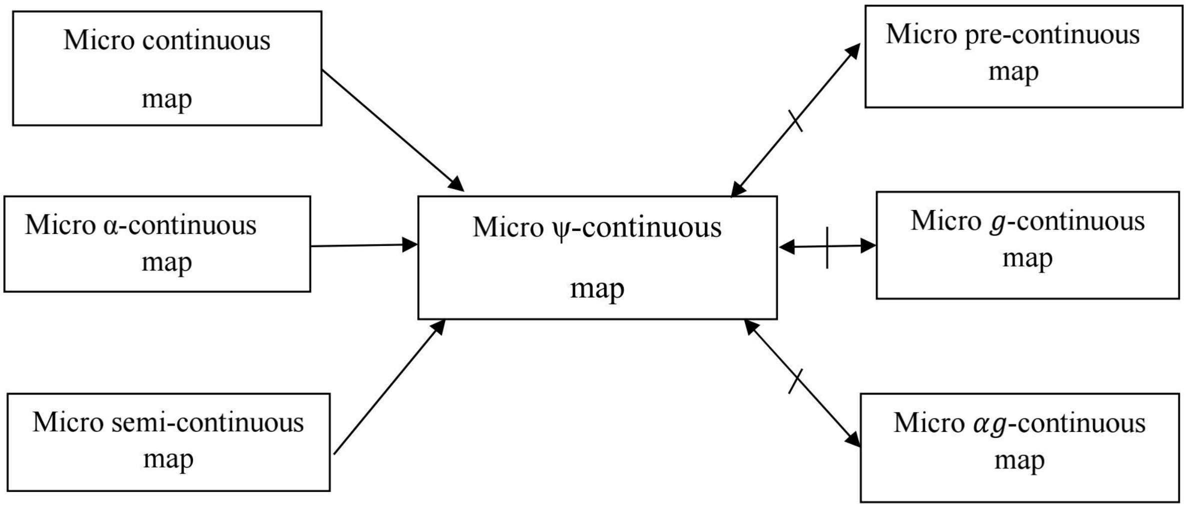
Let $U = \{a, b, c, d\}, U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{d\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{d\}, \{a, b\}, \{a, b\}, \{a, b, d\}\}$. Micro αg-closed sets in U are $\phi, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, U$. Micro ψ-closed sets in U are $\phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, U$. Let $V = \{a, b, c, d\}, V/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $Y = \{c, d\} \subseteq V$. Then $\tau_R(Y) = \{V, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$. Micro closed sets in V are $\phi, \{b\}, \{a, b\}, \{b, c, d\}, V$ .Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a, f(b) = c, f(c) = b, f(d) = d$. Then f is Micro αg-continuous but not Micro ψ-continuous, since for the Micro closed set $\{a, b\}$ in V, $f^{-1}(\{a, b\}) = \{a, c\}$ is not Micro ψ-closed in U.

Example 3.2.17

Let $U = \{a, b, c\}, U/R = \{\{a\}, \{b, c\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{b\}, \{a, b\}\}$. Micro αg-closed sets in U are $\phi, \{c\}, \{a, c\}, \{b, c\}, U$. Micro ψ-closed sets in U are $\phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, U$. Let $V = \{a, b, c\}, V/R = \{\{c\}, \{a, b\}\}$. Let $Y = \{a, b\} \subseteq V$. Then $\tau_R(Y) = \{V, \phi, \{a, b\}\}$. Let $\mu = \{c\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V, \phi, \{c\}, \{a, b\}\}$. Micro closed sets in V are $\phi, \{c\}, \{a, b\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = b, f(b) = c, f(c) = a$. Then f is Micro ψ-continuous but not Micro αg-continuous, since for the Micro closed set $\{c\}$ in V, $f^{-1}(\{c\}) = \{b\}$ is not Micro αg-closed in U.

Remark 3.2.18

The following diagram shows the dependency and independency relations of Micro ψ-continuous map with already existing various Micro continuous maps.



Remark 3.2.19

The composition of two Micro ψ-continuous maps need not be a Micro ψ-continuous map as seen from the following example.

Example 3.2.20

Let $U = \{a, b, c, d\}, U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{c, d\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$. Micro ψ-closed sets in U are $\phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, U$. Let $V = \{a, b, c, d\}$, $V/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $Y = \{a, b\} \subseteq V$. Then $\tau_R(Y) = \{V, \phi, \{a, b\}\}$. Let $\mu = \{d\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V, \phi, \{d\}, \{a, b\}, \{a, b\}, \{a, b, d\}\}$. Micro closed sets in V are $\phi, \{c\}, \{c, d\}, \{a, b, c\}, V$. Micro ψ-closed sets in V are $\phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, V$. Let $W = \{a, b, c, d\}, W/R = \{\{a\}, \{c\}, \{b, d\}\}$. Let $Z = \{b, d\} \subseteq W$. Then $\tau_R(Z) = \{W, \phi, \{b, d\}\}$. Let $\mu = \{b\} \notin \tau_R(Z)$. Then $\mu_R(Z) = \{W, \phi, \{b\}, \{b, d\}\}$. Micro closed sets in W are $\phi, \{a, c\}$, $\{a, c, d\}, W$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ and $g: (V, \tau_R(Y), \mu_R(Y)) \to (W, \tau_R(Z), \mu_R(Z))$ be the maps defined by $f(a) = a, f(b) = c, f(c) = b, f(d) = d$ and $g(a) = a, g(b) = c, g(c) = d, g(d) = b$. Then both f and g are Micro ψ-continuous but their composition $g \circ f: (U, \tau_R(X), \mu_R(X)) \to (W, \tau_R(Z), \mu_R(Z))$ is not Micro ψ-continuous, since for the Micro closed set $\{c\}$ in W, $(g \circ f)^{-1}(\{a, c\}) = f^{-1}(g^{-1}(\{a, c\})) = f^{-1}\{a, b\} = \{a, c\}$ is not Micro ψ-closed in U.

Theorem 3.2.21

Let $f: \big(U, \tau_R(X), \mu_R(X)\big) \to \big(V, \tau_R(Y), \mu_R(Y)\big)$ be a Micro ψ-continuous map and $g: \big(V, \tau_R(Y), \mu_R(Y)\big) \to \big(W, \tau_R(Z), \mu_R(Z)\big)$ be a Micro continuous map, then $g \circ f: \big(U, \tau_R(X), \mu_R(X)\big) \to \big(W, \tau_R(Z), \mu_R(Z)\big)$ is a Micro ψ-continuous map.

Proof: Let K be a Micro closed set in $\big(W, \tau_R(Z), \mu_R(Z)\big)$. Since g is Micro continuous, $g^{-1}(K)$ is Micro closed in $\big(V, \tau_R(Y), \mu_R(Y)\big)$. Since f is Micro ψ-continuous, $(g \circ f)^{-1}(K) = f^{-1}\big(g^{-1}(K)\big)$ is Micro ψ-closed. Hence $g \circ f$ is Micro ψ-continuous.

Theorem 3.2.22

Let $f: \big(U, \tau_R(X), \mu_R(X)\big) \to \big(V, \tau_R(Y), \mu_R(Y)\big)$ be a Micro ψ-continuous map and if $\big(U, \tau_R(X), \mu_R(X)\big)$ is a Micro semi-$T_{1/3}$ -space then f is a Micro semi continuous.

Proof: Let K be a Micro closed set in $\big(V, \tau_R(Y), \mu_R(Y)\big)$. Since f is Micro ψ-continuous, $f^{-1}(K)$ is Micro ψ-closed in $\big(U, \tau_R(X), \mu_R(X)\big)$. Since $\big(U, \tau_R(X), \mu_R(X)\big)$ is a Micro semi-$T_{1/3}$ space, $f^{-1}(K)$ is Micro semi-closed in $\big(U, \tau_R(X), \mu_R(X)\big)$. Hence f is Micro semi-continuous.

Theorem 3.2.23

Let $f: \big(U, \tau_R(X), \mu_R(X)\big) \to \big(V, \tau_R(Y), \mu_R(Y)\big)$ be a Micro ψ-continuous map and if $\big(U, \tau_R(X), \mu_R(X)\big)$ is a Micro semi-$T_{1/2}$ -space then f is a Micro semi continuous.

Proof: Let K be a Micro closed set in $\big(V, \tau_R(Y), \mu_R(Y)\big)$. Since f is Micro ψ-continuous, $f^{-1}(K)$ is Micro ψ-closed in $\big(U, \tau_R(X), \mu_R(X)\big)$. Since every Micro ψ-closed set is Micro sg-closed and $\big(U, \tau_R(X), \mu_R(X)\big)$ is a Micro semi-$T_{1/2}$-space, $f^{-1}(K)$ is Micro semi-closed in $\big(U, \tau_R(X), \mu_R(X)\big)$. Hence f is Micro semi-continuous.

Theorem 3.2.24

Let $f: \big(U, \tau_R(X), \mu_R(X)\big) \to \big(V, \tau_R(Y), \mu_R(Y)\big)$ be a Micro ψ-continuous map and if $\big(U, \tau_R(X), \mu_R(X)\big)$ is a Micro ${}_{\psi}T_c$-space then f is a Micro continuous.

Proof: Let K be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro ψ-continuous, $f^{-1}(K)$ is Micro ψ-closed in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro $_{\psi}T_c$-space, $f^{-1}(K)$ is Micro closed in $(U, \tau_R(X), \mu_R(X))$. Hence f is Micro continuous.

Theorem 3.2.25

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be any two maps. Then $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ is Micro ψ-continuous if and only if $f^{-1}(B)$ is Micro ψ-open in $(U, \tau_R(X), \mu_R(X))$ whenever B is a Micro open set in $(V, \tau_R(Y), \mu_R(Y))$.

Proof: Let f be a Micro ψ-continuous map and B be Micro open in $(V, \tau_R(Y), \mu_R(Y))$. Then B^c is Micro closed in $(V, \tau_R(Y), \mu_R(Y))$. By hypothesis $f^{-1}(B^c)$ is Micro ψ-closed in $(U, \tau_R(X), \mu_R(X))$, i.e.,[$f^{-1}(B)$]^c is a Micro ψ-closed set in $(U, \tau_R(X), \mu_R(X))$. Hence $f^{-1}(B)$ is a Micro ψ-open in $(U, \tau_R(X), \mu_R(X))$. Conversely, suppose $f^{-1}(B)$ is a Micro ψ-open set in $(U, \tau_R(X), \mu_R(X))$ whenever B is Micro open in $(V, \tau_R(Y), \mu_R(Y))$. Let H be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Then H^c is Micro open set in $(V, \tau_R(Y), \mu_R(Y))$. By assumption $f^{-1}(H^c)$is Micro ψ-open in $(U, \tau_R(X), \mu_R(X))$, i.e., $[f^{-1}(H)]^c$ is a Micro ψ-open set in $(U, \tau_R(X), \mu_R(X))$. Then $f^{-1}(H)$ is Micro ψ-closed in $(U, \tau_R(X), \mu_R(X))$. Hence f is a Micro ψ-continuous map.

3.3 Micro ψ-Irresolute Maps and its Properties

This section presented the definition and properties of Micro ψ-irresolute maps.

Definition 3.3.1

A map $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ is called Micro ψ-irresolute if $f^{-1}(K)$ is Micro ψ-closed in U for every Micro ψ- closed set K in V.

Example 3.3.2

Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$.Let$X = \{b, d\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{b, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Micro ψ-closed sets in $U$are $\phi, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, U$. Let $V =$

$\{a, b, c, d\}$, $V/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let Y=$\{c, d\} \subseteq V$. Then $\tau_R(Y) = \{V, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$. Micro ψ-closed sets in V are $\phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a, f(b) = c, f(c) = b, f(d) = d$. Therefore for every Micro closed set K in V, $f^{-1}(K)$ is Micro ψ-closed in U. Hence f is a Micro irresolute map.

Proposition 3.3.3

Every Micro ψ-irresolute map is Micro ψ-continuous but not conversely.

Proof: Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro ψ-irresolute map. Let K be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since every Micro closed set is Micro-ψ-closed and f is Micro ψ-irresolute, $f^{-1}(K)$ is Micro ψ-closed. Hence f is Micro ψ-continuous.

Example 3.3.4

Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{b\}, \{c, d\}$. Let $X = \{c, d\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$. Micro closed sets in U are $\phi, \{b\}, \{a, b\}, \{b, c, d\}, U$. Micro ψ-closed sets in $U$are $\phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}\{a, b, d\}, \{b, c, d\}, U$. Let $V = \{a, b, c, d\}, V/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $Y = \{a, b\} \subseteq V$. Then $\tau_R(Y) = \{V, \phi, \{a, b\}\}$. Let $\mu = \{d\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V, \phi, \{d\}, \{a, b\}, \{a, b, d\}\}$. Micro closed sets in V are $\phi, \{c\}, \{c, d\}, \{a, b, c\}, V$. Micro ψ-closed sets in V are $\phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a, f(b) = c, f(c) = b, f(d) = d$. Then f is Micro ψ-continuous but not Micro ψ-irresolute, since for the Micro ψ-closed set $\{d\}$ in V, $f^{-1}(\{d\}) = \{d\}$ is not Micro ψ-closed in U.

Theorem 3.3.5

Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro ψ-irresolute map and $g: (V, \tau_R(Y), \mu_R(Y)) \to (W, \tau_R(Z), \mu_R(Z))$ be a Micro ψ-irresolute map then $g \circ f: (U, \tau_R(X), \mu_R(X)) \to (W, \tau_R(Z), \mu_R(Z))$ is a Micro ψ-irresolute map.

Proof: Let K be a Micro ψ-closed set in $(W, \tau_R(Z), \mu_R(Z))$. Since g is Micro ψ-irresolute, $g^{-1}(K)$ is Micro ψ-closed in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro ψ-irresolute, $(g \circ f)^{-1}(K) = f^{-1}(g^{-1}(K))$ is Micro ψ-closed. Hence $g \circ f$ is Micro ψ-irresolute.

Theorem 3.3.6

Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro ψ-irresolute map and $g: (V, \tau_R(Y), \mu_R(Y)) \to (W, \tau_R(Z), \mu_R(Z))$ be a Micro ψ-continuous map then $g \circ f: (U, \tau_R(X), \mu_R(X)) \to (W, \tau_R(Z), \mu_R(Z))$ is a Micro ψ-continuous map.

Proof: Let K be a Micro closed set in $(W, \tau_R(Z), \mu_R(Z))$. Since g is Micro ψ-continuous, $g^{-1}(K)$ is Micro ψ-closed in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro ψ-irresolute, $(g \circ f)^{-1}(K) = f^{-1}(g^{-1}(K))$ is Micro ψ-closed. Hence $g \circ f$ is Micro ψ-continuous.

Theorem 3.3.7

Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro ψ-irresolute map and if $(U, \tau_R(X), \mu_R(X))$ is Micro semi-$T_{1/3}$ -space then f is a Micro semi continuous map.

Proof: Let K be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since every Micro closed set is Micro ψ-closed and f is Micro ψ-irresolute, $f^{-1}(K)$ is Micro ψ-closed in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/3}$-space, $f^{-1}(K)$ is Micro semi-closed in $(U, \tau_R(X), \mu_R(X))$. Hence f is Micro semi-continuous.

Theorem 3.3.8

Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro ψ-irresolute map and if $(U, \tau_R(X), \mu_R(X))$ is Micro semi-$T_{1/2}$ -space then f is a Micro semi continuous map.

Proof: Let K be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since every Micro closed set is Micro ψ-closed and f is Micro ψ-irresolute, $f^{-1}(K)$ is Micro ψ-closed in $(U, \tau_R(X), \mu_R(X))$. Since every Micro ψ-closed set is Micro sg-closed and

$(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/2}$-space, $f^{-1}(K)$ is Micro semi-closed in $(U, \tau_R(X), \mu_R(X))$. Hence f is Micro semi-continuous.

Theorem 3.3.9

Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro ψ-irresolute map and if $(U, \tau_R(X), \mu_R(X))$ is a Micro ${}_{\psi}T_c$ -space then f is a Micro continuous map.

Proof: Let K be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since every Micro closed set is Micro ψ-closed and f is Micro ψ-irresolute, $f^{-1}(K)$ is Micro ψ-closed in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro ${}_{\psi}T_c$-space, $f^{-1}(K)$ is Micro closed in $(U, \tau_R(X), \mu_R(X))$. Hence f is Micro continuous.

SUMMARY AND CONCLUSION

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Preliminary definitions which are required in the course of study are presented in **Chapter 1.**

In **Chapter 2**, Micro ψ-closed sets and Micro ψ-open sets are introduced in Micro topological spaces. Properties of the defined sets and interrelations between Micro ψ-closed sets with other existing Micro closed sets are derived. Further, Micro ψ-closure and Micro ψ-interior operators are defined and their properties are analyzed. As an application of Micro ψ-closed sets four new Micro spaces namely, Micro semi-$T_{1/3}$-space, Micro semi-$T_{1/2}$-space, Micro T_b-space and Micro $_{\psi}T_c$-space are introduced and their properties and interrelations are studied.

In **Chapter 3**, Micro ψ-continuous maps and Micro ψ-irresolute maps in Micro topological spaces are introduced. Properties and characterizations are derived with respect to the defined maps. Also we have analyzed composition of mappings relevant to the defined maps.

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