CHAPTER - VI

CHAPTER VI

INTERVAL-VALUED FUZZY TOPSIS METHOD

In this chapter the concept of the TOPSIS method has been extended for interval-valued fuzzy data.

Interval-Valued Fuzzy Sets (IVFSs) and Intuitionistic Fuzzy Sets (IFSs) are equipollent generalizations of the concept of ordinary fuzzy sets. Therefore it can be easily seen that definitions 1.7 and 1.9 of chapter 1 are equivalent.

That is,

$$M_{A_i}(x) = [M^-_{A_i}(x), M^+_{A_i}(x)]$$
$$= [\mu_{A_i}(x), 1 - \nu_{A_i}(x)]$$
$$= [\mu_{A_i}(x), \mu_{A_i}(x) + \pi_{A_i}(x)]$$

Interval $[M^-_{A_i}(x), M^+_{A_i}(x)]$ shows all possible degrees of membership and the decision maker is hesitated to the extent $\pi_{A_i}(x)$.

A canonical Multi-Criteria Decision Making problem is concisely expressed in a decision matrix, whose element x_{ij}'s indicates the evaluation or value of the i^{th} alternative, A_i, with respect to the j^{th} criteria, x_j. In the method, the canonical matrix format is extended to interval-valued fuzzy decision matrix D.

Consider i = 1, 2,..., m; j = 1, 2,..., n; and $[M^-_{A_i}(x), M^+_{A_i}(x)]$ (or equivalently, $(\mu_{A_i}(x_j), \nu_{A_i}(x_j)\pi_{A_i}(x_j))$ by IFS notation) representing the performance measure of the i^{th} alternative in terms of the j^{th} attribute. The interval-valued fuzzy decision matrix D is

$$D = \begin{bmatrix} [M^-_{A_1}(x), M^+_{A_1}(x)] & [M^-_{A_1}(x), M^+_{A_1}(x)] & \cdots & [M^-_{A_1}(x), M^+_{A_1}(x)] \\ [M^-_{A_2}(x), M^+_{A_2}(x)] & [M^-_{A_2}(x), M^+_{A_2}(x)] & \cdots & [M^-_{A_2}(x), M^+_{A_2}(x)] \\ \vdots & \vdots & \ddots & \vdots \\ [M^-_{A_m}(x), M^+_{A_m}(x)] & [M^-_{A_m}(x), M^+_{A_m}(x)] & \cdots & [M^-_{A_m}(x), M^+_{A_m}(x)] \end{bmatrix}$$

$$= \begin{bmatrix} (\mu_{A_1}(x_1), \nu_{A_1}(x_1), \pi_{A_1}(x_1)) & (\mu_{A_1}(x_2), \nu_{A_1}(x_2), \pi_{A_1}(x_2)) & \cdots & (\mu_{A_1}(x_n), \nu_{A_1}(x_n), \pi_{A_1}(x_n)) \\ (\mu_{A_2}(x_1), \nu_{A_2}(x_1), \pi_{A_2}(x_1)) & (\mu_{A_2}(x_2), \nu_{A_2}(x_2), \pi_{A_2}(x_2)) & \cdots & (\mu_{A_2}(x_n), \nu_{A_2}(x_n), \pi_{A_2}(x_n)) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{A_m}(x_1), \nu_{A_m}(x_1), \pi_{A_m}(x_1)) & (\mu_{A_m}(x_2), \nu_{A_m}(x_2), \pi_{A_m}(x_2)) & \cdots & (\mu_{A_v}(x_n), \nu_{A_m}(x_n), \pi_{A_m}(x_n)) \end{bmatrix}$$

Since all attributes cannot be assumed to be of equal importance, a set of grades of importance, denoted as W, from the decision maker. The Interval-Valued Fuzzy Set is expressed as the subjective importance of decision criteria during the decision maker's evaluation process. An Interval-Valued Fuzzy Set W in X is an object having the form

$$W = \{\langle x, \mu_W(x)\rangle | x \in X\}$$
$$= \{\langle x, \mu_W(x), \nu_W(x)\rangle | x \in X\}$$

where $M_W : X \rightarrow \mathrm{Int}([0, 1])$, such that $x \rightarrow M_W(x) = [M_W(x), M_W(x)]$. In addition, $\mu_W(x) : X \rightarrow [0, 1]$ and $\nu_W(x) : X \rightarrow [0, 1]$ define the degree of importance and the degree of unimportance for an attribute.

For each $x \in X$, the uncertainty index toward the importance of an attribute is $\pi_w(x) = 1 - \mu_w(x) - \nu_w(x)$.

According to the definition given by Atanassov [6], for two IVFSs A_i and W,

$$A_i.W = \{\langle x, M_{A_i \cdot W}(x)\rangle | x \in X\}$$
$$= \{<x, \mu_{A_i}(x)\cdot\mu_w(x), \nu_{A_i}(x) + \nu_w(x) - \nu_{A_i}(x)\cdot\nu_w(x)> | x \in X\}$$

And

$$\pi_{A_i.w}(x) = 1 - \mu_{A_i}(x)\cdot\mu_w(x) - \nu_{A_i}(x) - \nu_w(x) + \nu_{A_i}(x)\cdot\nu_w(x)$$

The interval-valued fuzzy weighted decision matrix D' is then equal to

$$D' = \begin{bmatrix} [M^-_{A_1 \cdot w}(x_1), M^+_{A_1 \cdot w}(x_1)] & [M^-_{A_1 \cdot w}(x_n), M^+_{A_1 \cdot w}(x_n)] \\ [M^-_{A_m \cdot w}(x_1), M^+_{A_m \cdot w}(x_1)] & [M^-_{A_m \cdot w}(x_n), M^+_{A_m \cdot w}(x_n)] \end{bmatrix}$$

$$= \begin{bmatrix} (\mu_{A_1 \cdot w}(x_1), \nu_{A_1 \cdot w}(x_1), \pi_{A_1 \cdot w}(x_1)) & \cdots & (\mu_{A_1 \cdot w}(x_n), \nu_{A_1 \cdot w}(x_n), \pi_{A_1 \cdot w}(x_n)) \\ \vdots & \ddots & \vdots \\ (\mu_{A_m \cdot w}(x_1), \nu_{A_m \cdot w}(x_1), \pi_{A_m \cdot w}(x_1)) & \cdots & (\mu_{A_m \cdot w}(x_n), \nu_{A_m \cdot w}(x_n), \pi_{A_m \cdot w}(x_n)) \end{bmatrix}$$

Let J_1 be a collection of benefit attributes (i.e., the larger x_j , the greater preference) and J_2 be a collection of cost attributes (i.e., the smaller x_j , the greater preference). The interval-valued positive-ideal solution, denoted as A^+, and the interval-valued negative-ideal solution, denoted as A^-, are defined as

$$A^+ = \Big\{ \Big\langle x_j, \Big((\max_i \mu_{A_i \cdot w}(x_j) | j \in J_1), (\min_i \mu_{A_i \cdot w}(x_j) | j \in J_2) \Big),$$

$$\Big((\min_i \mu_{A_i \cdot w}(x_j) | j \in J_1), (\max_i \mu_{A_i \cdot w}(x_j) | j \in J_2) \Big) \Big\rangle \Big| i = 1,2,\ldots, m \Big\}$$

$$= \Big\{ \Big\langle x_1, \mu_{A^+ \cdot w}(x_1), \nu_{A^+ \cdot w}(x_1) \Big\rangle, \Big\langle x_2, \mu_{A^+ \cdot w}(x_2), \nu_{A^* \cdot w}(x_2) \Big\rangle$$

$$, \ldots, \Big\langle x_n, \mu_{A^+ \cdot w}(x_n), \nu_{A^+ \cdot w}(x_n) \Big\rangle \Big\}$$

$$A^- = \Big\{ \Big\langle x_j, \Big((\min_i \mu_{A_i \cdot w}(x_j) | j \in J_1), (\max_i \mu_{A_i \cdot w}(x_j) | j \in J_2) \Big),$$

$$\Big((\max_i \mu_{A_i \cdot w}(x_j) | j \in J_1), (\min_i \mu_{A_i \cdot w}(x_j) | j \in J_2) \Big) \Big\rangle \Big| i = 1,2,\ldots, m \Big\}$$

$$= \Big\{ \Big\langle x_1, \mu_{A^- \cdot w}(x_1), \nu_{A \ \cdot w}(x_1) \Big\rangle, \Big\langle x_2, \mu_{A^- \cdot w}(x_2), \nu_{A^* \cdot w}(x_2) \Big\rangle$$

$$, \ldots, \Big\langle x_n, \mu_{A^- \cdot w}(x_n), \nu_{A^- \ \cdot w}(x_n) \Big\rangle \Big\}$$

The separation measures, S_{i^+} and S_{i^-}, of each alternative from the interval-valued positive-ideal and negative-ideal solutions, respectively, are derived from:

(A) Seperation measures based on the Euclidean distance:

The definition according to Grzegorzewski [24]

$$S_{i^+} = \left[\sum_{j=1}^{n} \max\left\{ (\mu_{A_i \cdot W}(x_j) - \mu_{A^+ \cdot W}(x_j))^2, (\nu_{A_i \cdot W}(x_j) - \nu_{A^+ \cdot W}(x_j))^2 \right\} \right]^{\frac{1}{2}}$$

$$S_{i^-} = \left[\sum_{j=1}^{n} \max\left\{ (\mu_{A_i \cdot W}(x_j) - \mu_{A^- \cdot W}(x_j))^2, (\nu_{A_i \cdot W}(x_j) - \nu_{A^- \cdot W}(x_j))^2 \right\} \right]^{\frac{1}{2}}$$

(B) Seperation measures based on the normalized Euclidean distance:

The definition according to Grzegorzewski [24]

$$S_{i^+} = \left[\frac{1}{n} \sum_{j=1}^{n} \max\left\{ (\mu_{A_i \cdot W}(x_j) - \mu_{A^+ \cdot W}(x_j))^2, (\nu_{A_i \cdot W}(x_j) - \nu_{A^+ \cdot W}(x_j))^2 \right\} \right]^{\frac{1}{2}}$$

$$S_{i^-} = \left[\frac{1}{n} \sum_{j=1}^{n} \max\left\{ (\mu_{A_i \cdot W}(x_j) - \mu_{A^- \cdot W}(x_j))^2, (\nu_{A_i \cdot W}(x_j) - \nu_{A^- \cdot W}(x_j))^2 \right\} \right]^{\frac{1}{2}}$$

And the separation between alternatives can also be calculated by using equations (1) to (6) and (13) to (16) in chapter V.

The relative closeness of an alternative A_i with respect to the interval-valued positive-ideal solution A^+ is defined as

$$C_{i^+} = \frac{S_{i^-}}{S_{i^+} + S_{i^-}} \text{ where } 0 \le C_{i^+} \le 1 \text{ and } i=1,2,\ldots,m.$$

Then, the preference order of alternatives can be ranked according to the descending order of C_{i^+}'s.

ALGORITHM:

Step 1: Construct the interval-valued fuzzy decision matrix D.

Step 2: Construct the interval-valued fuzzy weighted decision matrix D'.

Step 3: Determine the interval-valued positive-ideal and interval-valued negative-ideal solutions

Step 4: Calculate the separation measures.

Step 5: Calculate the relative closeness to the interval-valued ideal solution.

Step 6: Rank the preference orders.