Introduction

INTRODUCTION

Real world decision making problems are very often uncertain or vague in a number of ways. In 1965, Zadeh [55] introduced the concept of Fuzzy Set Theory to meet those problems. The fuzzyness can be represented by different ways. One of the most useful representation is the membership function. Depending on the nature of the membership function the fuzzy numbers can be classified in different forms, such as Triangular Fuzzy Numbers (TFNs), Trapezoidal Fuzzy Numbers (TRFNs), Internal Fuzzy Numbers (IFNs), etc. Fuzzy Numbers are widely applied in many fields such as Operation Research, Control Theory, Management Sciences etc. Fuzzy matrices play an important role in scientific development. Fuzzy matrices were introduced in 1977 by Thomason, M.G. [48]. In 2007, Shyamal, A.K. and Pal, M. introduced Triangular Fuzzy Number Matrices [4]. Pal, M. and Bhowmik [36] introduced the concept of circulant triangular fuzzy number matrices and studied their properties such as adjoint, determinant etc. In 2010, Vijayalakshmi, V. and Sattanathan, R. [52] introduced trapezoidal fuzzy number matrices and circulant trapezoidal fuzzy number matrices. Madhumangal Pal, Gobinda Murmu and Anita Pal [30] introduced Interval Fuzzy Numbers and Interval Fuzzy Number Matrices.

The main aim of my thesis is to study different forms of Fuzzy Numbers and Fuzzy Number Matrices with applications to real world problems.

The plan of study is as follows :

1. Triangular Fuzzy Numbers and Triangular Fuzzy Number Matrices
2. Trapezoidal Fuzzy Numbers and Trapezoidal Fuzzy Number Matrices
3. Interval Fuzzy Numbers and Interval Fuzzy Number Matrices
4. Circulant Triangular Fuzzy Number Matrices
5. Circulant Trapezoidal Fuzzy Number Matrices
6. Some New Operators on Triangular Fuzzy Numbers and Triangular Fuzzy Number Matrices

7. Some New Operators on Interval Fuzzy Numbers and Interval Fuzzy Number Matrices
8. α-cuts of Triangular Fuzzy Numbers and α-cuts of Triangular Fuzzy Number Matrices
9. Application of Triangular Fuzzy Numbers in solving Fuzzy Linear programming problem
10. Application of Interval Fuzzy Number Matrices in the analysis of the factors influencing high scores in Higher Secondary Examinations.

"A triangular fuzzy number denoted by $\tilde{M} = < m, \alpha, \beta >$ has the membership function.

$$\mu_{\tilde{M}}(x) = \begin{cases} 0 \ \text{for } x \le m-\alpha \\ 1-\dfrac{m-x}{\alpha} \text{for } m-\alpha < x < m \\ 1 \ \text{for } x = m \\ 1-\dfrac{x-m}{\beta} \text{for } m < x < m+\beta \\ 0 \ \text{for } x \ge m+\beta \end{cases}$$

A trapezoidal fuzzy number denoted by $\tilde{A} = < a, b, c, d >$ has the membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a < x < b \\ 1, & b < x < c \\ \dfrac{d-x}{d-c}, & c \le x < d \\ 0, & x > d \end{cases}$$

An interval fuzzy number is defined as $\hat{A} = [a_L, a_R] = \{a : a_L \le a \le a_R\}$ where $a_L, a_R \in [0, 1]$".

"A matrix is said to be a triangular fuzzy number matrix (respectively trapezoidal fuzzy number matrix and interval fuzzy number matrix) if all its elements are triangular fuzzy numbers (respectively trapezoidal fuzzy numbers and interval fuzzy numbers)".

The first chapter is devoted to the study of triangular fuzzy numbers (TFNs) and triangular fuzzy number matrices (TFNMs). In this chapter, some elementary operators on triangular fuzzy number matrices introduced by Shyamal, A.K. and Pal, M. are studied. Some special types of triangular fuzzy number matrices (e.g. pure and fuzzy triangular, symmetric, pure and fuzzy skew symmetric, singular, semi-singular, constant) are also defined and a number of properties of these triangular fuzzy number matrices are presented. Finally, Generalized Triangular Fuzzy Number Matrices are defined and a systematic process of finding the distance between generalized triangular fuzzy number matrices is proposed.

The second chapter deals with Trapezoidal Fuzzy Numbers and Trapezoidal Fuzzy Number Matrices. In this chapter, some arithmetic properties of trapezoidal fuzzy numbers are explored. Basic mathematical operations are reviewed and formulated on trapezoidal fuzzy numbers which have direct applications in fuzzy linear and non linear equations. Advantages of this form of trapezoidal fuzzy numbers over others is also discussed and a category of fuzzy numbers that are neither positive nor negative is investigated in detail with a discussion of some of its interesting properties and applications. Finally, Generalized Trapezoidal Fuzzy Number Matrices are defined and a systematic process of finding the distance between generalized trapezoidal fuzzy number matrices is introduced.

Chapter III deals with Interval Fuzzy Numbers and Interval Fuzzy Number Matrices. In this chapter, basic operations of interval fuzzy numbers and interval fuzzy number matrices are defined. Using these operations, the orthogonality of interval fuzzy number matrices are introduced and some properties are discussed.

A circulant matrix is a special kind of square matrix where each row after the first is obtained from its predecessor by a cyclic shift. A circulant matrix of order n is a square matrix of the form,

$$C = \begin{pmatrix} c_1 & c_2 & \cdots & c_n \\ c_n & c_1 & \cdots & c_{n-1} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ c_2 & c_3 & \cdots & c_1 \end{pmatrix} = \text{circ}\,(c_1, c_2, \ldots, c_n)$$

Circulant matrices and their generalizations have important applications in Physics, Image Processing, Probability, Number Theory, Geometry and Numerical Analysis. Circulant triangular fuzzy number matrices (circulant TFNM) and circulant trapezoidal fuzzy number matrices (circulant TRFNM) are studied respectively in chapters IV and V. Some operations on these circulant fuzzy number matrices are presented. Some properties of determinant and adjoint of these circulant matrices are also studied.

The following are the important theorems proved regarding these circulant fuzzy number matrices :

1. An n x n TFNM A is circulant if and only if A C_n = C_n A, where C_n is the permutation matrix of unit TFNM (Theorem 4.7).
2. For the circulant TFNMs A and B

    (i) A + B is a circulant TFNM

    (ii) A′ is a circulant TFNM

    (iii) A B is also a circulant TFNM. In particular, A^k is also a circulant TFNM.

    (iv) A A′ is circulant TFNM.

    (v) If A and B are circulant TFNMs then A B = B A (Theorem 4.9, 4.10).

3. A circulant triangular fuzzy number matrix (resp. circulant trapezoidal fuzzy number matrix) A is symmetric iff $\tilde{A}_{1i} = \tilde{A}_{1(n-i+2)}$ for every $i \in \{1, 2, \ldots, n\}$. (Theorem 4.11, 5.3).
4. Adj A of a circulant triangular fuzzy number matrix (resp. circulant trapezoidal fuzzy number matrix) is also circulant TFNM (resp. circulant TRFNM) (Theorem 4.13, 5.2).
5. If a TFNMs A is circulant, then E A is symmetric where is a permutation matrix of unit TFNM (Theorem 4.12).

From the review of literature available, one can find various operators for fuzzy numbers and fuzzy matrices defined by different authors. In 2004, Amiya K. Shyamal and Madhumangal Pal introduced two new binary operators for fuzzy matrices [3]. Using these operators, some new operators on Triangular Fuzzy Numbers, Triangular Fuzzy Number Matrices, Interval Fuzzy Numbers, Interval Fuzzy Number Matrices are introduced by the author of this thesis.

It is worth mentioning that the author of this thesis published three articles related to this topic as detailed below :

1. "Some new operators on triangular fuzzy numbers and triangular fuzzy number matrices", Proceedings of International Conference on Mathematics and its Applications – A New Wave (ICMANW), December 21-22 [21].
2. "α-cuts of Triangular Fuzzy Numbers and α-cuts of Triangular Fuzzy Number Matrices", International Journal of Mathematical Sciences and Applications [22].
3. "Some new operators on Interval Fuzzy Numbers and Interval Fuzzy Number Matrices", International Journal of Mathematical Archive [23].

In chapter VI, some new elementary operators on Triangular Fuzzy Numbers (TFNMs) (Definition 6.1) and some new operators on Triangular Fuzzy Number Matrices (TFNMs) (Definition 6.2) are introduced with interesting examples. Using these operators, some important properties of TFNs and TFNMs are presented (Theorems 6.5 and 6.6).

In chapter VII, some new elementary operators on Interval Fuzzy Numbers (IFNs) (Definition 7.1) and some new operators on Interval Fuzzy Number Matrices (IFNMs) (Definition 7.2) are introduced and using these operators, some important properties are proved (Theorems 7.3, 7.5, 7.6 and 7.7).

Chapter VIII deals with α-cuts of Triangular Fuzzy Numbers and α-cuts of Triangular Fuzzy Number Matrices.

"For $\alpha \in [0, 1]$, the upper α-cut of the triangular fuzzy number $\tilde{M} = < m, \rho, \beta >$ is defined as

$\tilde{M}^{(\alpha)} = < m^{(\alpha)}, \rho^{(\alpha)}, \beta^{(\alpha)} >$ and the lower α-cut of $\tilde{M}$ is defined as

$\tilde{M}_{(\alpha)} = < m_{(\alpha)}, \rho_{(\alpha)}, \beta_{(\alpha)} >$ where for x, $\alpha \in [0, 1]$.

$$x^{(\alpha)} = \begin{cases} 1, & \text{if } x \geq \alpha \\ 0, & \text{if } x < \alpha \end{cases} \quad \text{and}$$

$$x_{(\alpha)} = \begin{cases} x, & \text{if } x \geq \alpha \\ 0, & \text{if } x < \alpha \end{cases}$$

The upper α-cut of a triangular fuzzy number matrix $M = (\tilde{M}_{ij})_{mxn}$ is defined as

$M^{(\alpha)} = (\tilde{M}_{ij}^{(\alpha)})_{mxn}$ and the lower α-cut of M is defined as

$M_{(\alpha)} = (\tilde{M}_{ij(\alpha)})_{mxn}$".

The following are the important properties proved in this chapter :

For any two triangular fuzzy number matrices M and N,

(i) $(M \ominus N)^{(\alpha)} \geq M^{(\alpha)} \ominus N^{(\alpha)}$

(ii) $(M \vee N)^{(\alpha)} = M^{(\alpha)} \vee N^{(\alpha)}$

(iii) $(M \oplus N)^{(\alpha)} \geq M^{(\alpha)} \oplus N^{(\alpha)}$

(iv) $(M \vee N)_{(\alpha)} = M_{(\alpha)} \vee N_{(\alpha)}$

(v) $(M \ominus N)_{(\alpha)} = M_{(\alpha)} \ominus N_{(\alpha)}$

(vi) $(M \oplus N)_{(\alpha)} \geq M_{(\alpha)} \oplus N_{(\alpha)}$

One major application of the fuzzy number arithmetic is solving linear systems whose parameters are all or partially represented by fuzzy numbers and those systems are named as Fully Fuzzy Linear Systems. Nagoor Gani, A. and Mohamed Assarudeen, S.N., defined a new operation on triangular fuzzy numbers and illustrated its application in solving Fully Fuzzy Linear Programming problem. The details of this problem is given in chapter IX.

Interval Mathematics is an important mathematical tool for dealing with uncertainties. It has a rich potential for application in solving real life problems.

Chapter X deals with Application of Interval Fuzzy Number Matrices in the analysis of the Factors influencing high scores in higher secondary examinations. Higher Scores in the Higher Secondary Examinations have become the high order priority in the academic life of a student in the present scenario. The parents have more concern and they leave no stone unturned to get their wards a pass with comparatively higher score which enables them to join a course, particularly a professional course, in a reputed institution. Many students perform upto the expectations, though not all.

As an application of interval fuzzy number matrices the author of this thesis made an attempt to analyze the factors contributing high scores in the higher secondary examination and also to identify the prime factors by collecting the data from students, academicians and parents. The details are given in chapter X.

Regarding this application, the author of this thesis published an article entitled "Application of Interval Fuzzy Number Matrices and Interval Valued Fuzzy Soft Sets in the Analysis of the Factors Influencing High Scores in Higher Secondary Examinations" in International Journal of Mathematical Sciences and Applications [24].